Breit-wigner enhancement with kinetic decoupling

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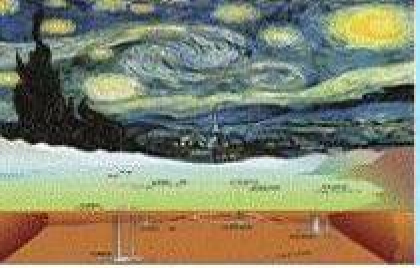
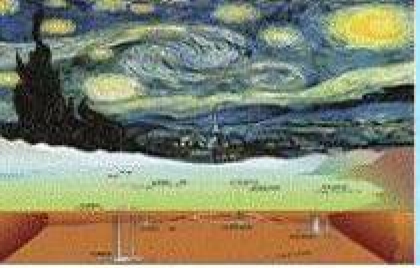


Outline

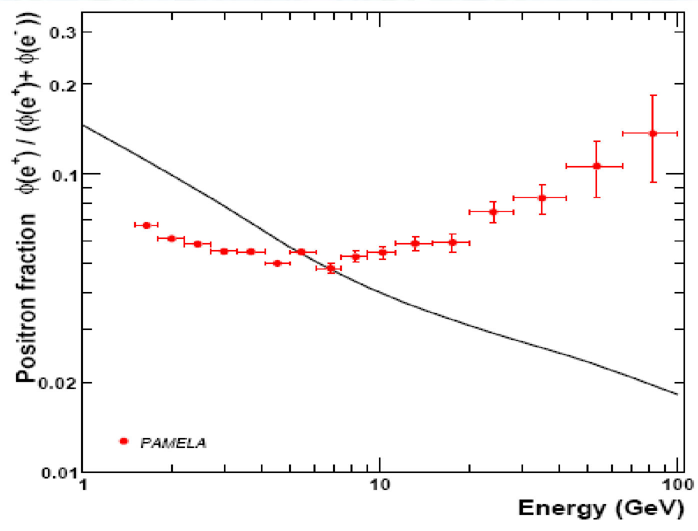
- Breit-Wigner enhancement

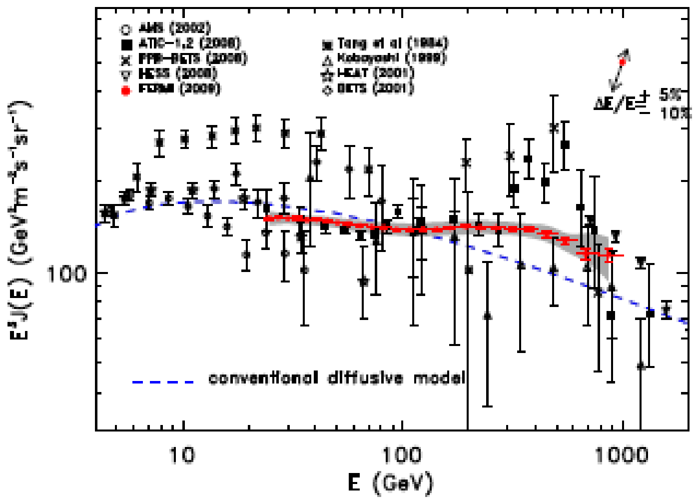
- Kinetic decoupling

- Breit-Wigner enhancement with Kinetic decoupling



I. Large Boost factor





Dark matter implication

- annihilating DM
- decaying DM

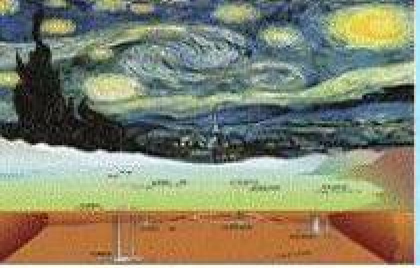
annihilating dark matter implication

- Large positron flux-> large $\langle\sigma v\rangle$ in the galaxy halo ~**10^{-23} cm^{3}s^{-1}**
- Thermal relic density-> $\langle\sigma v\rangle$ in the epoch of freeze-out ~**10^{-26} cm^{3}s^{-1}**

Boost factor

$$B \equiv \frac{\langle\sigma v\rangle|_{\rm halo}}{3\times10^{-26}cm^3s^{-1}}$$

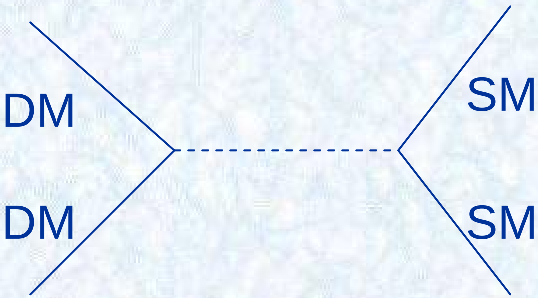
- Non-thermal production
- DM substructure
- Sommerfeld enhancement — **Feldman, Liu, Nath 2008**; **Ibe, Murayama, Yanagida 2008**
- Breit-Wigner enhancement — **Guo, Wu 2009**



Cross section via resonance

s-channel resonance

$$\sigma = \frac{16\pi}{s\bar{\beta}_i\beta_i}\frac{M^2\Gamma^2}{(s-M^2)^2+M^2\Gamma^2}B_iB_f$$



$$s=4\mathrm{m}^2+m^2v^2$$

$$M^2=4\mathrm{m}^2(1-\delta) \qquad \gamma=\Gamma/M$$

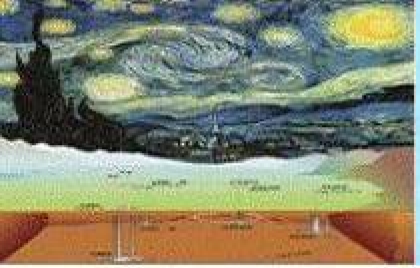
Resonance mass — Resonance decay width

Parameterize the cross section as

$$\sigma v = \sigma_0 \frac{\delta^2+\gamma^2}{(\delta+z)^2+\gamma^2}$$

$\sigma v \to \sigma_0$ **, if** $v \to 0$

$$s \equiv 4\mathrm{m}^2(1+z) \qquad \sigma_0 = \frac{32\pi B_iB_f}{M^2\bar{\beta}_i}\frac{\gamma^2}{\delta^2+\gamma^2}$$



Solving Boltzmann equation

Boltzmann equation

$$\frac{dY}{dx} = -\lambda' x^{-2} \langle \sigma v \rangle \, (Y^2 - Y_{eq}^2) \qquad x \equiv m/T \qquad Y \equiv n/s$$

$$s(x) = \frac{2\pi^2 g_{*s}}{45} \frac{m^3}{x^3} \qquad H(x) = \sqrt{\frac{4\pi^3 g_*}{45 m_{pl}^2}} \frac{m^2}{x^2} \qquad \lambda' \equiv s/H|_{x=1} \simeq 0.264 (g_{*s}/g_*^{1/2}) m_{pl} m_x$$

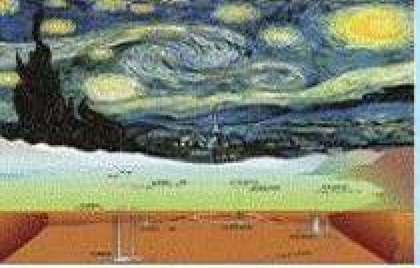
The DM relic density can be obtained

$$\Omega_X h^2 = 2.74 \times 10^8 \frac{m}{GeV} Y_\infty \sim 0.1$$

$\langle \sigma v \rangle$ **can be parameterized as** $\sigma_0 x^{-n}$ $(v \propto T_x^{1/2})$ **, associate with partial waves s,p, …with powers of n=0,1,…**

As long as $\Gamma \sim H$ **then** $Y \sim Y_{eq}$**. The particles will freeze out if the annihilation rate is smaller than universe expansion rate**

$$Y_\infty \simeq (n+1) x_f^{n+1} / \lambda' \sigma_0 \qquad x_f \sim \ln[c(c+2) a \lambda' \sigma_0]$$



Boost factor predicted by Breit-Wigner effect

Rewrite Boltzmann equation as

$$\frac{dY}{dx} = -\frac{\lambda}{x^2}\frac{\delta^2+\gamma^2}{(\delta+\xi x^{-1})^2+\gamma^2}(Y^2 - Y_{eq}^2) \qquad x = m/T \sim v^{-2}$$

The cross section of DM increase as temperature drops, DM continue to annihilate after x_f until the cross section reaches maximum value

$$x_b \sim \xi^{-1}\max[\delta,\gamma]^{-1}$$

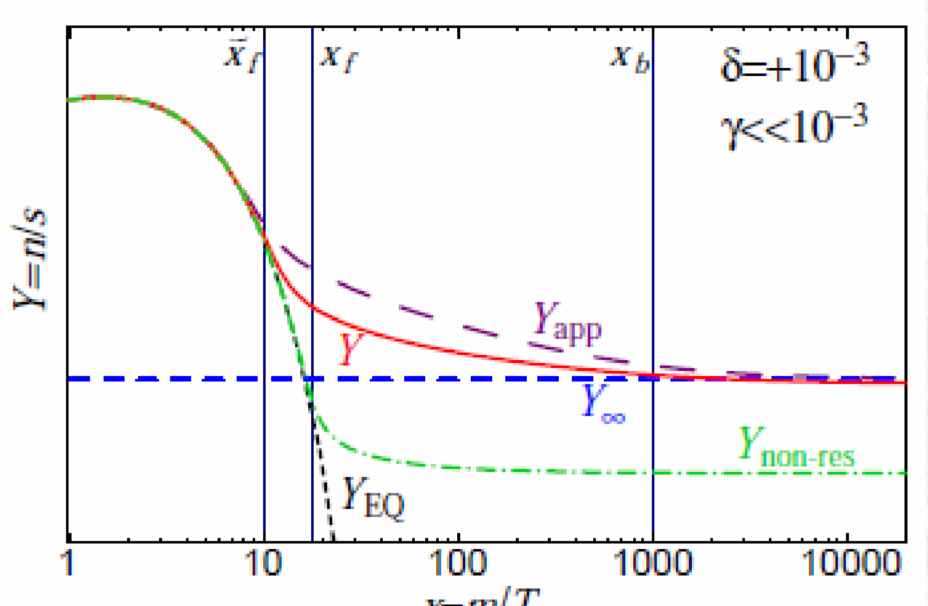
The solution of Boltzmann equation can be obtained as

$$\frac{dY}{dx} = -\frac{\lambda}{x^2}Y^2$$

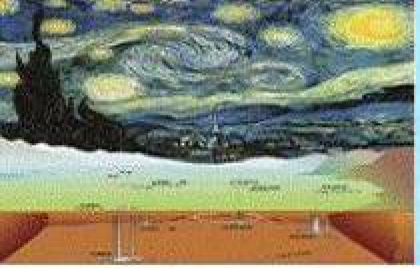
$$Y_\infty \sim \frac{1}{\lambda}x_b \sim \frac{1}{\lambda}\max[\delta,\gamma]^{-1} \qquad Y_\infty \sim \frac{1}{\lambda_f}x_f$$

Boost factor

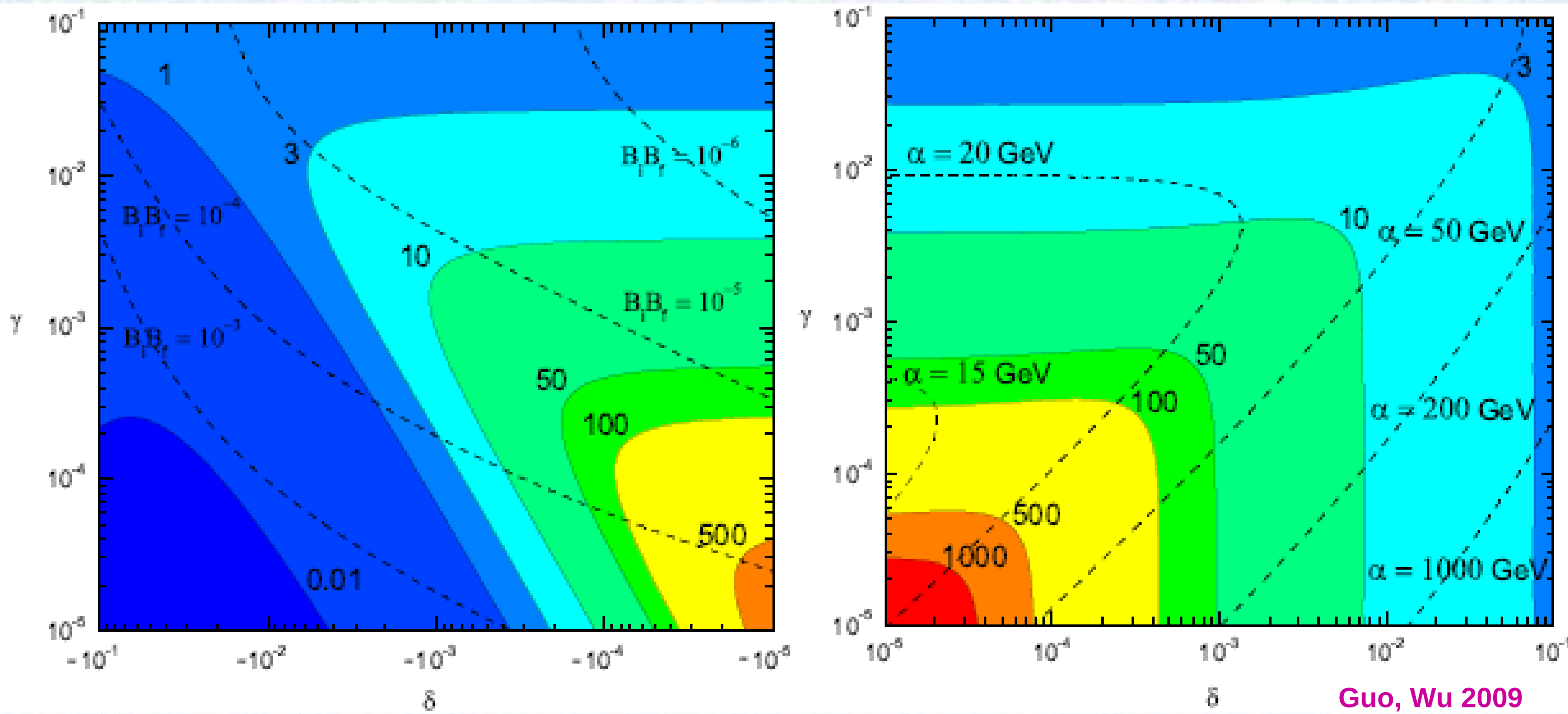
$$B \simeq \frac{\lambda}{\lambda_f} = \frac{x_b}{x_f} \sim \frac{\max[\delta,\gamma]^{-1}}{o(10)}$$



Ibe, Murayama, Yanagida 2008

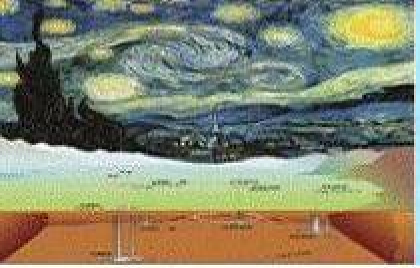


Numerical results



Guo, Wu 2009

- Large boost factor for very small parameters
- Large boost factor for both physical pole and unphysical pole
- Different behaviors due to different evolution of cross sections

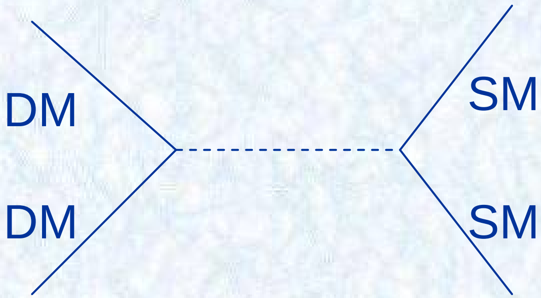


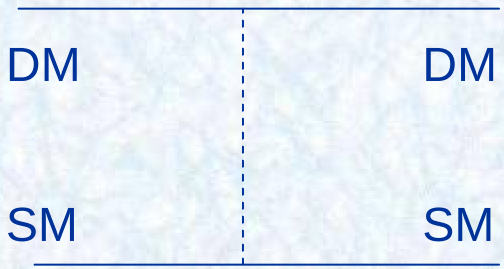
II. Kinetic decoupling

- **Chemical equilibrium: particle number reaction rate is fast** Gelmini 2010

  Chemical decoupling: particle number reaction is not efficient

  $$\Gamma(T_f) \sim H(T_f) \qquad \Gamma(T_f) = n\sigma v|_{T_f}$$





- **kinetic equilibrium: momentum exchange rate is fast**

  kinetic decoupling: the exchange of momentum with the radiation is not efficient

  $$\Gamma(T_{kd}) \sim H(T_{kd})$$

  It is important to study kinetic decoupling for structure formation.

  For example, what is the smallest mass scale of DM structures

DM temperature

- **Define temperature of DM**

$$T_x \equiv \frac{1}{3m_x n_x}\int \frac{d^3p}{(2\pi)^3} p^2 f(p) \sim \frac{v_0^2}{3} \qquad x_X \simeq \frac{m}{T_X} = \frac{3}{v_0^2}$$

- **If the DM particles keep in kinetic equilibrium with SM radiation**

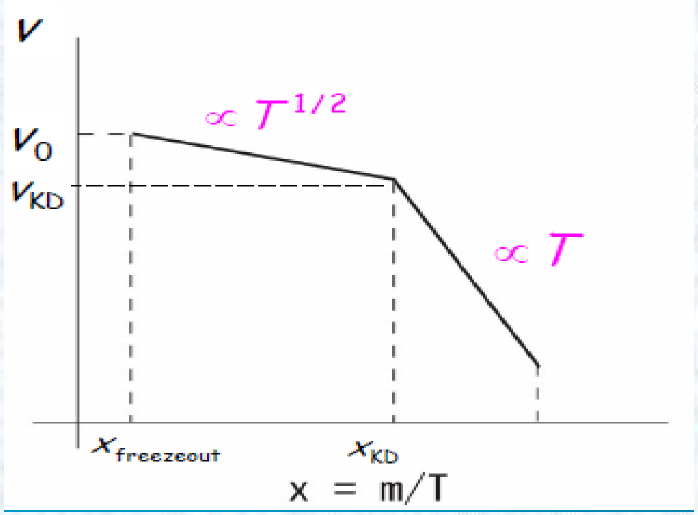
$$T_x = T \propto R^{-1} \qquad v \propto T^{-1/2}$$

- **After kinetic decoupling, DM temperature decrease as**

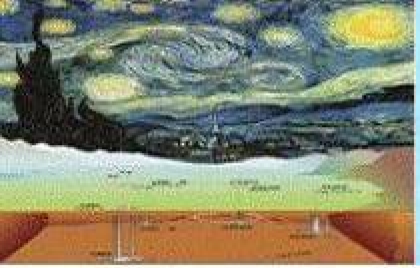
$$T_x \propto R^{-2} \qquad v \propto T^{-1}$$

- **DM temperature is determined by**

$$T_x = \begin{cases} T, T_x > T_{kd} \\ T^2/T_{kd}, T_x \le T_{kd} \end{cases}$$



Dent, Dutta, Scherrer 2009



More precise results

- **Solving equation**

$$\frac{dT_x}{dT} - [2 + aA(\frac{T}{m})^{n+2}]\frac{T_x}{T} = -a(\frac{T}{m})^{n+2}$$

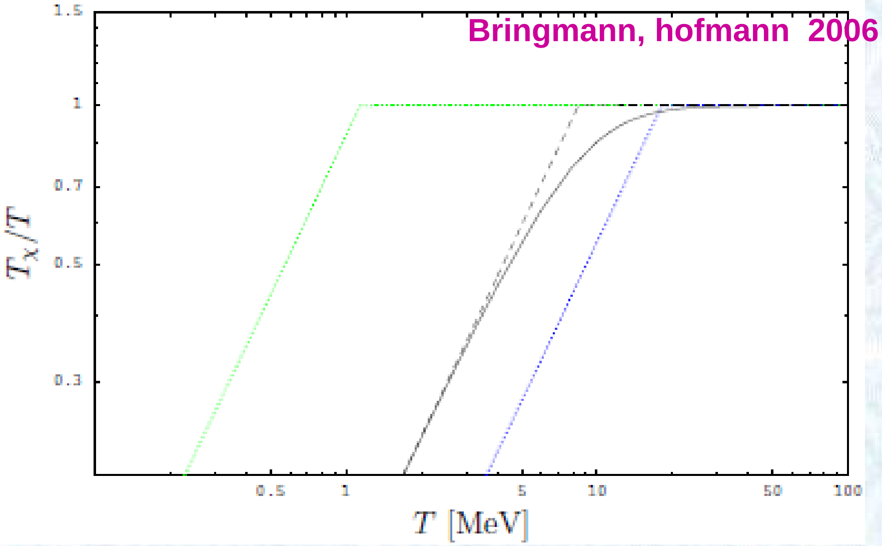
- **If A=1**

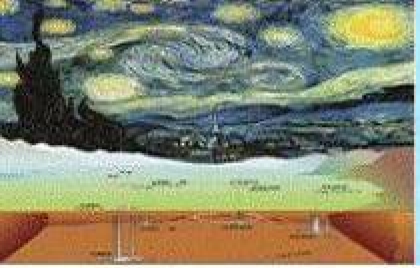
$$T_x \to T \qquad T \to \infty$$

$$T_x \to (\frac{a}{n+2})^{1/(n+2)}\Gamma[\frac{n+1}{n+2}]\frac{T^2}{m} \qquad T \to 0$$

- **Kinetic decoupling temperature**

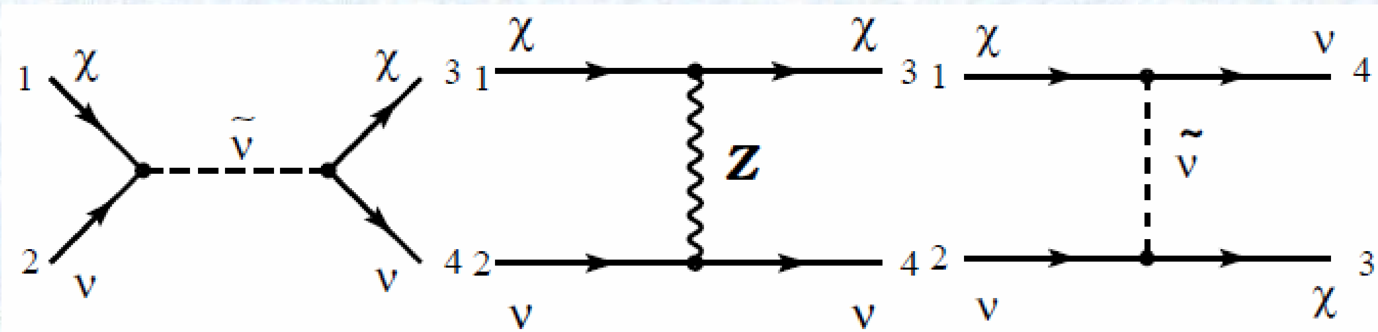
$$T_{kd} = m((\frac{a}{n+2})^{1/(n+2)}\Gamma[\frac{n+1}{n+2}])^{-1}$$





Kinetic decoupling of SUSY particles

- **Calculate elastic neutralino-SM fermion cross section**



- **Momentum transfer** $\frac{\Delta p_x}{p_x} \sim \frac{T}{m}$

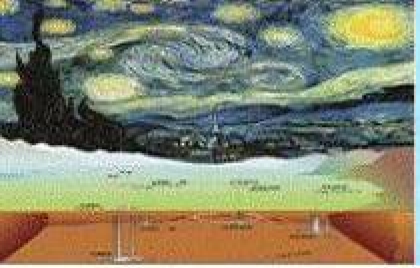
  momentum transfer rate $n \langle \sigma v_s \rangle \frac{T}{m}$

- **Kinetic decoupling temperature** $T_{kd} \simeq [0.0012 \frac{m_{pl}}{m_x (m_{\tilde{l}}^2 - m_x^2)}]^{-\frac{1}{4}}$

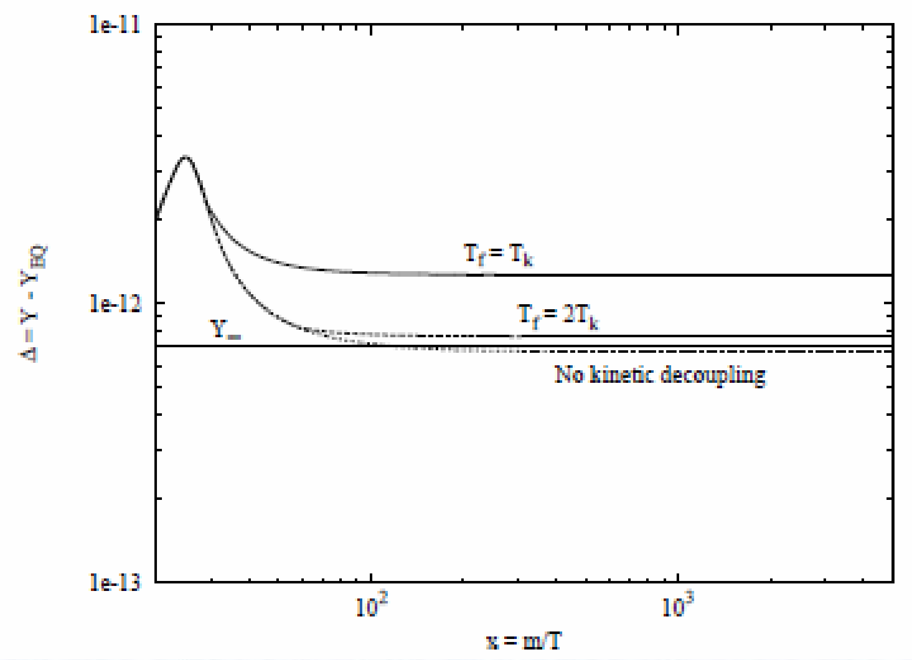
  typical value O(10)~O(10^2) Mev

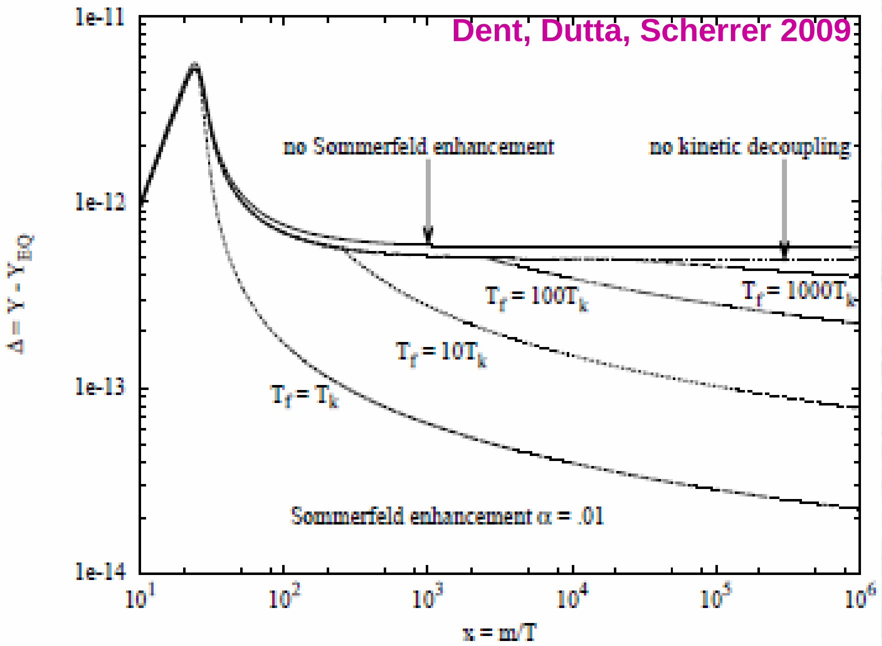
Chen, Kamionkowski, Zhang 2001

Hofmann, Schwarz, Stocker 2001

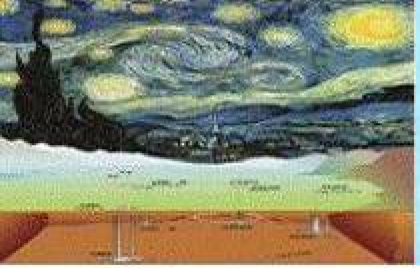


Kinetic decoupling effect for velocity-dependent interaction





- **P wave scattering and Sommerfeld effect**



III Resonance cross section with kinetic decoupling

- **DM temperature**

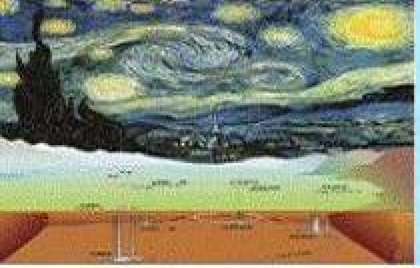
$$T_x = \begin{cases} T, T_x > T_{kd} \\ T^2 / T_{kd}, T_x \le T_{kd} \end{cases}$$

- **Thermal averaged cross section**

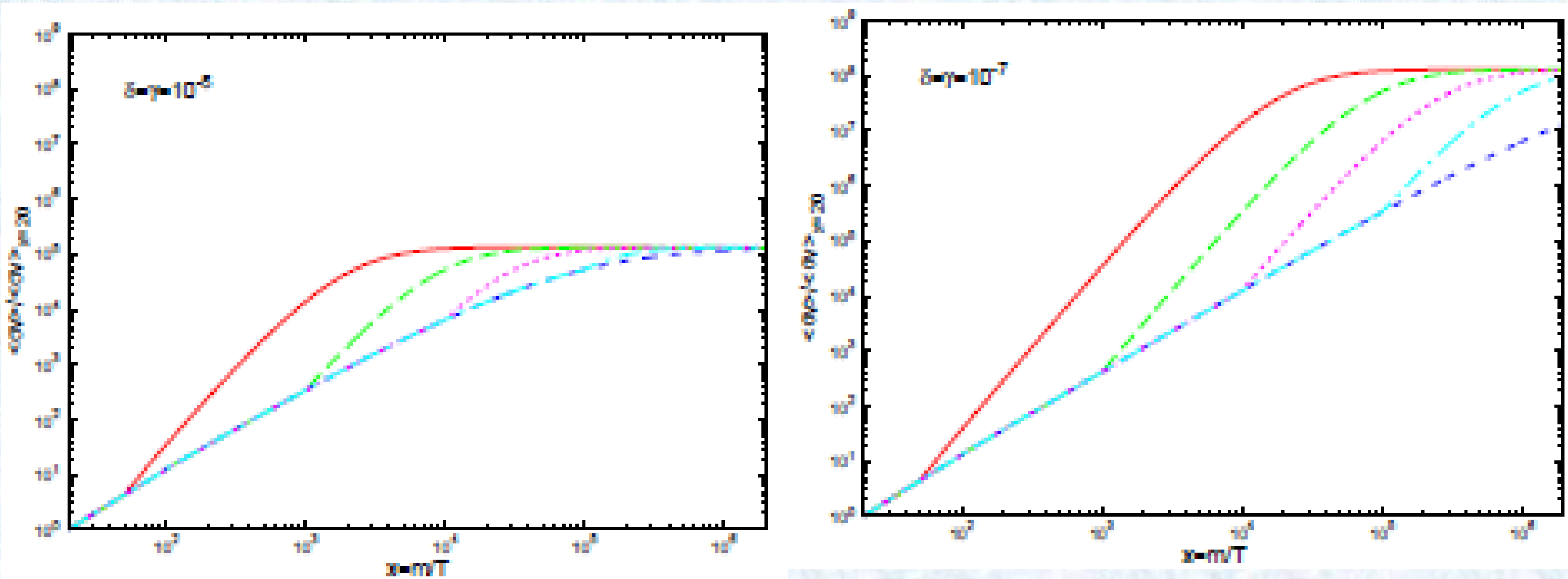
$$\langle \sigma v \rangle = \frac{1}{n_{eq}^2} \frac{m g_i^2}{32\pi^4 x_x} \int_{4m^2}^{\infty} p_{eff} w_{eff} K_1\left(\frac{x_x \sqrt{s}}{m}\right) ds$$

$$n_{eq} = \frac{g_i}{2\pi^2} \frac{m^3}{x_x} K_2(x_x)$$

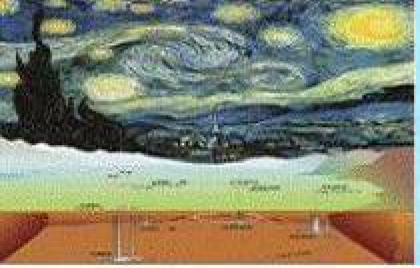
$$w_{eff} = 4E_1 E_2 \sigma v \qquad p_{eff} = \frac{1}{2}\sqrt{s - 4m^2}$$



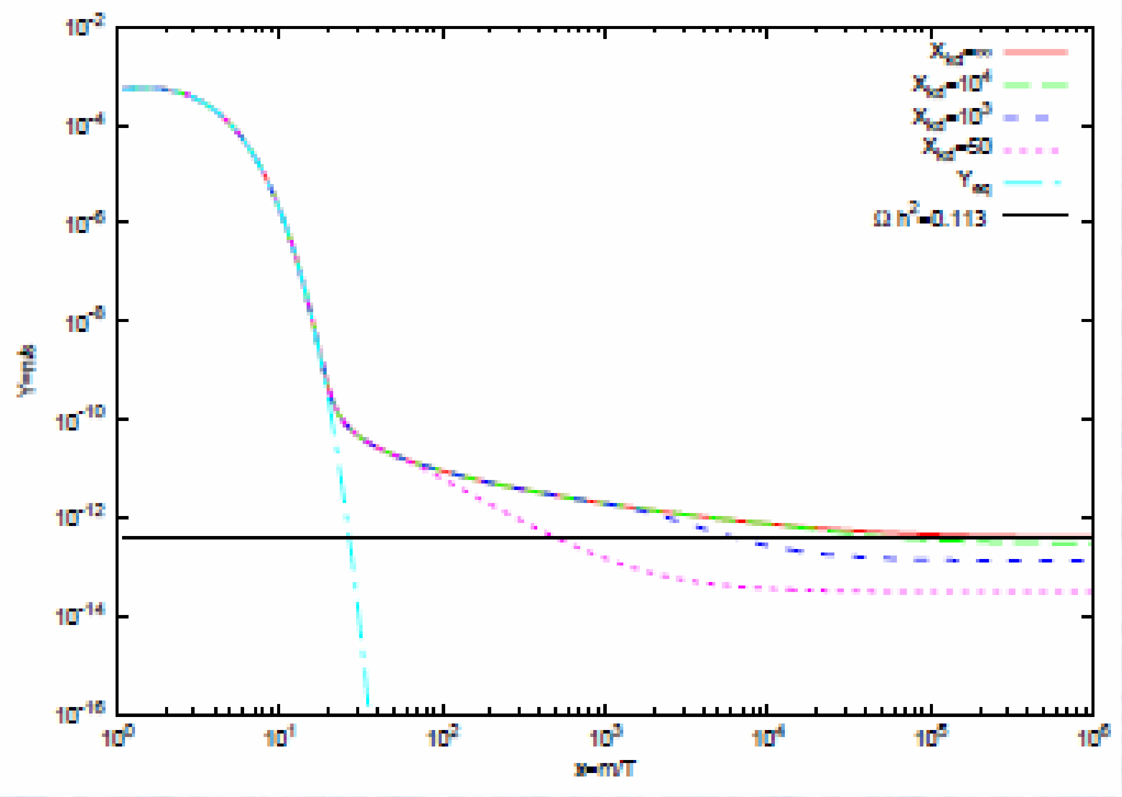
Modified cross section



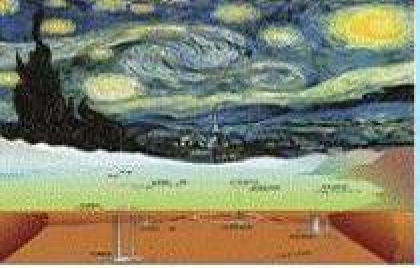
- After kinetic decoupling, DM temperature decreases rapidly and the cross section increases significantly
- The cross section for Small x_{kd} increases more quickly
- The maximum value of cross section increases as δ, γ decrease



Relic density



- **Small x_{kd} will decrease DM relic density significantly, and only allow small boost factor**



DM elastic cross section

- We need the Xf->Xf elastic scattering cross section
- Assume effective interaction Lagrangian

$$\frac{d\sigma}{d\Omega}=\frac{1}{64\pi^2 s}\sqrt{\frac{s}{s-4m^2}}\,g_A^2 g_B^2\,|R_{0a}|^2|M_a'|^2$$

for annihilation

$$\sigma_0=\frac{1}{32\pi m^2}\frac{g_A^2 g_B^2}{M^4(\delta^2+\gamma^2)}\int\frac{|M_a'|^2}{4\pi}d\Omega$$

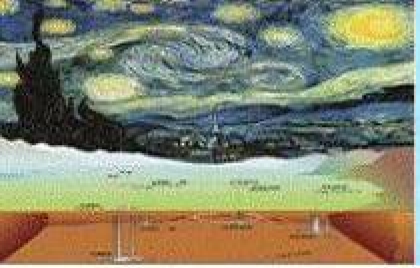
for elastic scattering

$$\sigma v_s=\frac{1}{16\pi m^2}\frac{g_A^2 g_B^2}{M^4}\int\frac{|M_s'|^2}{4\pi}d\Omega$$

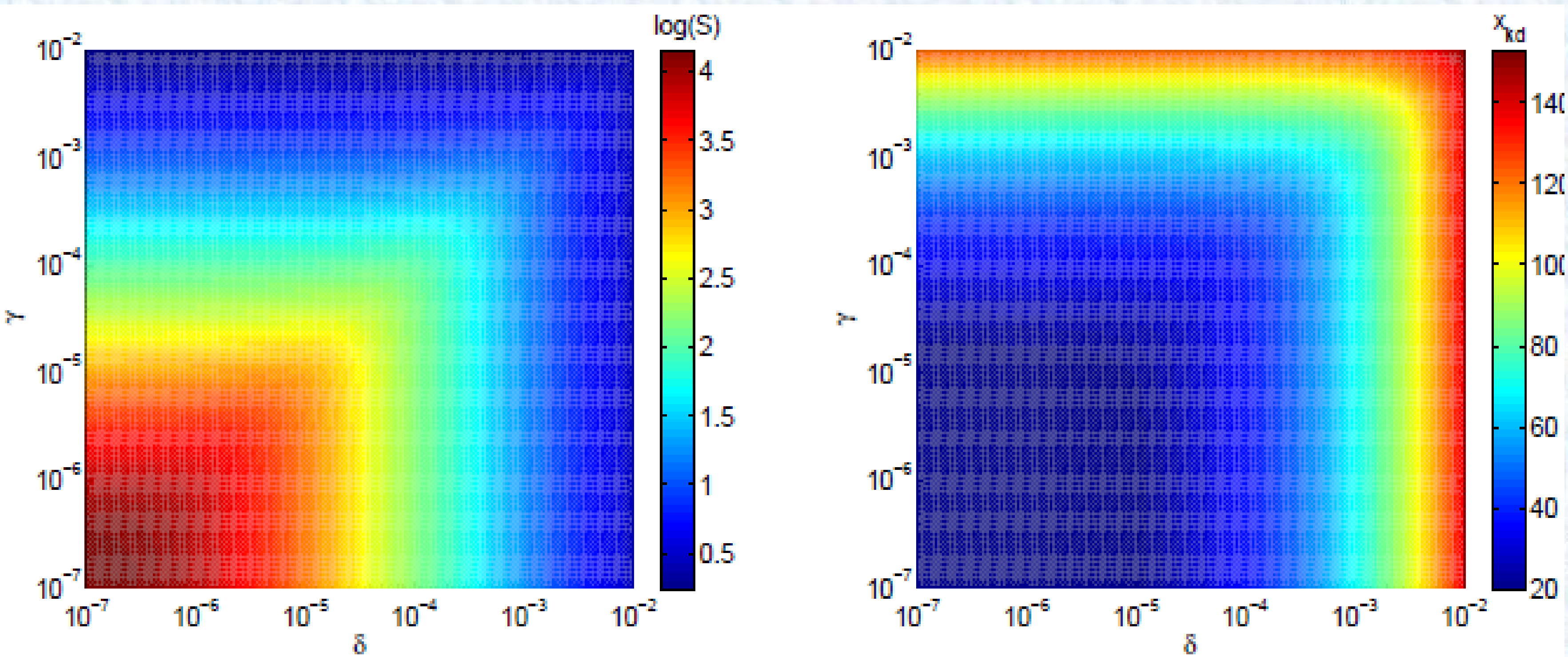
we can express σv_s by σ_0 . Finally we achieve

$$\sigma v_s=2\sigma_0(\delta^2+\gamma^2)\frac{\int|M_s'|^2\,d\Omega}{\int|M_a'|^2\,d\Omega}\sim a\sigma_0(\delta^2+\gamma^2)\frac{T^2}{m^2}$$

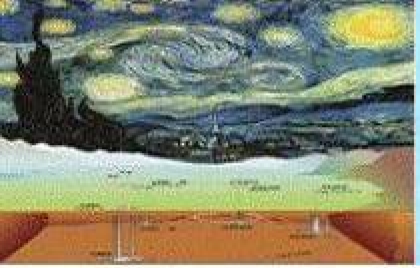
For example, for Z' model $|M_a'|^2=32m^4$, $|M_s'|^2=8m^2E_l^2\sim 8m^2T^2$



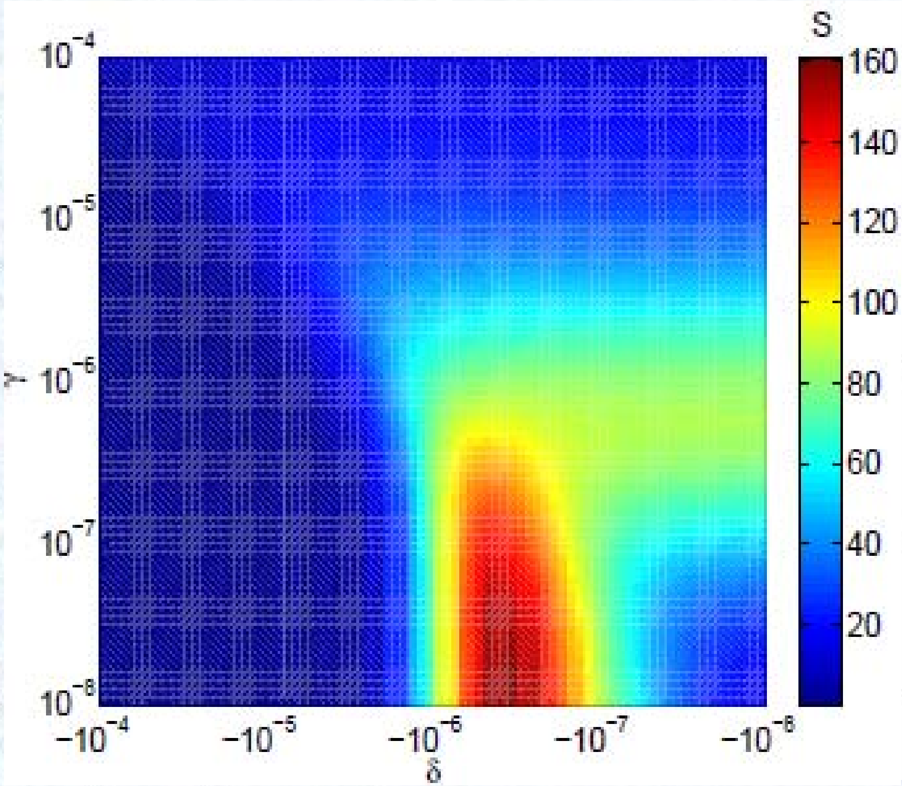
Kinetic decoupling temperature

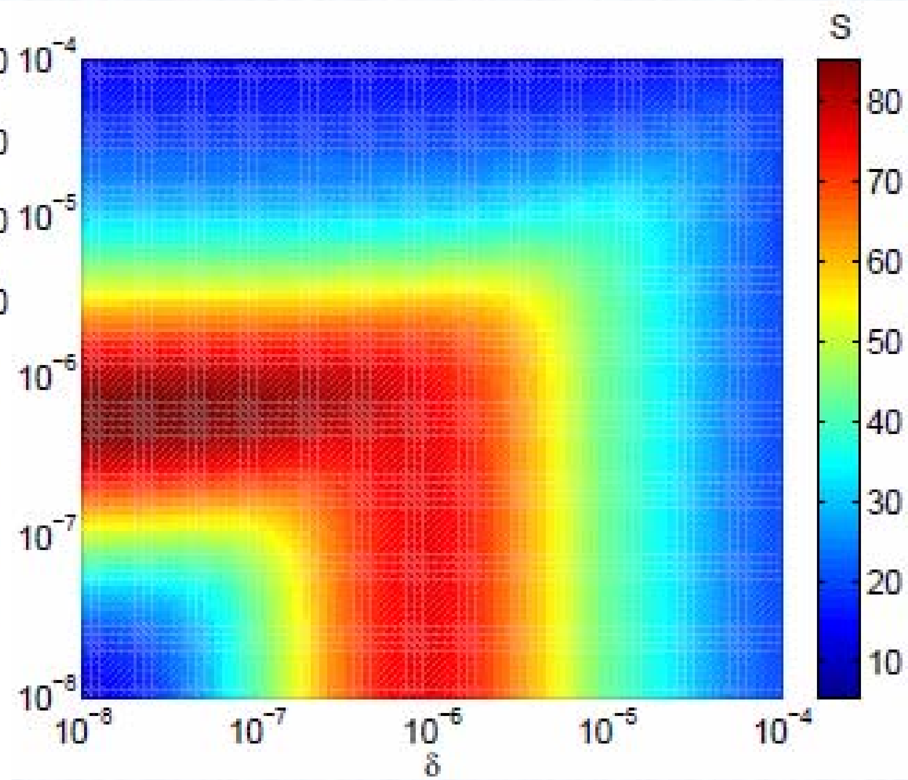


$$T_{kd} \sim \left[\frac{9.088 \cdot g_*^{1/2} m^3}{a_k \sigma_0 (\delta^2 + \gamma^2) m_{pl}}\right]^{\frac{1}{4}} \sim 110.5 \left[\frac{0.5}{a_k}\right]^{\frac{1}{4}} \left[\frac{10^{-6} GeV^{-2}}{\sigma_0}\right]^{\frac{1}{4}} \left[\frac{10^{-10}}{\delta^2 + \gamma^2}\right]^{\frac{1}{4}} \left[\frac{g_*}{100}\right]^{\frac{1}{8}} \left[\frac{m}{1TeV}\right]^{\frac{3}{4}}$$



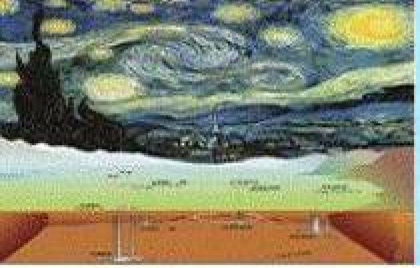
Numerical results of boost factor





Boost factor

$$B \equiv \frac{\langle \sigma v \rangle|_{\text{halo}}}{3\times10^{-26}\,cm^3 s^{-1}} = \frac{\langle \sigma v \rangle|_{\text{v}=10^{-3}c}}{3\times10^{-26}\,cm^3 s^{-1}} \neq \frac{\langle \sigma v \rangle|_{\text{v}=0}}{3\times10^{-26}\,cm^3 s^{-1}}$$



discussion

After kinetic decoupling, rewrite Boltzmann equation as

$$\frac{dY}{dx} = -\frac{\lambda}{x^2} \frac{\delta^2 + \gamma^2}{(\delta + \xi x_{kd} x^{-2})^2 + \gamma^2} (Y^2 - Y_{eq}^2)$$

$$x_b \sim \max[\delta, \gamma]^{-1/2} \sqrt{x_{kd}}$$

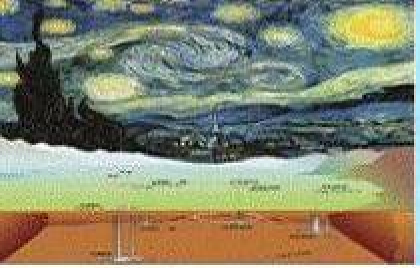
$$B \sim x_b / x_f \sim \max[\delta, \gamma]^{-1/2} \sqrt{x_{kd}} / x_f$$

It seems we could still achieve a arbitrary large boost factor, but it is not the case

for $\delta \simeq \gamma \le O(10^{-6})$ $\quad \sigma v_{halo} \sim \sigma_0 (\max[\delta, \gamma] / v_{halo}^2)^2 < \sigma_0$

To understand maximum boost factor, we set $z \sim v_{halo}^2 \sim 10^{-6}$

$$\sigma v = \sigma_0 \frac{\delta^2 + \gamma^2}{(\delta + z)^2 + \gamma^2} \qquad B \sim \frac{\max[\delta, \gamma]^{\frac{3}{2}}}{\max[\delta + v_{halo}^2, \gamma]^2} \frac{\sqrt{x_{kd}}}{x_{f0}}$$



Summary

- If the kinetic decoupling occur at nearly the same epoch as chemical decoupling, the DM annihilation becomes more important and reduces the DM relic density
- It is difficult to achieve large boost factor to explain PAMELA/ATIC in the simplest model
- The damping mass of subhalo maybe smaller than usual WIMP model due to high kinetic decoupling temperature. Need more detailed studies

$$M_d \sim 10^{-6} M_\odot (m/100GeV)^{-3/2}(T_{kd}/30MeV)^{-3/2}$$

- Some complicated models could still work. For example, there exist a s-channel resonance to enhance Xf->xf scattering, the kinetic decoupling temperature may be as low as no effect on reducing DM relic density

Thanks