Dispersion relations explaining OPERA data from deformed Lorentz symmetry

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Gang Guo and Xiao-Gang He. arXiv:1111.6330 [hep-ph]

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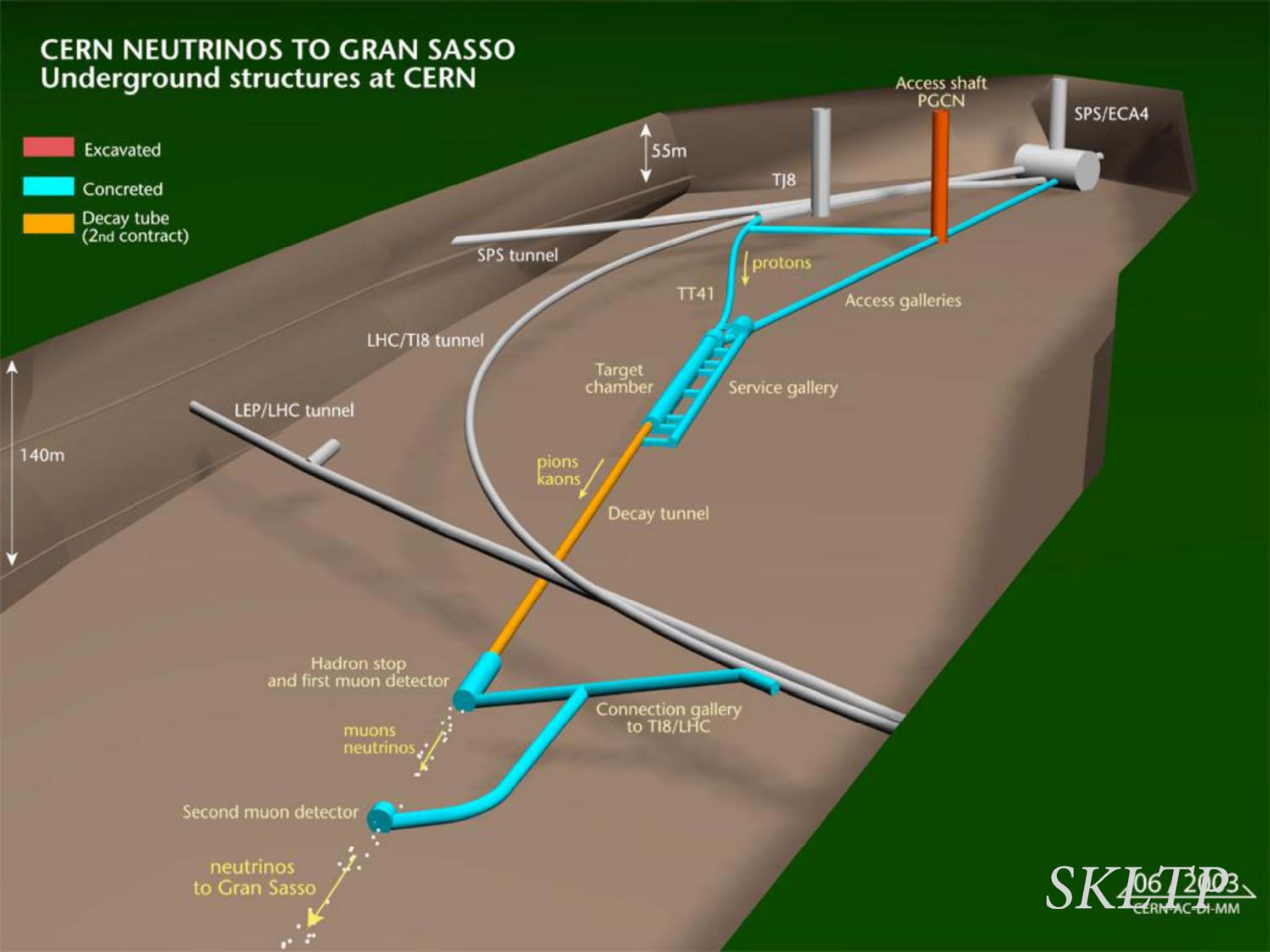
CERN NEUTRINOS TO GRAN SASSO
Underground structures at CERN
Excavated
Concreted
Decay tube
(2nd contract)
55m
140m
Access shaft
PGCN
SPS/ECA4
TJ8
SPS tunnel
protons
TT41
Access galleries
LHC/TI8 tunnel
Target
chamber
Service gallery
LEP/LHC tunnel
pions
kaons
Decay tunnel
Hadron stop
and first muon detector
Connection gallery
to TI8/LHC
muons
neutrinos
Second muon detector
neutrinos
to Gran Sasso
SKLTP
CERN-AC-DI-MM

Results of OPERA

$L \sim 730Km$ with a precision of 20cm

The firstly released results:

$\delta v = (v - c)/c = 2.48 \pm 0.28(stat.) \pm 0.30(sys.)) \times 10^{-5}$ $< E >$=17GeV

$\delta v = (v - c)/c = 2.16 \pm 0.76(stat.) \pm 0.30(sys.)) \times 10^{-5}$ $< E >$=13.9GeV

$\delta v = (v - c)/c = 2.74 \pm 0.74(stat.) \pm 0.30(sys.)) \times 10^{-5}$ $< E >$=42.9GeV

The new results with short-pulse proton beam used:

$\delta v = (v - c)/c = 2.37 \pm 0.32(stat.)^{+0.34}_{-0.24}(sys.)) \times 10^{-5}$ $< E >$=17GeV

$\delta v = (v - c)/c = 2.24 \pm 0.75(stat.)^{+0.30}_{-0.28}(sys.)) \times 10^{-5}$ $< E >$=13.8GeV

$\delta v = (v - c)/c = 2.94 \pm 0.78(stat.)^{+0.30}_{-0.28}(sys.)) \times 10^{-5}$ $< E >$=40.7GeV

T. Adam et al., [OPERA Collaboration], arXiv:1109.4897

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GPS time ?

It's natural to doubt about the GPS time from the very beginning:

- Gravitational effects
- Special relativity effects
- Doppler's effect

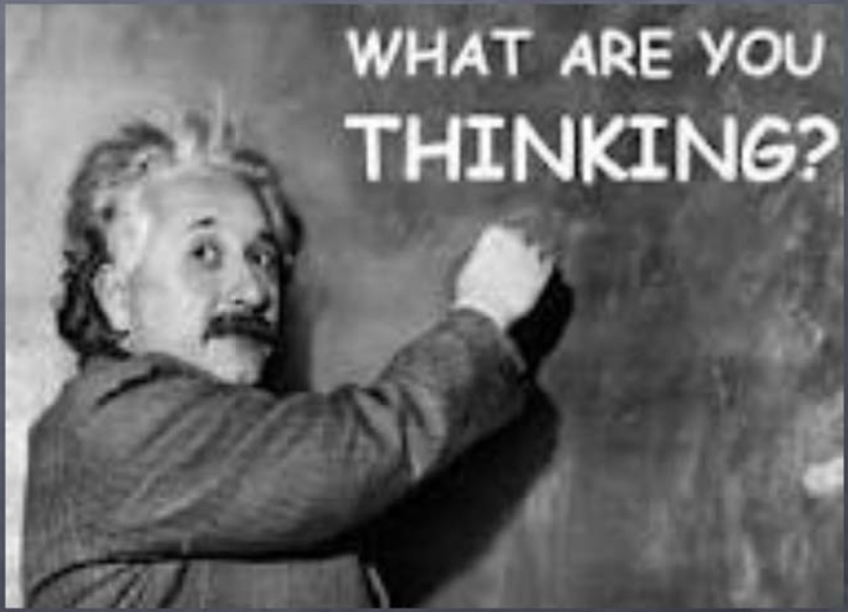
  ……..

Up to now , there is no confirmative answer to the GPS problem !

So if we assume OPERA is true

- Lorentz violation
- Tachyon neutrinos
- Extra dimension
- Sterile neutrinos

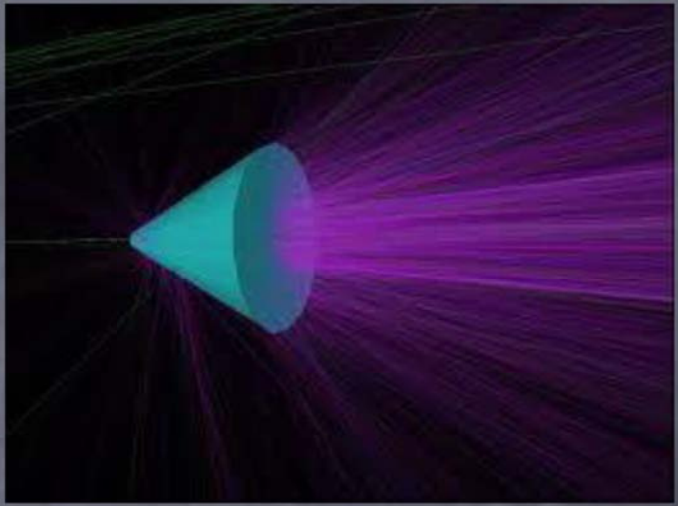
  ....



Theoretical constraints on LV(a)

Cherenkov analogous process

$$\nu_\mu \longrightarrow \begin{cases} \nu_\mu + \gamma \\ \nu_\mu + \nu_e + \overline{\nu}_e \\ \nu_\mu + e^+ + e^- \end{cases}$$



For dispersion relation like : $E^2 = m^2 + p^2 + \epsilon p^2$

$$\Gamma = k' \frac{G_F^2}{192\pi^3} E^5 \delta^3$$

$$\frac{dE}{dx} = -k \frac{G_F^2}{192\pi^3} E^6 \delta^3$$

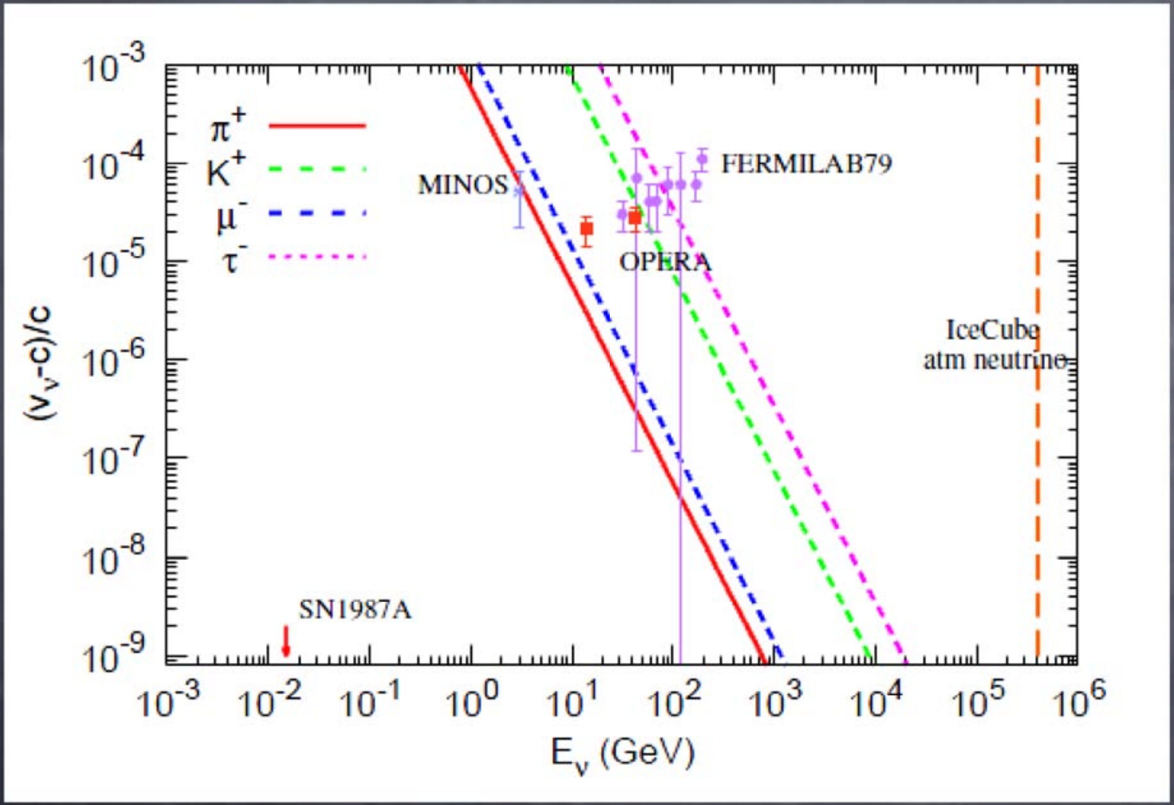
A.G. Cohen, S. L. Glashow, PRL 107 (2011) 181803 ; arXiv:1109.6562.

The superluminal neutrinos that reach Gran Sasso will not have energy in excess of 12.5GeV due to the rapidly energy loss.

Theoretical constraints on LV(b)

The main channel for muon neutrino production in OPERA : $\pi^+ \rightarrow \mu^+ + \nu_\mu$

$$E^2 = m^2 + p^2 + \epsilon p^2$$



threshold: $m_\pi \geq m_\mu + m_\upsilon^{eff}$

$$E_\nu \leq (m_\pi - m_\nu)\sqrt{1 + 1/\epsilon}$$

For $\delta v = 2.37 \times 10^{-5}$, neutrinos produced from pion decay can't have energies exceeding 5GeV.

X.-J. Bi, P.-F. Yin, Z.-H. Yu, Q. Yuan. Phys. Rev. Lett. 107, 241802(2011). [arXiv:1109.6667v3[hep-ph]]

L. Gonzalez-Mestres. arXiv:1109.6630[hep-ph]

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Ways out of it

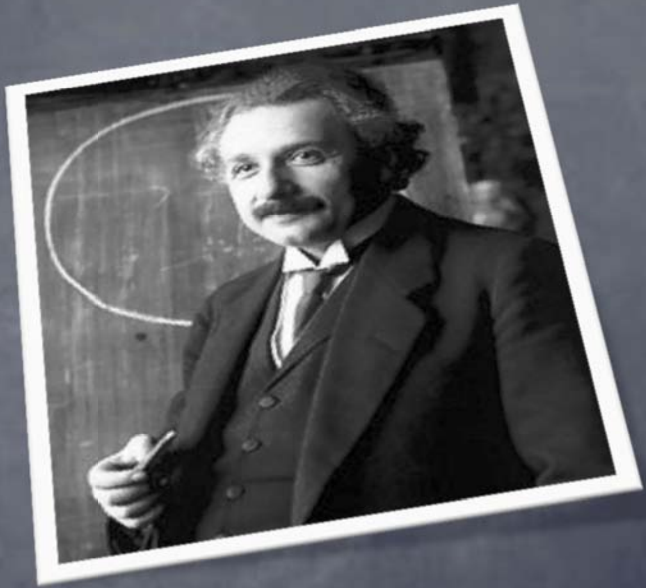
------ deformed Lorentz symmetry

Basic idea of DLS:

- ◆ Violate ordinary Lorentz symmetry to induce superluminality.

- ◆ Maintain a deformed(modified) Lorentz symmetry to keep the relativity of inertial frames.

- ◆ Evade the constraints from Cohen and Bi .



G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman, L. Smolin, arXiv:1110.0521[hep-ph]
G. Amelino-Camelia et al., arXiv:1111.0993[hep-ph]
Yi Ling, arXiv:1111.3716[hep-ph]
Yunjie Hou et al., arXiv:1111.4994[hep-ph]

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A general formulism of DLS

A general dispersion relation can be written as : $E^2 f^2(E,p) - p^2 g^2(E,p) = m^2$, which can be rewritten without loss of generality as:

$$F^2(E,p)E^2 - p^2 = m^2$$

Where, $F^2(E,p) = f^2(E,p) - (g^2(E,p) - 1)p^2/E^2$.

We will write F as F(p, m) since E can be solved as a function of p and m ,and have :

$$F(p=0,m) = 1$$

The generator of Lorentz boost N_i is modified in such a way that:

$$[N_i, F^2(p,m)E^2 - p^2] = 0,$$

The above commutation relation can be achieved by

$$[N_i, p_j] = F(p,m)E\delta_{ij}, \qquad [N_i, E] = p_i\left(\frac{1}{F(p,m)} - 2E^2\frac{\partial F(p,m)}{\partial p^2}\right)$$

The conserved quantities within deformed Lorentz are modified to :

$$F(p,m)E, \quad \vec{p}$$

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Superluminality without price

For $\pi^+ \rightarrow \mu^+ \nu_\mu$

Energy-momentum conservation laws :

$$\overrightarrow{p_\pi} = \overrightarrow{p_\mu} + \overrightarrow{p_\upsilon} \qquad E_\pi = E_\mu + F(p,m)E_\nu$$

From deformed Lorentz symmetry , $F(p,m)E_\nu = \sqrt{\vec{p}_\nu^2 + m_\nu^2}$, we get:

$$\overrightarrow{p_\pi} = \overrightarrow{p_\mu} + \overrightarrow{p_\upsilon} \qquad \sqrt{\vec{p}_\pi^2 + m_\pi^2} = \sqrt{\vec{p}_\mu^2 + m_\mu^2} + \sqrt{\vec{p}_\nu^2 + m_\nu^2}$$

$\pi^+ \rightarrow \mu^+ \nu_\mu$ is not forbidden in any inertial frame in DLS .

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Dispersion relation Vs Superluminality

For a general dispersion relation $F^2(p,m)E^2 - p^2 = m^2$, we can get the group velocity:

$$v = \frac{dE}{dp} = \frac{p}{E}\left(\frac{1}{F^2(p,m)} - 2\frac{E^2}{F(p,m)}\frac{dF(p,m)}{dp^2}\right)$$
$$= \frac{p}{\sqrt{p^2+m^2}}\frac{1}{F(p,m)} + \sqrt{p^2+m^2}\frac{d}{dp}\frac{1}{F(p,m)};$$

For neutrinos with a neglected mass, we have :

$$\delta v = \frac{1}{F(p,0)} + p\frac{d}{dp}\frac{1}{F} - 1\,.$$

For example, for the dispersion relation $E^2 - p^2 = m^2 + 2E^2p^2/M^2$, ($F^2 = 1 - 2\,p^2/M^2$)

$$\delta v = v - 1 = \frac{1}{(1-2p^2/M^2)^{3/2}} - 1$$

G. Amelino-Camelia et al., arXiv:1110.0521

Conversely, once $\delta v(p)$ is known , we can obtain the corresponding F(p) via:

$$\frac{1}{F} = \frac{1}{p}\left(\int^p (1+\delta v(p'))dp' + c\right) ,$$

with a boundary condition $F(0) = 1$

Fitting the superluminal data

$$\delta v(p) = \frac{1}{(1-2p^2/M^2)^{3/2}} - 1$$

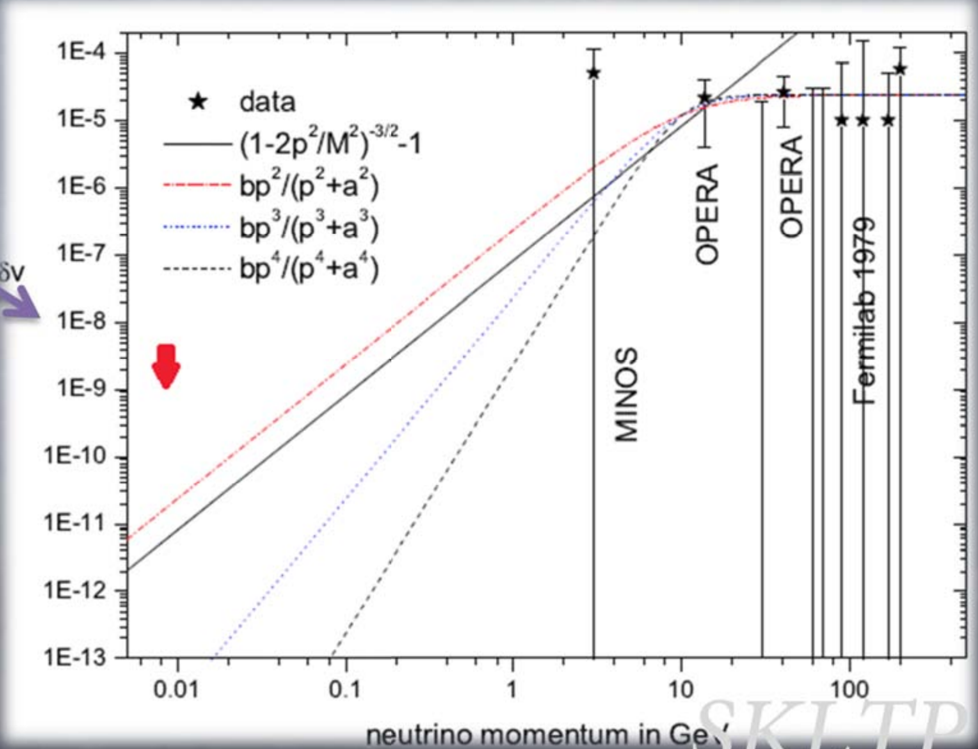
(failed)

$$\delta v(p) = b\frac{p^n}{a^n + p^n}$$

$$(a = 10GeV \ \& \ b = 2.37 \times 10^{-5})$$

anti-electron neutrinos from SN1987a

We assume no energy dependence of superluminality due to observed neutrino oscillations.



Since $\dfrac{1}{F} = \dfrac{1}{p}\left(\int^{p}(1+\delta v(p'))dp' + c\right)$, $F(0) = 1$

For $\delta v(p) = b\,\dfrac{p^n}{a^n + p^n}$

$$
\begin{aligned}
\frac{1}{F_n(p)} &= 1 + b \\
&- \frac{ab}{p}\begin{cases} \frac{2}{n}\sum_{k=0}^{n/2-1}[Q_k \sin(\frac{(2k+1)\pi}{n}) - P_k \cos(\frac{(2k+1)\pi}{n})] \ , \quad \mathrm{n = even} \\ \frac{1}{n}\ln((1+\frac{p}{a}) + \frac{2}{n}\sum_{k=0}^{(n-3)/2}[Q_k \sin(\frac{(2k+1)\pi}{n}) - P_k \cos(\frac{(2k+1)\pi}{n})] - C \ , \quad \mathrm{n = odd}\ . \end{cases}
\end{aligned}
$$

where

$$
\begin{aligned}
P_k &= \frac{1}{2}\ln[\frac{p^2}{a^2} - 2\frac{p}{a}\cos(\frac{(2k+1)\pi}{n}) + 1] \ , \\
Q_k &= \arctan\left(\frac{\frac{p}{a} - \cos(\frac{(2k+1)\pi}{n})}{\sin(\frac{(2k+1)\pi}{n})}\right) \ , \\
C &= \sum_{k=0}^{(n-3)/2}(\frac{(2k+1)\pi}{n} - \frac{\pi}{2})(\sin\frac{(2k+1)\pi}{n}) \ .
\end{aligned}
$$

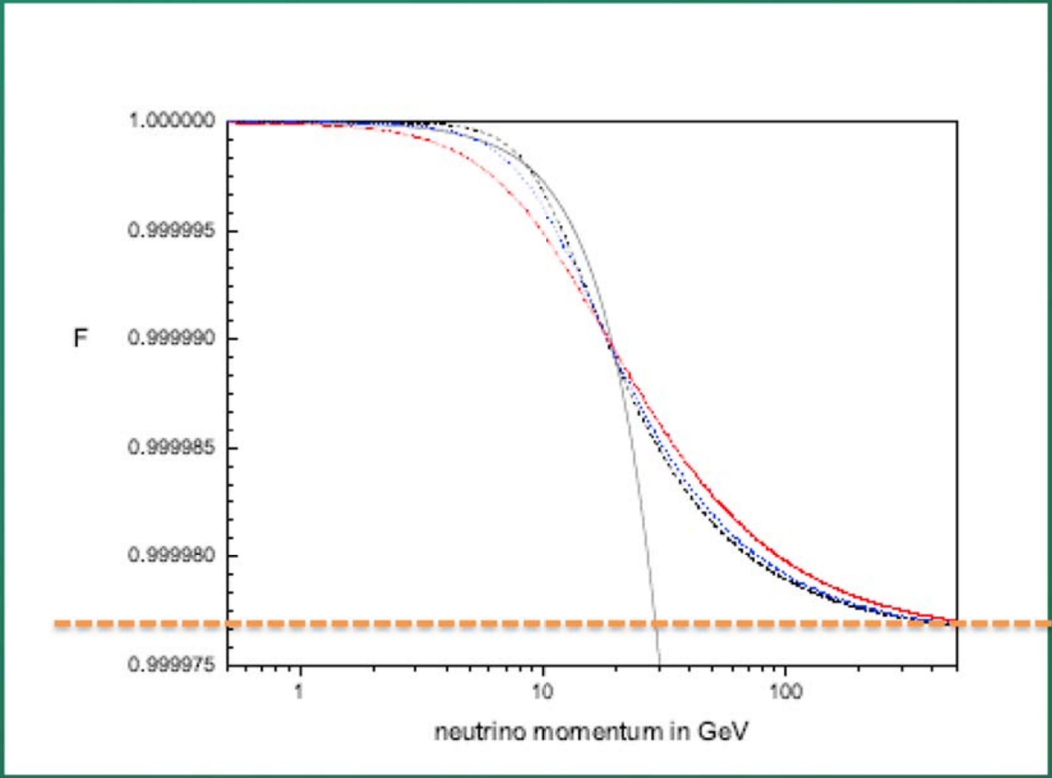
For $n = 2$ and $n = 3$, we have

$$
F_2(p) = \frac{p/a}{(1+b)p/a - d\arctan(p/a)} \ ,
$$

$$
F_3(p) = \frac{6\sqrt{3}p/a}{6\sqrt{3}(1+b)p/a - b(\pi + 6\arctan[(2p-a)/\sqrt{3}a] + \sqrt{3}\ln[(a+p)^2/(a^2 - ap + p^2)])} \ .
$$

Modified conservation Law

To remind you , F(p,m)E is conserved



Up to now, it's still hard for us to test these modified energy conservation laws.

Of order 10^{-5}

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Summary

- Deformed Lorentz symmetry can explain the OPERA data without suffering the theoretical constraints.
- Energy conservation law should be modified within Deformed Lorentz Symmetry and that can be tested in future neutrino experiment with high precision energy measurement.
- Some underlying theories are required to explain the deformed Lorentz symmetry.

Thanks