Theoretical Implications of the OPERA Experiment

Tianjun Li
Institute of Theoretical Physis, Chinese Academy of Sciences

I. Introduction

II. Generic Discussions on BDLV

III. Type IIB String Theory

IV. Deformed Lorentz Invariance

V Conclusion

arXiv:1101.0451; arXiv:1101.3451; arXiv:1111.4994; arXiv:1112.0264

ITP, CAS, December 20, 2011

I. Introduction

Lorentz Symmetry:

- Space-Time Symmetry: $SO(3,1)$
- Special Relativity: the speed of light in vacuum is invariant under all the observers.
- General Relativity/Riemannian Geometry : local Lorentz covariance in an infinitesimal region of spacetime at every point.
- Quantum Field Theory: Lorentz Invariance.

Quantum Gravity:

- Gravity is not renormalizable
- Quantum Gravity: String Theory???
- Lorentz Symmetry Violation: Non-Commutative Geometry; D-Brane; ...
- How to Probe the Quantum Gravity? Cosmology and Astrophysics.

OPERA Experiment [a]:

- Muon neutrinos with mean energy 17 GeV from CERN to CNGS (730 km)

$$\delta t = \left(57.8 \pm 7.8\,(\text{stat.})^{+8.3}_{-5.9}(\text{sys.})\right)\,\text{ns}\,,$$
$$\delta v_\nu \equiv \frac{v_\nu - c}{c} = \left(2.37 \pm 0.32\,(\text{stat.})^{+0.34}_{-0.24}\,(\text{sys.})\right) \times 10^{-5}\,.$$

- No significant energy dependence for δt:

$$\delta t_1 = \left(54.7 \pm 18.4\,(\text{stat.})^{+7.3}_{-6.9}\,(\text{sys.})\right)\ \text{ns}\,,$$
$$\delta t_2 = \left(68.1 \pm 19.1\,(\text{stat.})^{+7.3}_{-6.9}\,(\text{sys.})\right)\ \text{ns}\,.$$

[a] T. Adam *et al.* [OPERA Collaboration], arXiv:1109.4897.

- MINOS experiment: muon neutrino with a spectrum peaking at about 3 GeV, and a tail extending above 100 GeV [a]

$$\delta v_\nu = (5.1 \pm 2.9) \times 10^{-5} \qquad \text{(MINOS)} .$$

- The earlier short-baseline experiments have set the upper bounds on $|\delta v_\nu|$ around 4×10^{-5} in the energy range from 30 GeV to 200 GeV [b].

δv_ν **may be up to around** 10^{-5}**.**

[a] P. Adamson *et al.* [MINOS Collaboration], Phys. Rev. D 76 (2007) 072005 [arXiv:0706.0437].
[b] G. R. Kalbfleisch, N. Baggett, E. C. Fowler and J. Alspector, Phys. Rev. Lett. 43 (1979) 1361.

OPERA Experiment:

- The technical issues such as pulse modelling, timing and distance measurement deserve further experimental scrutiny.
- Other experiments like MINOS and T2K are also needed to to do independent measurements.

Assuming that the OPERA experiment is correct, can we explain both the OPERA experiment and the other phenomenological constrants on LV simultaneous?

The theoretical challenges of the OPERA Experiment

- Bremsstrahlung effects [a]. The superluminal muon neutrinos with δv_ν would lose energy rapidly via Cherenkov-like processes on their ways from CERN to LNGS, and the most important process is $\nu_\mu \to \nu_\mu + e^+ + e^-$. Thus, the OPERA experiment can not observe the muon neutrinos with energy in excess of 12.5 GeV.

- Pion decays [b]. The superluminal muon neutrinos with δv_ν can not have energy larger than about 10 GeV from pion decays, for example, $\pi^+ \to \mu^+ \nu_\mu$ and $\mu \to \nu_\mu + e + \bar{\nu}_e$.

---

[a] A. G. Cohen, S. L. Glashow, arXiv:1109.6562 [hep-ph].

[b] L. Gonzalez-Mestres, arXiv:1109.6630 [physics.gen-ph]; X. -J. Bi, P. -F. Yin, Z. -H. Yu, Q. Yuan, arXiv:1109.6667 [hep-ph]; R. Cowsik, S. Nussinov, U. Sarkar, arXiv:1110.0241 [hep-ph].

One possible solution [a]:

These anomalous processes are forbidden if the Lorentz symmetry is deformed, preserving the relativity of inertial frames. These deformations add non-linear terms to the energy-momentum relations, conservation laws, and Lorentz transformations in a way which is consistent with the relativity of inertial observers.

---

[a]G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman, L. Smolin, arXiv:1110.0521 [hep-ph].

The SN1987a observations:

- The Irvine-Michigan-Brookhaven (IMB) [a], Baksan [b], and Kamiokande II [c] experiments collected $8+5+11$ neutrino events (presumably mainly $\bar{\nu}_e$) with energies between $7.55\ \mathrm{MeV}$ and $395\ \mathrm{MeV}$ within 12.4 seconds. The neutrinos arrived on the Earth about 4 hours before the corresponding light. Thus, the upper limit on δv_ν is

$$|\delta v_\nu(15\ \mathrm{MeV})| \leq 2 \times 10^{-9}\ .$$

- We can employ the time coincidence of these events to constrain the velocity differences for neutrinos with various energies

$$|\delta v_\nu(30\ \mathrm{MeV}) - \delta v_\nu(10\ \mathrm{MeV})| < 5 \times 10^{-13} \text{ at 95\% CL}\ .$$

[a] R. M. Bionta *et al.* [IMB Collaboration], Phys. Rev. Lett. 58 (1987) 1494.
[b] E. N. Alekseev, L. N. Alekseeva, I. V. Krivosheina and V. I. Volchenko, Phys. Lett. B 205 (1988) 209.
[c] K. Hirata *et al.* [KAMIOKANDE-II Collaboration], Phys. Rev. Lett. 58 (1987) 1490.

Time Delay of High Energy Photons due to QG

Refractive Index

$$n = 1 + \frac{E_\gamma^n}{M_{QGn}^n} \,.$$

MAGIC Experiment: [a]

- Gamma Rays from AGN Mkn 501.
- A time lag of about four minutes was found for the maximum of the time profile envelope for photons in the 1.2-10 TeV energy band relative to those in the range 0.25-0.6 TeV. The difference between the mean eneries in these two bands is about 2 TeV.

$$
\begin{aligned}
M_{\rm QG1} &> 0.21 \times 10^{18}\ \text{GeV} \ \text{ at } 95\%\ \text{C.L.},\\
M_{\rm QG2} &> 0.26 \times 10^{11}\ \text{GeV} \ \text{ at } 95\%\ \text{C.L.}
\end{aligned}
$$

[a] J. Albert *et al.* [MAGIC Collaboration and Other Contributors Collaboration], Phys. Lett. B **668**, 253 (2008) [arXiv:0708.2889 [astro-ph]].

HESS Experiment: [a]

- Flare from the Active Galaxy PKS 2155-304.
- No time lag found.
- The best fit of the MAGIC and HESS data gives the Quantum Gravity scale

$$M_{QG1} = (0.98^{+0.77}_{-0.30}) \times 10^{18} \ .$$

[a] F. Aharonian *et al.*, Phys. Rev. Lett. **101**, 170402 (2008) [arXiv:0810.3475 [astro-ph]].

FERMI (I): [a]

- Gamma rays from the GRB 080916C.
- There is a 4.5-second time-lag between the onsets of high- (> 100 MeV) and low-energy (< 100 KeV) emissions. Moreover, the highest-energy photon with an energy $E = 13.2^{+0.70}_{-1.54}$ GeV was detected $\Delta t = 16.5$ s after the start of the burst.
- Time delay for the highest-energy photon can also be explained for $M_{QG1} > 1.50 \pm 0.20 \times 10^{18}$ GeV. The quadrative refractive index does not work.
- The 4.5 second time delay can not be explained and might be due to source mechanism.

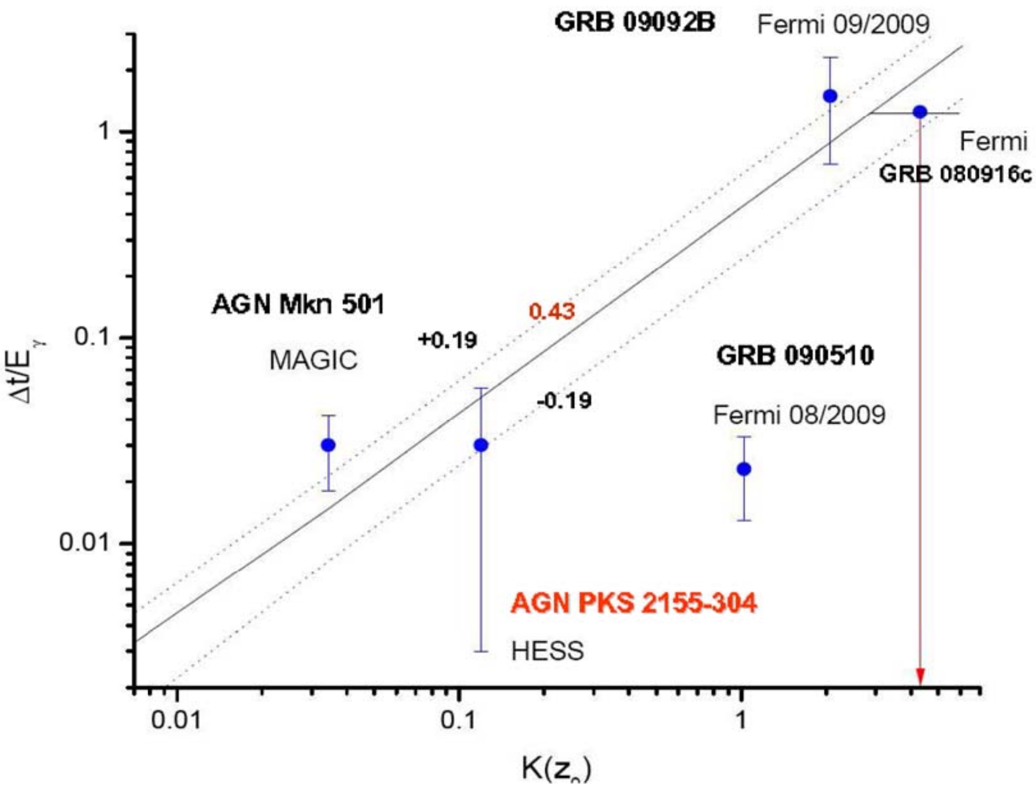
---

[a] A. A. Abdo *et al.* [Fermi LAT and Fermi GBM Collaborations], Science **323**, 1688 (2009).

FERMI (II): [a]

- Gamma rays from the GRB 090510.
- The observed short delays of this burst can be explained on the basis of linearly modified dispersion relations only if the quantum gravity scale is *larger* than $M_{\rm QG\,1} \simeq 1.2 M_P$ where $M_P = 1.22 \times 10^{19}$ GeV is the Planck mass.
- Not Consistent with the above three cases.

[a] A. A. Abdo *et al.* [Fermi LAT and Fermi GBM Collaborations], Nature 462, 331-334.



The fit for the data from two AGNs, Mkn 501 and PKS 2155-304, and three GRBs observed by the Fermi satellite, 080916C, 090510, 09092B [a].

[a] A. A. Abdo *et al.* [The Fermi/GBM collaboration and The Fermi/LAT Collaborations and The S Collaboration], Astrophys. J. **706**, L138 (2009) [arXiv:0909.2470 [astro-ph.HE]].

Time Delay of High Energy Photons due to QG:

Refractive Index:

$$n = 1 + \frac{E_\gamma}{M_{QG}} .$$

The delays observed by the MAGIC, HESS and FERMI (I) Collaborations are compatible with each other for the following $M_{\rm QG}$ [a]

$$M_{\rm QG} \simeq 0.98 \times 10^{18}\ {\rm GeV} .$$

All the MAGIC, HESS, FERMI (I), and FERMI (II) results may be compatible with each other by considering a redshift dependent D-particle density [b].

How to explain the linear refractive index?

[a] J. Ellis, N. E. Mavromatos, D. V. Nanopoulos, Phys. Lett. **B674**, 83-86 (2009).
[b] J. Ellis, N. E. Mavromatos, D. V. Nanopoulos, Int. J. Mod. Phys. **A26**, 2243-2262 (2011).

δv_π **for pions :**

- The superluminal π^+ will lose energy quickly via the radiative emission process $\pi^+ \rightarrow \pi^+\gamma$.
- The IceCube experiment has measured the atmospheric neutrino spectrum up to ~ 400 TeV, which agrees pretty well with the model calculations [a].
- The high energy charged pions do not lose much energy before they decay to neutrinos, which gives a strong constraint on the process $\pi^+ \rightarrow \pi^+\gamma$.
- The threshold of this process is $E_\pi > m_\pi/\sqrt{v_\pi^2 - c^2} \simeq m_\pi/\sqrt{2\delta v_\pi}$, and the maximal pion energy is about 2 PeV, we obtain an upper bound on $\delta v_\pi < 2 \times 10^{-14}$.

[a] R. Abbasi *et al.* [IceCube Collaboration], Phys. Rev. **D83**, 012001 (2011) [arXiv:1010.3980 [astro-ph.HE]].

Astrophysical constraints on Lorentz violation for charged leptons:

- The constraint from the Crab Nebula synchrotron radiation observations on the electron dispersion relation [a]. For δv_e linearly dependent on its energy, we obtain the electron dispersion relation

$$E^2 = p^2 + m_e^2 - \frac{p^3}{M'_{\rm QG}} , \quad M'_{\rm QG} \geq 1 \times 10^{24}\,{\rm GeV} .$$

- The electron vacuum Cherenkov radiation via the decay process $e \to e\gamma$ becomes kinematically allowed for $E_e > m_e/\sqrt{\delta v_e}$. Note that the cosmic ray electrons have been detected up to 2 TeV, we have $\delta v_e < 10^{-13}$ from cosmic ray experiments [b].

---

[a] L. Maccione, S. Liberati, A. Celotti and J. G. Kirk, JCAP **0710**, 013 (2007).

[b] C. D. Carone, M. Sher and M. Vanderhaeghen, Phys. Rev. D 74 (2006) 077901 [arXiv:hep-ph/0609150].

- The high-energy photons are absorbed by CMB photons and annihilate into the electron-positron pairs. The process $\gamma\gamma \to e^+e^-$ becomes kinematically possible for $E_{\rm CMB} > m_e^2/E_\gamma + \delta v_e E_\gamma/2$. Because it has been observed to occur for photons with energy about $E_\gamma = 20$ TeV, we have $\delta v_e < 2m_e^2/E_\gamma^2 \sim 10^{-15}$ from cosmic ray experiments [a].

- The process, where a photon decays into e^+e^-, becomes kinematically allowed at energies $E_\gamma > m_e\sqrt{-2\delta v_e}$. As the photons have been observed up to 50 TeV, we have $-\delta v_e < 2 \times 10^{-16}$ from cosmic ray experiments [b]. The analogous bound for the muon is $-\delta v_\mu < 10^{-11}$ [c].

---

[a] F. W. Stecker and S. L. Glashow, Astropart. Phys. 16 (2001) 97 [arXiv:astro-ph/0102226].
[b] C. D. Carone, M. Sher and M. Vanderhaeghen, Phys. Rev. D 74 (2006) 077901 [arXiv:hep-ph/0609150].
[c] B. Altschul, Astropart. Phys. 28 (2007) 380 [arXiv:hep-ph/0610324].

The relevant constraints on Lorentz violation from the experiments done on the Earth:

- The agreement between the observation and the theoretical expectation of electron synchrotron radiation as measured at LEP gives the stringent bound on isotropic Lorentz violation $|\delta v_e| < 5 \times 10^{-15}$ [a]. Here, we emphasize that the electrons and positrons propagated in the vacuum tunnel at the LEP experiment.

- For the electron neutrinos at the KamLAND experiment, the non-trivial energy dependence of the neutrino survival probability implies that the Lorentz violating off-diagonal elements of the δv_ν matrix in the flavor space are smaller than about 10^{-20} [b]. Thus, if the Lorentz violation can not realize the flavour independent couplings naturally, we do need to fine-tune the relevant couplings.

---

[a] B. Altschul, Phys. Rev. D 80 (2009) 091901 [arXiv:0905.4346].

[b] S. Abe *et al.* [KamLAND Collaboration], Phys. Rev. Lett. **100**, 221803 (2008); A. Strumia and F. Vissani, arXiv:hep-ph/0606054.

II. Generic Discussions on BDLV

Assumptions [a]**:**

- The Lorentz violation for all the SM particles is not constant in the space time in principle.
- The Lorentz violation for all the SM particles on the Earth is much larger than those on the interstellar scale or in the vacuum.

[a] G. Dvali, A. Vikman, arXiv:1109.5685 [hep-ph]; J. Alexandre, J. Ellis, N. E. Mavromatos, arXiv:1109.6296 [hep-ph]; E. Ciuffoli, J. Evslin, J. Liu, X. Zhang, arXiv:1109.6641 [hep-ph]; TL and Nanopoulos, arXiv:1101.0451; arXiv:1101.3451.

Generic Study:

- Considering the effective field theory or string theory, we can parametrize the generic δv for a particle ϕ

$$\delta v_\phi \;=\; -\frac{m_\phi^2}{2p^2} + \sum_{n\geq 0} a_n^\phi \frac{p^n}{M_*^n} \,.$$

- In the effective theory, we can realize all the terms.
- In the string theory, we can obtain the a_1^ϕ term naturally.

OPERA and SN1987a:

- We denote the couplings a_n^ϕ on the Earth as $[a_n^\phi]^{\rm E}$, and the couplings a_n^ϕ on the interstellar scale as $[a_n^\phi]^{\rm IS}$.
- Assuming that only one $[a_n^\nu]^{\rm E}$ term generates OPERA δv_ν and satisfies the SN1987a constraints, we obtain

$$\frac{[a_0^\nu]^{\rm IS}}{[a_0^\nu]^{\rm E}} \leq 8.1 \times 10^{-5} \,, \quad \frac{[a_1^\nu]^{\rm IS}}{[a_1^\nu]^{\rm E}} \leq 2.3 \times 10^{-5} \,, \quad \frac{[a_2^\nu]^{\rm IS}}{[a_2^\nu]^{\rm E}} \leq 8.6 \times 10^{-3} \,.$$

- Considering the uncertainties in the SN1987a observations, we find that the ratio $[a_0^\nu]^{\rm IS}/[a_0^\nu]^{\rm E}$ is similar to the ratio $[a_1^\nu]^{\rm IS}/[a_1^\nu]^{\rm E}$.

- Because both the $[a_0^\nu]^{\rm E}$ term and the $[a_1^\nu]^{\rm E}$ term can be consistent with the OPERA results on weak energy dependence, the OPERA and SN1987a results may be generated by both terms.

Eddington's dictum: "Never believe an experiment until it has been confirmed by theory".

Effective Field Theory Approach:

- Type II seesaw mechanism: a triplet Higgs field Φ whose quantum number under $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetries is $(\mathbf{1}, \mathbf{3}, \mathbf{1})$.

- Lorentz violation: a unit vector $u^\mu = (1,\ 0,\ 0,\ 0)$.

- Effective operators:

$$\mathcal{L} \;=\; \frac{1}{1+\delta_{ij}} \left(i y_{jk} \frac{\Phi}{M_*} u^\mu L_j \partial_\mu L_k + y'_{jk} \frac{\Phi}{M_*^2} (u^\mu \partial_\mu L_j)(u^\nu \partial_\nu L_k) \right) \;,$$

- Superluminal neutrinos:

$$\delta v_\nu \;=\; \frac{y_{ii}^2 V_\Phi^2}{2M_*^2} + \frac{2 y_{ii} y'_{ii} V_\Phi^2 E}{M_*^3} + \frac{3 y'^2_{ii} V_\Phi^2 E^2}{2M_*^4} \;.$$

III. Type IIB String Theory with D3/D7 Branes

Set-Up:

- D3-branes are inside the D7-branes. The D3-branes wrap a three-cycle, and the D7-branes wrap a four-cycle.
- The D3-branes can be considered as point particles.
- The SM particles are on the D7-branes.

Subtle point:

- The ND particles: open strings end on both D3-branes and D7-branes. Their gauge couplings with the gauge fields on the D7-branes

$$\frac{1}{g_{37}^2} = \frac{V}{g_7^2} \to 0 \, .$$

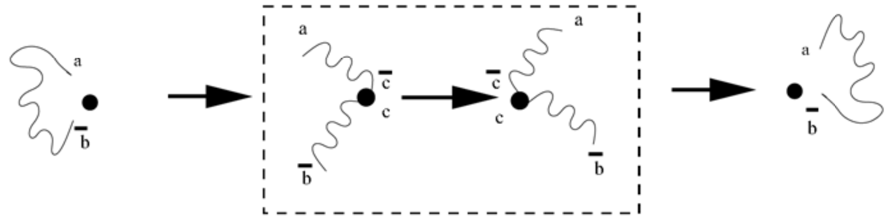
- The D3-branes are distributed *locally* uniformly in the Universe. In the string field theory, a D-brane is a fat object with thickness of the order of the string scale. The widths of the D-brane along the transverse dimensions are about $1.55\ell_s$, as follows from an analysis of the tachyonic lump solution in the string field theory [a]

$$\frac{1}{g_{37}^2} = \frac{V_{A3} R'}{(1.55\ell_s)^4} \frac{\ell_s^4}{g_7^2} = \frac{V_{A3} R'}{(1.55)^4} \frac{1}{g_7^2} \, .$$

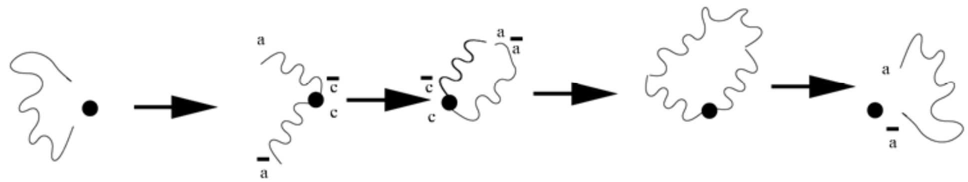
[a] N. Moeller, A. Sen and B. Zwiebach, JHEP **0008**, 039 (2000), and references therein.

SM particles:

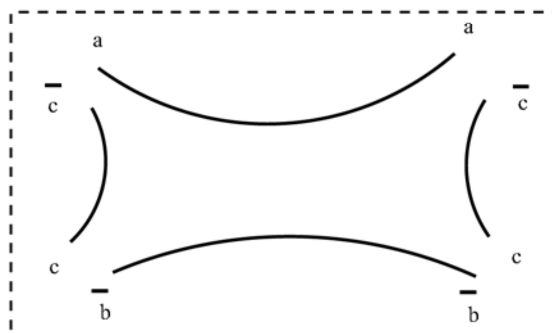
- A generic SM particle is an open string $a\bar{b}$ with both ends on the D7-branes.
- The gauge fields related to the Cartan subalgebras of the SM gauge groups, and their supersymmetric partners (gauginos), $a = b$.
- The other particles such as $W^{\pm}$ and electron, $a \neq b$.


a
$\bar{b}$
a
$\bar{c}$
c
$\bar{b}$
$\bar{c}$
c
a
$\bar{b}$
a
$\bar{b}$

(a)


a
$\bar{c}$
c
$\bar{a}$
$\bar{c}$
c
a
$\bar{a}$
a
$\bar{a}$

(b)


a
a
$\bar{c}$
$\bar{c}$
c
c
$\bar{b}$
$\bar{b}$

(c)

Time Delay/Advance: The Four-Fermion (or Scalar) Scattering Amplitude

$$\mathcal{A}_{\rm total} \equiv \mathcal{A}(1,2,3,4) + \mathcal{A}(1,3,2,4) + \mathcal{A}(1,2,4,3) \ .$$

$$\mathcal{A}(1,2,3,4) \equiv A(1,2,3,4) + A(4,3,2,1) \ .$$

$A(1,2,3,4)$ **is the standard four-point ordered scattering amplitude**

$$(2\pi)^4\delta^{(4)}(\sum_a k_a)A(1,2,3,4) = \frac{-i}{g_s l_s^4}\int_0^1 dx$$
$$\left\langle \mathcal{V}^{(1)}(0,k_1)\mathcal{V}^{(2)}(x,k_2)\mathcal{V}^{(3)}(1,k_3)\mathcal{V}^{(4)}(\infty,k_4)\right\rangle \ .$$

In terms of the Mandelstam variables: $s = -(k_1 + k_2)^2$, $t = -(k_2 + k_3)^2$ and $u = -(k_1 + k_3)^2$, the ordered four-point amplitude $\mathcal{A}(1, 2, 3, 4)$ is given by

$$\mathcal{A}(1_{j_1 I_1}, 2_{j_2 I_2}, 3_{j_3 I_3}, 4_{j_4 I_4}) =$$
$$-g_s l_s^2 \int_0^1 dx\ x^{-1-s\, l_s^2}\ (1-x)^{-1-t\, l_s^2}\ \frac{1}{[F(x)]^2}\ \times$$
$$\left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}(1-x) + \bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)} x\right]$$
$$\times\{\eta\delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3}\sum_{m\in\mathbf{Z}}\ e^{-\pi\tau\, m^2\, \ell_s^2/R'^2}$$
$$+\delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3}\sum_{n\in\mathbf{Z}} e^{-\pi\tau n^2\, R'^2/\ell_s^2}\}\ ,$$

where $F(x) \equiv F(1/2;1/2;1;x)$ is the hypergeometric function, $\tau(x) = F(1-x)/F(x)$, j_i and I_i with $i = 1,\ 2,\ 3,\ 4$ are indices on the D7-branes and D3-branes, respectively, and η is

$$\eta = \frac{(1.55\ell_s)^4}{V_{A3}R'}\ .$$

Also, u is a fermion polarization spinor, and the dependence of the appropriate Chan-Paton factors has been made explicit.

Taking $F(x) \simeq 1$, we obtain

$$\mathcal{A}(1,2,3,4) \propto g_s \ell_s^2 \Big(t\ell_s^2 \overline{u}^{(1)} \gamma_\mu u^{(2)} \overline{u}^{(4)} \gamma^\mu u^{(3)} + s\ell_s^2 \overline{u}^{(1)} \gamma_\mu u^{(4)} \overline{u}^{(2)} \gamma^\mu u^{(3)} \Big) \times \frac{\Gamma(-s\ell_s^2)\Gamma(-t\ell_s^2)}{\Gamma(1+u\ell_s^2)} ,$$

$$\mathcal{A}(1,3,2,4) \propto g_s \ell_s^2 \Big(t\ell_s^2 \overline{u}^{(1)} \gamma_\mu u^{(3)} \overline{u}^{(4)} \gamma^\mu u^{(2)} + u\ell_s^2 \overline{u}^{(1)} \gamma_\mu u^{(4)} \overline{u}^{(3)} \gamma^\mu u^{(2)} \Big) \times \frac{\Gamma(-u\ell_s^2)\Gamma(-t\ell_s^2)}{\Gamma(1+s\ell_s^2)} ,$$

$$\mathcal{A}(1,2,4,3) \propto g_s \ell_s^2 \Big(u\ell_s^2 \overline{u}^{(1)} \gamma_\mu u^{(2)} \overline{u}^{(3)} \gamma^\mu u^{(4)} + s\ell_s^2 \overline{u}^{(1)} \gamma_\mu u^{(3)} \overline{u}^{(2)} \gamma^\mu u^{(4)} \Big) \times \frac{\Gamma(-s\ell_s^2)\Gamma(-u\ell_s^2)}{\Gamma(1+t\ell_s^2)} .$$

The Particles with $j_1 = \bar{j}_2$: the SM gauge fields and gauginos

- Time delays or advances arise from the amplitude $\mathcal{A}(1,2,3,4)$ by considering backward scattering $u = 0$ [a].

- Noting that $s + t + u = 0$, the first term in $\mathcal{A}(1,2,3,4)$ for $u = 0$ is proportional to

$$
\begin{aligned}
t\ell_s^2\Gamma(-s\ell_s^2)\Gamma(-t\ell_s^2) &= -s\ell_s^2\Gamma(-s\ell_s^2)\Gamma(s\ell_s^2) \\
&= \frac{\pi}{\sin(\pi s\ell_s^2)} \,.
\end{aligned}
$$

- It has pole at $s = n/\ell_s^2$, which implies a resonance corresponding to an intermediate string state.

[a] N. Seiberg, L. Susskind and N. Toumbas, JHEP **0006**, 044 (2000).

Time Delay:

- Using the correct ϵ prescription replacing $s \to s + i\epsilon$, which shift the poles off the real axis, we obtain

$$\frac{1}{\sin(\pi s \ell_s^2)} = -i \sum_{n \geq 0} e^{i(2n+1)\pi s \ell_s^2} + \mathcal{O}(\epsilon) .$$

- Noting that $s = E^2$, we obtain the time delay due to one D3-brane

$$\Delta t = (2n+1)\pi E \ell_s^2 , \quad \text{where } n \geq 0 .$$

- The velocity of the particle is

$$v = \frac{1}{1 + \frac{(2n+1)\pi E}{\xi M_{\mathrm{St}}}} c , \quad \delta v \simeq -\frac{(2n+1)\pi E}{\xi M_{\mathrm{St}}} ,$$

$$M_{\mathrm{St}} \equiv \frac{1}{\ell_s} , \quad \xi \equiv M_{\mathrm{St}} \times V_{A3}^{1/3} .$$

Time Advance:

- Using the correct ϵ prescription replacing $s \to s - i\epsilon$, which shift the poles off the real axis, we obtain

$$\frac{1}{\sin(\pi s \ell_s^2)} = i \sum_{n \geq 0} e^{-i(2n+1)\pi s \ell_s^2} + \mathcal{O}(\epsilon) \, .$$

- Noting that $s = E^2$, we obtain the time advance due to one D3-brane

$$\Delta t = -(2n+1)\pi E \ell_s^2 \, , \quad \text{where } n \geq 0 \, .$$

- The velocity of the particle is

$$v = \frac{1}{1 - \frac{(2n+1)\pi E}{\xi M_{\mathrm{St}}}} c \, , \quad \delta v \simeq \frac{(2n+1)\pi E}{\xi M_{\mathrm{St}}} \, .$$

The Particles with $j_1 \neq \bar{j}_2$:

- Only the amplitude $\mathcal{A}(1,3,2,4)$ gives dominant contribution.
- Considering backward scattering with $u = 0$ and $s + t + u = 0$, we obtain only the pole terms and then no-time delays/advances at the leading order

$$\mathcal{A}(1,3,2,4) \;\propto\; g_s \ell_s^2 \left(\overline{u}^{(1)} \gamma_\mu u^{(3)} \overline{u}^{(4)} \gamma^\mu u^{(2)} - \frac{1}{s\ell_s^2} u \ell_s^2 \overline{u}^{(1)} \gamma_\mu u^{(4)} \overline{u}^{(3)} \gamma^\mu u^{(2)} \right) \; .$$

- At order η ($\mathcal{O}(\eta)$), we have time delays and advances for these particles:

$$\delta v \simeq \pm \frac{(2n+1)\eta\pi E}{\xi M_{\mathrm{St}}} \; .$$

Assumptions:

- $R' = 2.5\ell_s$, $n = 1$.
- To explain the MAGIC, HESS, and FERMI experiments, the photons must propagate subluminally.
- To explain the OPERA and MINOS experiments, the muon neutrinos must propagate superluminally.

OPERA Expriment:

- $(V_{A3}^{\mathrm{E}})^{1/3} = 2.5\ell_s$, $\eta^{\mathrm{E}} = 0.148$, and $\xi^{\mathrm{E}} = 2.5$.
- Using the OPERA mean muon neutrino energy 17 GeV and the central value for δv_ν, we obtain

$$M_{\mathrm{St}} \ = \ 1.27 \times 10^5 \ \mathrm{GeV} \ .$$

- The physical volume of the three extra dimensions transverse to the D7-branes is very large.

MAGIC, HESS, and FERMI Experiments:

- To have the effective quantum gravity scale $M_{\mathrm{QG}} \simeq 0.98 \times 10^{18}$ GeV, we get

$$
\begin{aligned}
(V_{A3}^{\mathrm{IS}})^{1/3} &= 1.9 \times 10^{8}\,[\mathrm{GeV}]^{-1}\,, \quad \eta^{\mathrm{IS}} = 1.6 \times 10^{-40}\,, \\
\xi^{\mathrm{IS}} &= 2.4 \times 10^{13}\,.
\end{aligned}
$$

- The FERMI observation of GRB 090510 may be explained by choosing smaller V_{A3}^{IS} in the corresponding space direction.

Summary:

- The additional contributions to the particle velosity δv from string theory is proportional to both the particle energy and the D3-brane number density, and is inversely proportional to the string scale.
- We can realize the background dependent Lorentz violation naturally by varying the D3-brane number density in space time.
- We can still explain the OPERA results and all the other phenomenological constraints and observations on Lorentz violation.

- How to test this model: measuring the energy dependent photon velocity.

Natural Solution to the Theoretical Challenges:

- The D3-branes are flavour blind, so we naturally explain the KamLAMD experiment.
- The time delays for the SM particles arise from their interactions with the D3-branes. Thus, when they do not interact with the D3-branes, for example, when they are away from the D3-branes, all the SM particles are just the conventional particles in the traditional SM without any Lorentz violation at all.

- We not only naturally avoid the LEP constraint on δv_e, but also naturally explain the theoretical challenges such as the Bremsstrahlung effects for muon neutrinos and the pion decays. In fact, the astrophysical Lorentz violation constraints on charged leptons are automatically escaped by the same reason.
- The only constraints on our model arise from the time delays in the MAGIC, HESS, and FERMI experiments, and the SN1987a observations.

Comments

- The D3-branes contribute to huge tadpoles: AdS flux vacua [a].
- The D3-brane distribution needs enormous fine-tunning.
- Why the photons are subluminal while the neutrinos are superluminal.

---

[a] C. M. Chen, T. Li and D. V. Nanopoulos, Int. J. Mod. Phys. A **24**, 2453 (2009); C. M. Chen, S. Hu, T. Li and D. V. Nanopoulos, arXiv:1107.3465 [hep-th].

IV. DEFORMED LORENTZ VILATION

δv_ν **is constant:**

$$E_A^2 \;=\; \vec{P}_A^2 + m_A^2 + \xi_P^A \vec{P}_A^2 + \xi_{PE}^A P_A E_A + \xi_E^A E_A^2 \,.$$

- ξ_P^A, ξ_{PE}^A, and ξ_E^A are universal functions of m_A for all the SM particles, *i.e.*, $\xi_P^A \equiv \xi_P(m_A)$, $\xi_{PE}^A \equiv \xi_{PE}(m_A)$, and $\xi_E^A \equiv \xi_E(m_A)$.
- Scenario I with only the $\xi_P^A \vec{P}_\nu^2$ term and Scenario II with only the $\xi_E^A E_\nu^2$ term.

Scenario I:

$$\xi_P^A \;=\; \alpha_P \frac{m_A^2 M_{\rm IR}^2}{m_A^4 + M_{\rm IR}^4}\;.$$

- Choosing $m_\nu = 0.05$ eV, $\xi_P^\nu = 4.74 \times 10^{-5}$, and $M_{\rm IR} = 2.3 \times 10^{-3}$ eV, we get $\alpha_P = 2.24 \times 10^{-2}$
- The electron mass is 0.511 MeV, we have $\xi_P^e = 4.54 \times 10^{-19}$.

Deformed Lorentz Invariance:

$$[N_i^\nu,\ E_\nu^2 - \vec{P}_\nu^2 - \xi_P^\nu \vec{P}_\nu^2] \ = \ 0 \ .$$

The Einstein special relativity should be realized at the $\xi_P^A = 0$ limit.

Thus, we obtain

$$[N_i^\nu,\ E_\nu] \ = \ \beta_1 (P_\nu)_i \ , \ \ [N_i^\nu,\ (P_\nu)_j] \ = \ \frac{1}{\beta_1} E_\nu \delta_{ij} \ ,$$

$$\beta_1 \ = \ \sqrt{1 + \xi_P^\nu} \ .$$

A generic physical process: the initial states include n neutrinos and n' other SM particles, and the final states include m neutrinos and m' other SM particles.

The momentum and energy conservation laws:

$$\sum_{k=1}^{n} \vec{P}^i_{\nu k} + \sum_{k=1}^{n'} \vec{P}^i_{k} = \sum_{k=1}^{m} \vec{P}^f_{\nu k} + \sum_{k=1}^{m'} \vec{P}^f_{k} ,$$

$$\frac{1}{\beta_1} \sum_{k=1}^{n} E^i_{\nu k} + \sum_{k=1}^{n'} E^i_{k} = \frac{1}{\beta_1} \sum_{k=1}^{m} E^f_{\nu k} + \sum_{k=1}^{m'} E^f_{k} .$$

The neutrino masses are tiny and can be neglected, so we get $E_\nu = \beta_1 P_\nu$. Thus, the energy conservation law can be rewritten as follows

$$\sum_{k=1}^{n} P^i_{\nu k} + \sum_{k=1}^{n'} E^i_k \; = \; \sum_{k=1}^{m} P^f_{\nu k} + \sum_{k=1}^{m'} E^f_k \; .$$

Therefore, the momentum conservation law and energy conservation law are the same as those in the traditional quantume field theory with Lorentz symmetry. And then all the theoretical challenges can be solved naturally.

Scenario II:

$$\xi_E^A = \alpha_E \frac{m_A^2 M_{\rm IR}^2}{m_A^4 + M_{\rm IR}^4} .$$

Deformed Lorentz Invariance:

$$[N_i^\nu, \; E_\nu^2 - \vec{P}_\nu^2 - \xi_E^\nu E_\nu^2] = 0 .$$

We obtain

$$[N_i^\nu, \; E_\nu] = \frac{1}{\beta_2}(P_\nu)_i \; , \; \; [N_i^\nu, \; (P_\nu)_j] = \beta_2 E_\nu \delta_{ij} .$$

$$\beta_2 = \sqrt{1 - \xi_E^\nu} .$$

The momentum and energy conservation laws:

$$\frac{1}{\beta_2}\sum_{k=1}^{n}\vec{P}^{i}_{\nu k}+\sum_{k=1}^{n'}\vec{P}^{i}_{k} \;=\; \frac{1}{\beta_2}\sum_{k=1}^{m}\vec{P}^{f}_{\nu k}+\sum_{k=1}^{m'}\vec{P}^{f}_{k}\;,$$

$$\sum_{k=1}^{n}E^{i}_{\nu k}+\sum_{k=1}^{n'}E^{i}_{k} \;=\; \sum_{k=1}^{m}E^{f}_{\nu k}+\sum_{k=1}^{m'}E^{f}_{k}\;.$$

Taking $P_\nu = \beta_2 E_\nu$, we have the momentum conservation law

$$\sum_{k=1}^{n}E^{i}_{\nu k}\vec{r}^{\,i}_{\nu k}+\sum_{k=1}^{n'}\vec{P}^{i}_{k} \;=\; \sum_{k=1}^{m}E^{f}_{\nu k}\vec{r}^{\,f}_{\nu k}+\sum_{k=1}^{m'}\vec{P}^{f}_{k}\;.$$

Bremsstrahlung effects [a]**:** $\nu_\mu \to \nu_\mu + e^+ + e^-$ in Scenario I

$$\vec{P}^i_\nu \;=\; \vec{P}^f_{\nu k} + \vec{P}^f_{e^+} + \vec{P}^f_{e^-} \,, \quad P^i_\nu \;=\; P^f_\nu + E^f_{e^+} + E^f_{e^-} \,.$$

Proof: let us suppose that this process is not forbidden.

$$(\vec{P}^i_\nu - \vec{P}^f_{\nu k})^2 \;=\; (\vec{P}^f_{e^+} + \vec{P}^f_{e^-})^2 \;<\; (E^f_{e^+} + E^f_{e^-})^2 \,.$$

$$(\vec{P}^i_\nu - \vec{P}^f_{\nu k})^2 \;<\; (P^i_\nu - P^f_\nu)^2 \,.$$

This inequality can not be satisfied obviously.

---

[a] A. G. Cohen, S. L. Glashow, arXiv:1109.6562 [hep-ph].

Pion decays:$\pi^+ \rightarrow \mu^+ + \nu_\mu$ in Scenario I

- Threshold condition [a]:

$$m_\pi \geq m_\mu + \sqrt{\xi_P^\nu} P_\nu \ .$$

- The energy and momentum conservation laws: [b]

$$\vec{P}_\pi \ = \ \vec{P}_\mu + \vec{P}_\nu \ , \quad E_\pi \ = \ E_\mu + \sqrt{1 + \xi_P^\nu} P_\nu \ .$$

This threshold condition is not valid and such laws are obviously different from the generic energy and momentum conservation laws in the deformed Lorentz invariance.

---

[a] L. Gonzalez-Mestres, arXiv:1109.6630 [physics.gen-ph]; X. -J. Bi, P. -F. Yin, Z. -H. Yu, Q. Yuan, arXiv:1109.6667 [hep-ph]

[b] R. Cowsik, S. Nussinov, U. Sarkar, arXiv:1110.0241 [hep-ph].

V. Conclusion

- Background Dependent Lorentz Violation.
- In the Type IIB string theory with D3-branes and D7-branes, we can evade all the constraints on Lorentz violation from all the known experiments, explain all the observations on Lorentz violation, and naturally explain all the theoretical challenges as well.
- Deformed Lorentz invariance.