Theory and phenomena on Lorentz Violation: I

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SKLTP

Einstein's Special Relativity

In 1905, Einstein published his famous paper "On the Electrodynamics of Moving Bodies".

Galileo Transformation in Classical Physics

$$x'_1 = x_1,$$

$$x'_2 = x_2,$$

$$x'_3 = x_3 - vt,$$

$$t' = t.$$

Wave Equation under Galileo Transformation

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 0.$$

$$\frac{\partial^2\phi}{\partial x_1'^2} + \frac{\partial^2\phi}{\partial x_2'^2} + \frac{\partial^2\phi}{\partial x_3'^2}\left(1 - \frac{v^2}{c^2}\right) + \frac{2v}{c^2}\frac{\partial^2\phi}{\partial x_3'\partial t'} - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t'^2} = 0.$$

Principles of Special Relativity

- Principle of Relativity: the equations describing the laws of physics have the same form in all admissible frames of reference.
- Principle of constant light speed: the speed of light is the same in all directions in vacuum in all reference frames, regardless whether the source of the light is moving or not.

Triumphs of Einstein's Relativity

- One of the foundations of modern physics.
- Proved to be valid at very high precision.

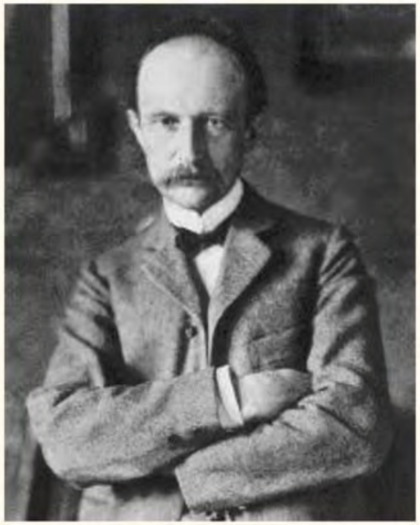
Lorentz Invariance, the basic theoretical foundation of relativity, states that

the equations describing the laws of physics have the same form in all admissible reference frames.

So why we seek for Lorentz Violation ?

Planck's *God-Given Unit System*

(Planck, 1899)



Planck, 1900

$$c,\ G,\ \hbar,\ k_B,\ \text{and}\ 1/4\pi\epsilon_0$$

units of length, mass, time, and temperature that would, independently of special bodies and substances, necessarily retain their significance for all times and all cultures, even extraterrestrial and extrahuman ones, and which may therefore be designated as natural units of measure. (Planck 1899, pp. 479–480)

Planck, M.: Über irreversible Strahlungsvorgänge. Sitzungsberichte der Königlich Preuischen Akademie der Wissenschaften zu Berlin **5**, 440 (1899)

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Basic units of the universe: Planck Units

$$l_P = \sqrt{\frac{G\hbar}{c^3}} = 1.61624(8) \times 10^{-35}\, m$$

$$t_{\rm P} \equiv \sqrt{G\hbar/c^5} \simeq 5.4 \times 10^{-44}\ {\rm s}$$

$$M_P = \sqrt{\frac{\hbar c}{G}} = 1.22089(6)\times 10^{19}\,\frac{\rm GeV}{\rm c^2} = 2.17644(11)\times 10^{-8}\,{\rm kg}$$

$$E_{\rm P} \equiv \sqrt{\hbar c^5/G} \simeq 2.0 \times 10^9\ {\rm J}$$

$$T_{\rm P} \equiv \sqrt{\hbar c^5/G k_B^2} \simeq 1.4 \times 10^{32}\ {\rm K}$$

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A physical argument of discrete space-time

Y.Xu & B.-Q.Ma, MPLA 26 (2011) 2101, arXiv: 1106.1778

- From two known entropy constraints:

$$S_{\rm matter} \le 2\pi E R, \qquad S_{\rm matter} \le \frac{A}{4},$$

- Combined with black-body entropy

$$S = \frac{4}{45}\pi^2 T^3 V = \frac{16}{135}\pi^3 R^3 T^3.$$

- We arrive at a minimum value of space

$$R \ge \left(\frac{128}{3645\pi}\right)^{\frac{1}{2}} l_{\rm P} \simeq 0.1 l_{\rm P},$$

We reveal from physical arguments that space-time is discrete rather than continuous.

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Proposal of a new fundamental length scale instead of the Newtonian constant

L.Shao & B.-Q.Ma, Sci.China Phys. Mech. Astro. 54 (2011) 1771, arXiv: 1006.3031

- If gravity is emergent, a new fundamental constant should be introduced to replace G.
- It is natural to suggest a fundamental length scale
- Such constant can be explained as the smallest length scale of quantum space-time.
- Its value can be measured through searches of Lorentz violation.

LV-Window of Quantum Gravity

- The typical scale of quantum gravity is Planck mass

$$M_P = \sqrt{\frac{\hbar c}{G}} = 1.22089(6)\times 10^{19}\,\frac{\mathrm{GeV}}{\mathrm{c}^2} = 2.17644(11)\times 10^{-8}\,\mathrm{kg}$$

Lorentz Violation

might be a relic probe for quantum gravity

Pioneers' study of Lorentz symmetry violation

The early discussion on the effects of Lorentz violation

- Dirac's æther and nonlinear electrodynamics
  P.A.M. Dirac, Nature **168**, 906 (1951).
- Goldstone boson associated to Spontaneous Lorentz symmetry breaking(SLSB)
  ▷Bjorken's earlier attempts: Photon as Goldstone boson associated to SLSB. J.D. Bjorken, Ann.Phys. **24**, 174 (1963).
  ▷Is Graviton also a Goldstone boson?
  P.R. Phillips, Phys. Rev. **146**, 966 (1966)....
- An universal length scale
  T.G. Pavlopoulos, Phys. Rev. **159**, 1106 (1967)
- Nielsen's renomalization group calculation of the beta-function for a non-covariant pure Yang-Mills theory
  H.B. Nielsen and M. Ninomiya, Nucl. Phys. B **141**, 153 (1978). ...

Many possible ways for Lorentz violation

- spacetime foam [Ellis et al.'08, PLB]
- loop gravity [Alfaro et al.'00, PRL]
- torsion in general gravity [Yan'83, TP]
- vacuum condensate of antisymmetric tensor fields in string theory [Kostelecky & Samuel'89 & '91, PRL]
- double special relativity [Amelino-Camelia'02, Nature & '02 IJMPD]

Various theories on Lorenz violation

- Effective Field Theory
  - Standard Model Extension
    an explicitly introduction of condensation of background tensor field
    $\mathcal{L}_{LV} \sim \frac{\lambda}{M^k_{\text{Planck}}} < T > \overline{\psi}\Gamma(i\partial)^k\chi$
    D. Colladay and V.A. Kostelecký, Phys. Rev. D **58**, 116002 (1998).
- Non EFT
  - Double Special Relativity with two universal invariants:
    photon limiting velocity c,
    Planck length scale $l_{\text{Planck}} = 1.616 * 10^{-33}\text{cm}$
  - Stringy spacetime foam model
  - Dynamical critical exponent of space and time scaling
    $t \to \lambda^z t, \quad \vec{r} \to \lambda\vec{r}$
    Lorentz symmetry emergent at low energies as $z \to 1$
    P. Horava, Phys. Rev. D **79**, 084008 (2009); JHEP **020**, 0903 (2009).

Effective Field Theory
SME

- The total Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \delta\mathcal{L}, \tag{4}$$

where $\delta\mathcal{L}$ denotes tiny LV parts.

- take QED as example

$$\delta\mathcal{L}_{\rm QED} = \delta\mathcal{L}_{\rm photon} + \delta\mathcal{L}_{\rm electron}, \tag{5}$$

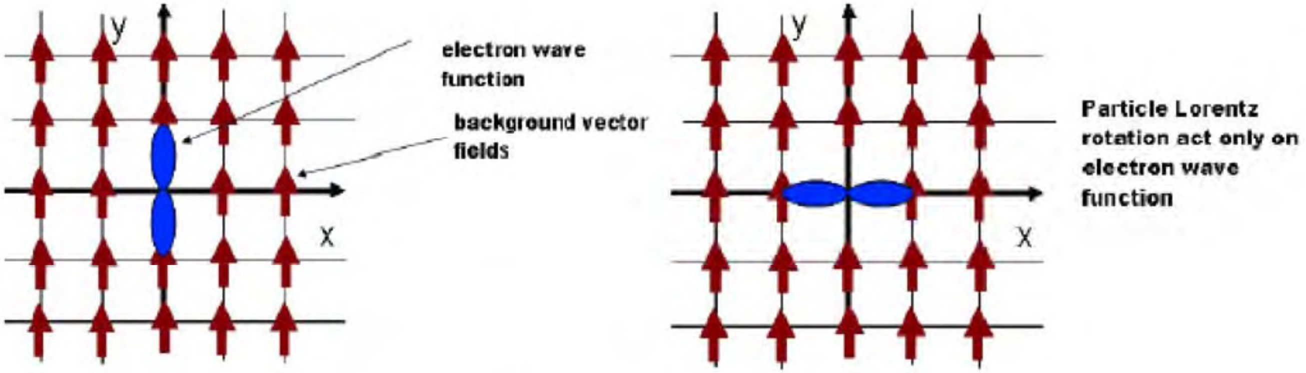
where

$$\delta\mathcal{L}_{\rm photon} \supset -\frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} + \frac{1}{2}(k_{AF})_\kappa \epsilon^{\kappa\lambda\mu\nu}A_\lambda F_{\mu\nu}, \tag{6}$$

$$\begin{aligned}\delta\mathcal{L}_{\rm electron} \supset\ & \frac{1}{2}i\overline{\psi}(\widetilde{c}^{(\nu\mu)}\gamma_\nu + \widetilde{d}^{\nu\mu}\gamma_5\gamma_\nu + \frac{1}{2}\widetilde{g}^{\lambda\nu\mu}\sigma_{\lambda\nu})\overleftrightarrow{D}_\mu\,\psi \\ & -\overline{\psi}(\widetilde{b}_\mu\gamma_5\gamma^\mu + \frac{1}{2}\widetilde{H}_{\mu\nu}\sigma^{\mu\nu})\psi.\end{aligned} \tag{7}$$

Effective Field Theory
SME

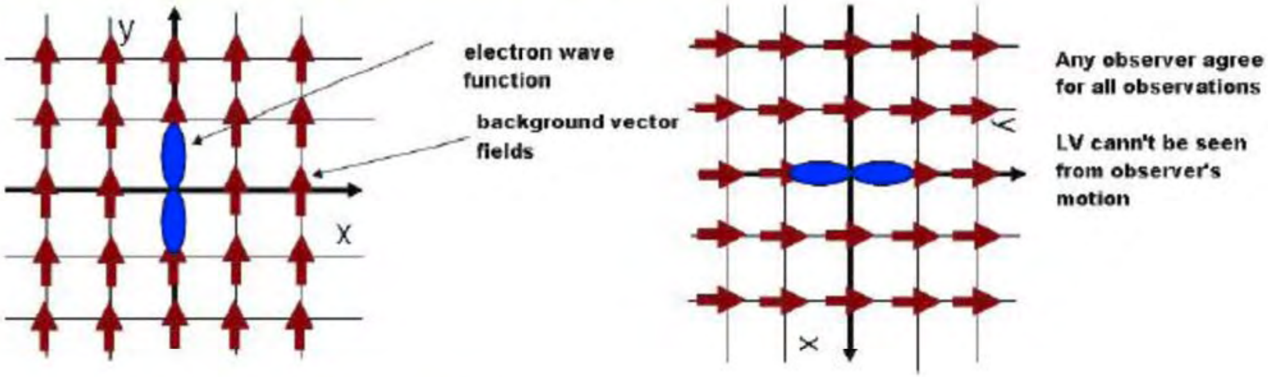
- Lorentz violation-conflict with covariance?
  - Particle (active) Lorentz rotation



Graph is obtained from T. Katori (MIT)

Where the background field is unalted under particle Lorentz rotation, as an tensor condensate.

- Lorentz violation-conflict with covariance?
    - Observer (passive) Lorentz rotation



Graph is obtained from T. Katori (MIT)

Effective Field Theory

SME

- Lorentz violation-conflict with covariance?

$$\overline{\psi}(x)(a^\nu\gamma_\nu)\psi(x) \to [U\overline{\psi}(x)U^{-1}]([Ua^\nu U^{-1}][U\gamma_\nu U^{-1}])[U\psi(x)U^{-1}]$$
$$= [\overline{\psi}(\Lambda x)S^{-1}](a^\nu[\Lambda^\rho_{\ \nu}\gamma_\rho])[S\psi(\Lambda x)] = \overline{\psi}(\Lambda x)(a^\nu\Lambda^\rho_{\ \nu}\gamma_\rho)\psi(\Lambda x)$$

So Lorentz violation can be incorporated in a covariant form

Lorentz invariance breaks;

But Lorentz covariance works.

So we call it Lorentz invariance violation (LV or LIV)

Consequences of Lorentz violation

- **Could provide explanation of neutrino oscillation without neutrino mass**

  S.Yang and B.-Q.Ma, IJMPA24(09)5861, arXiv:0910.0897

  Z.Xiao and B.-Q.Ma, IJMPA24(09)1539

- **Modified dispersion relation could increase threshold energy of photo-induced meson production of the proton: an increase of GZK cutoff energy**

  Z.Xiao and B.-Q.Ma, IJMPA24(09)1539

- **Modified dispersion relation may cause time lag of photons with different energies when they propagate in space from far-away astro-objects**

  Z.Xiao and B.-Q.Ma, PRD 80 (2009) 116005, arXiv:0909.4927,

  L.Shao, Z.Xiao and B.-Q.Ma, APP 33(2010)312, arXiv:0911.2276

- **Lorentz violation induced vacuum birefringence and its astrophysical consequences**

  L.Shao and B.-Q.Ma, PRD 83 (2011) 127702, arXiv:1104.4438

Standard model of neutrino oscillations

Neutrino oscillations

⇓

$$\Delta m^2 \neq 0$$

⇓

neutrinos have mass

A non-standard model for neutrino oscillations

Lorentz Violation (LV) in Neutrino Sector

- S. Coleman and S. L. Glashow, Phys. Lett. B 405 (1997) 249
- V. A. Kostelecky and M. Mewes, Phy. Rev. D 69 (2004) 016005
- N. Arkani-Hamed et al., JHEP 05 (2004) 074
- Y. Grossman et al., Phy. Rev. D 72 (2005) 125001
- P. Arias et al., Phys. Lett. B 650 (2007) 401
- D. Morgan and J. Brunner et al., hep-ph/0705.1987v2
- Z.Xiao and B.-Q.Ma, arXiv:0805.2012

Modified dispersion relations and neutrino oscillation

- LV may lead to the modifications of the dispersion relation

$$E^2 = p^2 + m^2 + \eta p^2 (E/E_p)^\alpha \qquad (2.1)$$

- The Hamiltonian in the mass basis, for two neutrino system, may be written, to first order in η as

$$H = \begin{pmatrix} \frac{m_1^2}{2E} + \frac{\eta_1 E^{\alpha+1}}{2} & 0 \\ 0 & \frac{m_2^2}{2E} + \frac{\eta_2 E^{\alpha+1}}{2} \end{pmatrix} \qquad (2.2)$$

- The oscillation probability is therefore (2.3)

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \cdot \sin^2[\frac{\Delta m^2 L}{4E} + \frac{\Delta \eta E^{\alpha+1} L}{4E_p^\alpha}]$$

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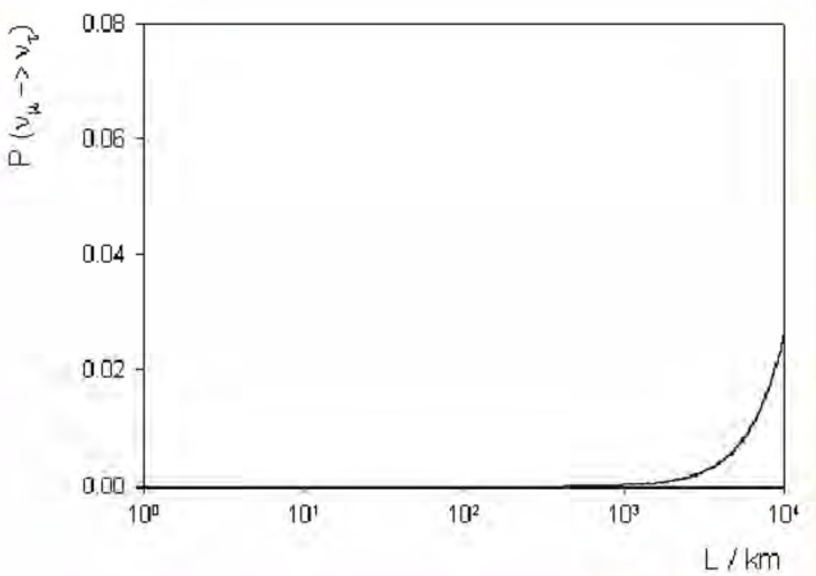
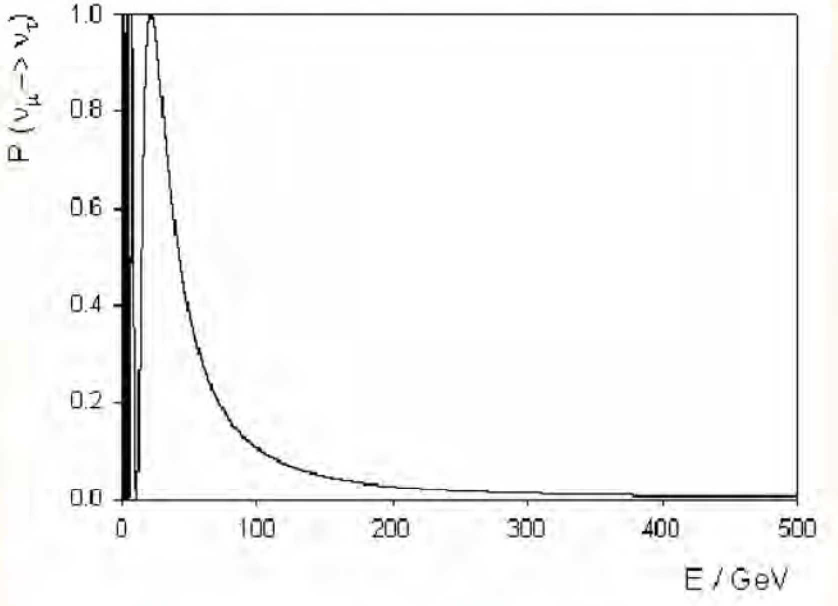
Modified dispersion relations and neutrino oscillation

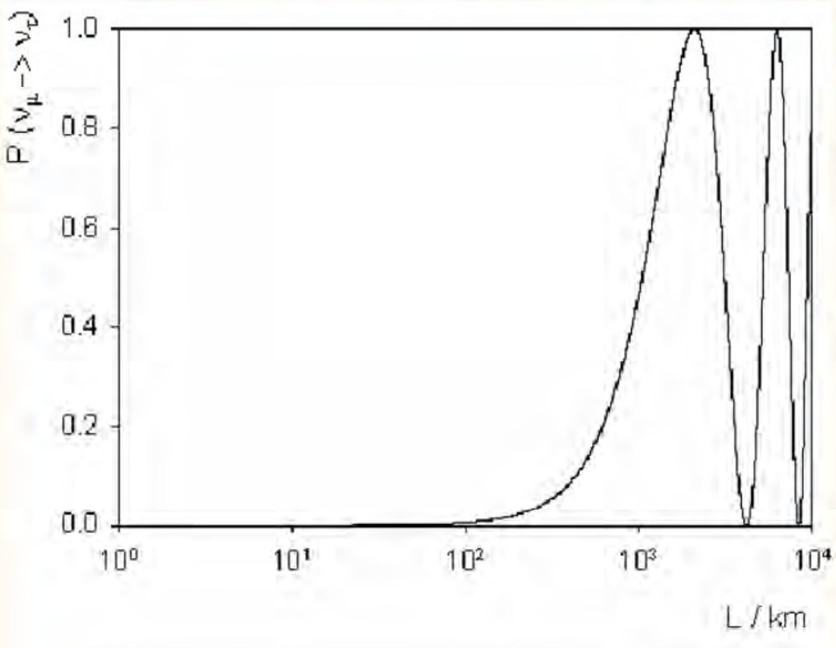
➢ Standard atmospheric neutrino oscillation probability

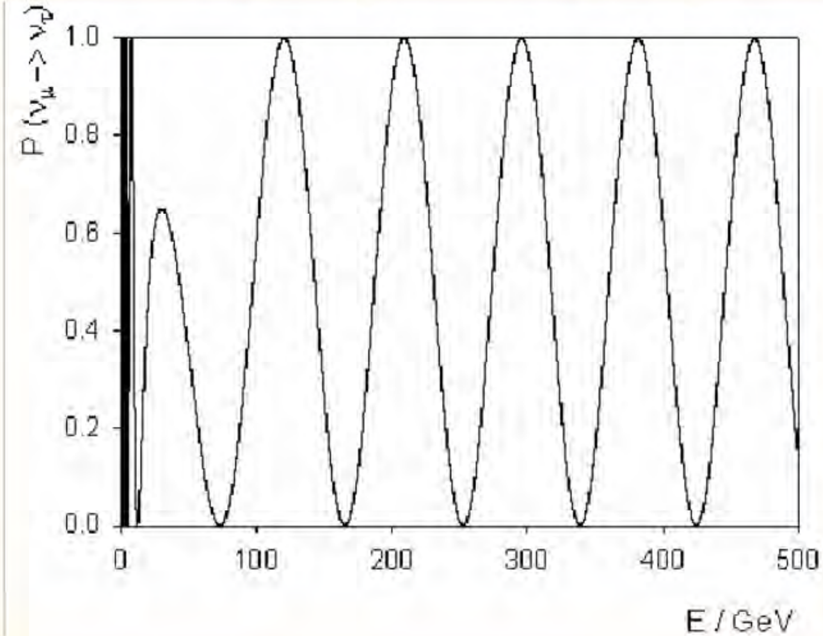




Modified dispersion relations and neutrino oscillation

- Atmospheric neutrino oscillation probabilities for $\alpha = 0$





S.Yang and B.-Q.Ma, IJMPA24(09)5861

Lorentz violation in three-family neutrino oscillations

The general equation for neutrino oscillation probabilities

- The Lagrangian density for neutrino sector

$$L = \frac{1}{2} i \bar{\nu}_A \gamma^\mu \overleftrightarrow{\partial}_\mu \nu_B \delta_{AB} + \frac{1}{2} i c^{\mu\nu}_{AB} \bar{\nu}_A \gamma^\mu \overleftrightarrow{\partial}^\nu \nu_B - a^\mu_{AB} \bar{\nu}_A \gamma^\mu \nu_B \qquad (3.1)$$

- From eq.(3.1), we figure out the Hamiltonian density

$$H = (-i \bar{\nu}_A \vec{\gamma} \cdot \nabla \delta_{AB} - i c^{\mu i}_{AB} \bar{\nu}_A \gamma^\mu \cdot \nabla_i) \nu_B + a^\mu_{AB} \bar{\nu}_A \gamma_\mu \nu_B \qquad (3.2)$$

- Transforming the description into quantum mechanics

$$\hat{H} = -i \gamma^0 \vec{\gamma} \cdot \nabla \delta_{AB} - i c^{\mu i}_{AB} \gamma^0 \gamma^\mu \cdot \nabla_i + a^\mu_{AB} \gamma^0 \gamma_\mu \qquad (3.3)$$

The general equation for neutrino oscillation probabilities

- Up to now, we have not detected right-handed neutrinos or left-handed anti-neutrinos, so we choose the basis vector as

$$(u_L(p), v_R(-p))^T \tag{3.4}$$

- We could get the dynaminal equation for neutrinos

$$\left(i\frac{\partial}{\partial t} - H_{AB} \right)\begin{pmatrix} a_B \\ b_B \end{pmatrix} = 0 \tag{3.5}$$

- The Hamiltonian matrix for neutrino given

$$H_{AB} = \begin{pmatrix} |p|\delta_{AB} + c_{AB}^{\mu j}\frac{p_\mu p_j}{|p|} + \frac{a_{AB}^{\mu} p_\mu}{|p|} & 0 \\ 0 & |p|\delta_{AB} + c_{AB}^{\mu j}\frac{p_\mu p_j}{|p|} + \frac{a_{AB}^{\mu} p_\mu}{|p|} \end{pmatrix} \tag{3.6}$$

The general equation for neutrino oscillation probabilities

- Diagonalizing H_{AB} to get the energy spectrum for neutrinos

$$\mathrm{E} = U^{+} H U \tag{3.7}$$

- The relationship between energy eignstates ν_i and flavor eigenstates ν_α

$$\left|\nu_\alpha\right\rangle = \sum_i U_{\alpha i}^{*} \left|\nu_i\right\rangle \tag{3.8}$$

- The general equation for neutrino oscillation

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}[(U^{+})_{i\alpha}^{*}(U^{+})_{i\beta}(U^{+})_{j\alpha}^{*}(U^{+})_{j\beta}]\sin^2[\frac{\Delta E_{ij}}{2}t]$$
$$+2\sum_{i>j} \mathrm{Im}[(U^{+})_{i\alpha}^{*}(U^{+})_{i\beta}(U^{+})_{j\alpha}^{*}(U^{+})_{j\beta}]\sin[\Delta E_{ij} t]$$

(3.9)

Special models

- Given

$$h = \begin{pmatrix} E & a & 0 \\ a & E+f & b \\ 0 & b & E \end{pmatrix} \quad (3.10)$$

$$a = c_{e\mu}^{0Z} E\cos\theta + a_{\mu\tau}^{0} \quad b = c_{\mu\tau}^{0Z} E\cos\theta + a_{\mu\tau}^{0} \qquad f = c_{\mu\mu}^{0Z} E\cos\theta + a_{\mu\mu}^{0}$$

- Diagonalizing h with unitary matrix U

$$\mathrm{E} = U^{+} h U \quad (3.11)$$

- where

$$\mathrm{E} = \begin{pmatrix} E & 0 & 0 \\ 0 & E + \frac{f}{2} - \frac{\sqrt{4a^2+4b^2+f^2}}{2} & 0 \\ 0 & 0 & E + \frac{f}{2} + \frac{\sqrt{4a^2+4b^2+f^2}}{2} \end{pmatrix} \quad (3.12)$$

Special models

- We work out the unitary matrix U

$$U^{+}=\begin{pmatrix} -\frac{b}{\sqrt{a^2+b^2}} & 0 & \frac{a}{\sqrt{a^2+b^2}} \\ \frac{a}{\mathrm{M}} & \frac{f-\sqrt{4a^2+4b^2+f^2}}{2\mathrm{M}} & \frac{b}{\mathrm{M}} \\ \frac{a}{\mathrm{N}} & \frac{f+\sqrt{4a^2+4b^2+f^2}}{\mathrm{N}} & \frac{b}{\mathrm{N}} \end{pmatrix} \qquad (3.13)$$

- where

$$\mathrm{M}=\sqrt{2(a^2+b^2)+\frac{f^2}{2}-\frac{f\sqrt{4a^2+4b^2+f^2}}{2}}$$

$$\mathrm{N}=\sqrt{2(a^2+b^2)+\frac{f^2}{2}+\frac{f\sqrt{4a^2+4b^2+f^2}}{2}}$$

Special models

- The probabilities for neutrino oscillations

$$P(\nu_e \to \nu_e) = 1$$

$$-\frac{4a^2b^2}{(a^2+b^2)[2(a^2+b^2)+\frac{f^2}{2}-\frac{f\sqrt{4(a^2+b^2)+f^2}}{2}]}\sin^2[(\frac{f}{4}-\frac{\sqrt{4(a^2+b^2)+f^2}}{4})t]$$

$$-\frac{4a^2b^2}{(a^2+b^2)[2(a^2+b^2)+\frac{f^2}{2}+\frac{f\sqrt{4(a^2+b^2)+f^2}}{2}]}\sin^2[(\frac{f}{4}+\frac{\sqrt{4(a^2+b^2)+f^2}}{4})t]$$

$$-\frac{4a^4}{(a^2+b^2)(4a^2+4b^2+f^2)}\sin^2[\frac{\sqrt{4a^2+4b^2+f^2}}{2}t] \qquad (3.14)$$

Special models

- The probabilities for neutrino oscillations

$$P(\nu_e \to \nu_\mu) = \frac{4a^2}{4a^2 + 4b^2 + f^2} \cdot \sin^2[\frac{\sqrt{4a^2 + 4b^2 + f^2}}{2} t] \quad (3.15)$$

$$P(\nu_\mu \to \nu_\mu) = 1 - \frac{4a^2 + 4b^2}{4a^2 + 4b^2 + f^2} \cdot \sin^2[\frac{\sqrt{4a^2 + 4b^2 + f^2}}{2} t] \quad (3.16)$$

$$P(\nu_\mu \to \nu_\tau) = \frac{4b^2}{4a^2 + 4b^2 + f^2} \cdot \sin^2[\frac{\sqrt{4a^2 + 4b^2 + f^2}}{2} t] \quad (3.17)$$

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Special models

$$P(\nu_e \to \nu_\tau) = \frac{4a^2b^2}{(a^2+b^2)[2(a^2+b^2)+\frac{f^2}{2}-\frac{f\sqrt{4(a^2+b^2)+f^2}}{2}]} \cdot \sin^2[(\frac{f}{4}-\frac{\sqrt{4(a^2+b^2)+f^2}}{4})t]$$

$$+\frac{4a^2b^2}{(a^2+b^2)[2(a^2+b^2)+\frac{f^2}{2}+\frac{f\sqrt{4(a^2+b^2)+f^2}}{2}]} \cdot \sin^2[(\frac{f}{4}+\frac{\sqrt{4(a^2+b^2)+f^2}}{4})t]$$

$$-\frac{4a^2b^2}{(a^2+b^2)(4a^2+4b^2+f^2)} \cdot \sin^2[\frac{\sqrt{4a^2+4b^2+f^2}}{2}t]$$

Experimental Results

- Kamland

$P(\nu_e \rightarrow \nu_e) = 61\%$ E=4.3MeV L=180km

- MINOS

$P(\nu_\mu \rightarrow \nu_\mu) = 76\%$ E=4.9GeV L=735km

- K2K

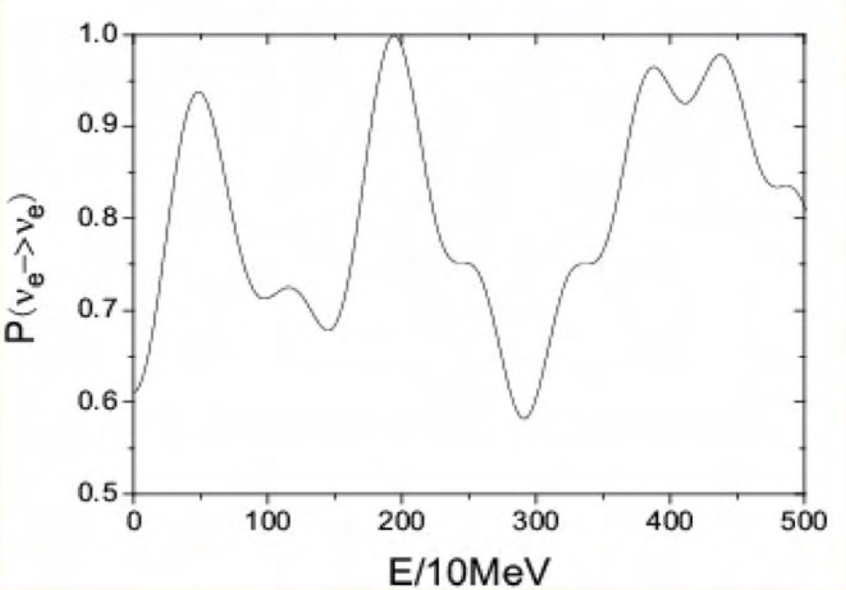
$P(\nu_\mu \rightarrow \nu_\tau) = 36\%$ E=1.8GeV L=250km

- K2K

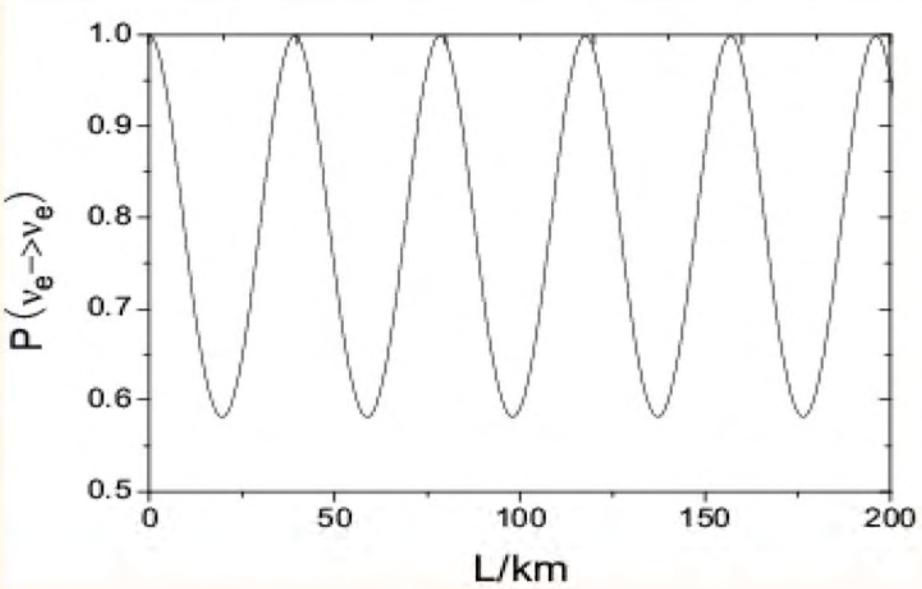
$\nu_\mu \rightarrow \nu_\tau$ $\sin^2 2\theta_{\mu e} < 0.13$

Model 1

- Given $a = a^{0}_{e\mu}$ $b = a^{0}_{\mu\tau}$ $f = c^{0Z}_{\mu\mu} E \cos\theta$



Propagation length L equal to 180km

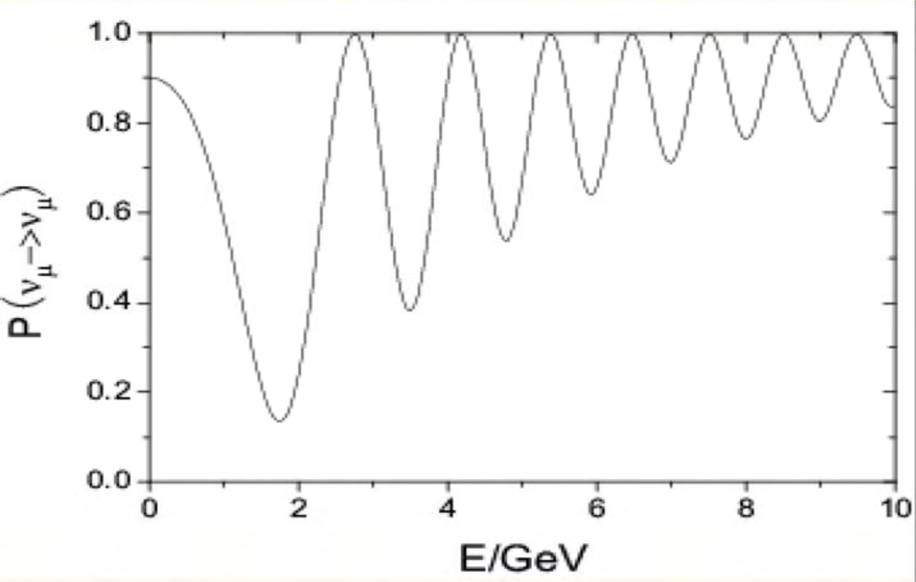


Neutrino energy equal to 10MeV

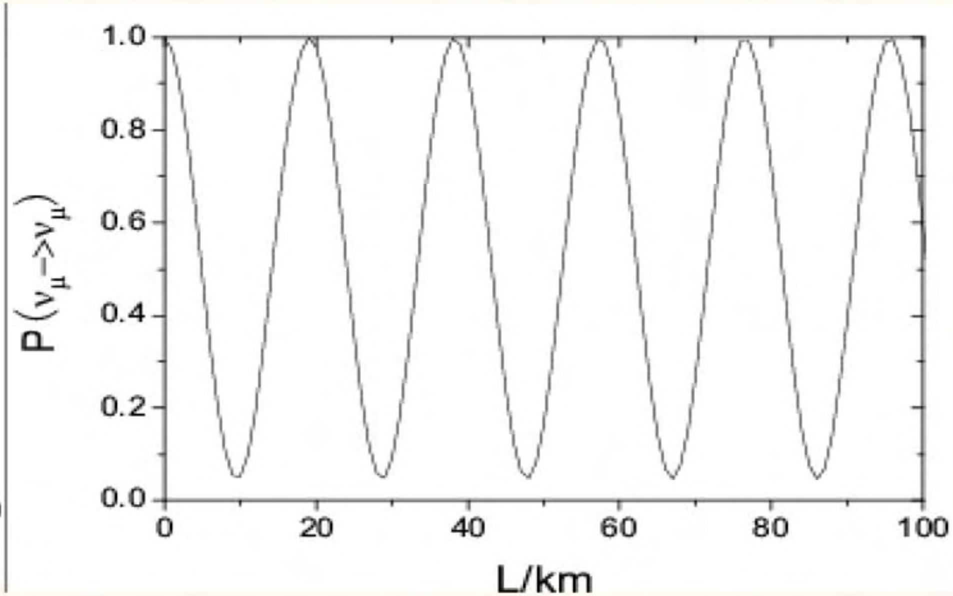
Model 1

- Given

$$a^0_{e\mu} = 1.09\times10^{-11} eV \qquad a^0_{\mu\tau} = 2.97\times10^{-11} eV \qquad c^{0Z}_{\mu\mu} = 1.42\times10^{-20}$$



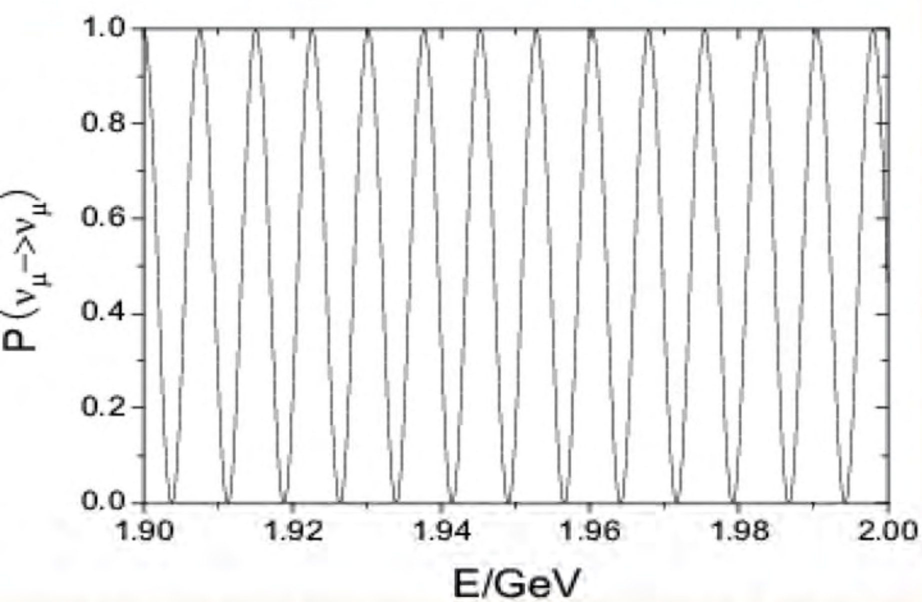
Propagation length L equal to 100km



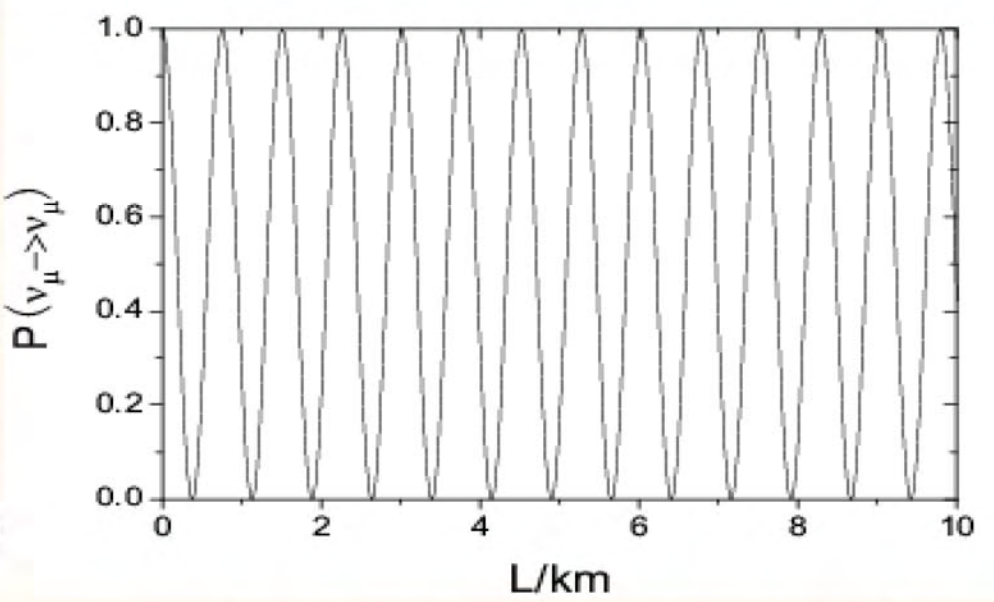
Neutrino energy equal to 1GeV

Model 2

- Given $a = c_{e\mu}^{0Z} E\cos\theta$ $b = c_{\mu\tau}^{0Z} E\cos\theta$ $f = a_{\mu\mu}^{0}$



Propagation length L equal to 200km

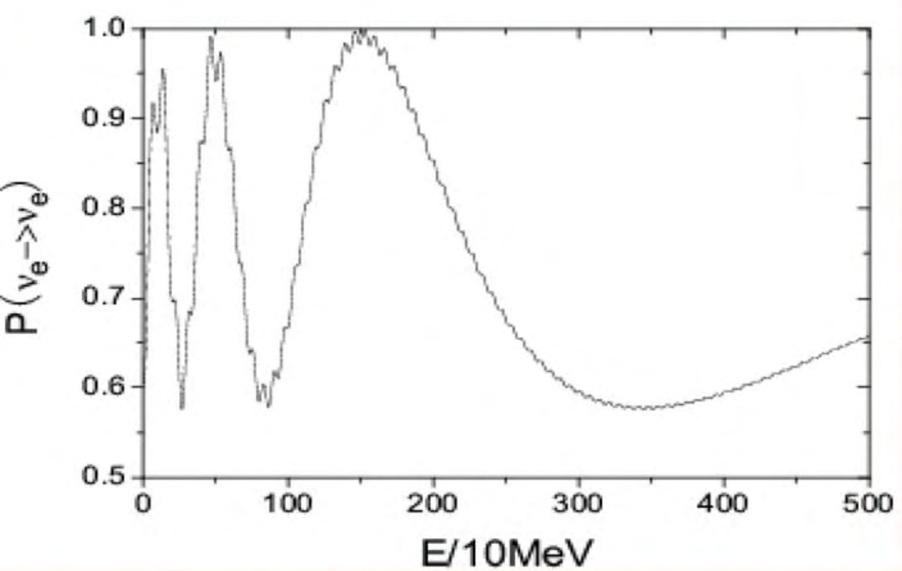


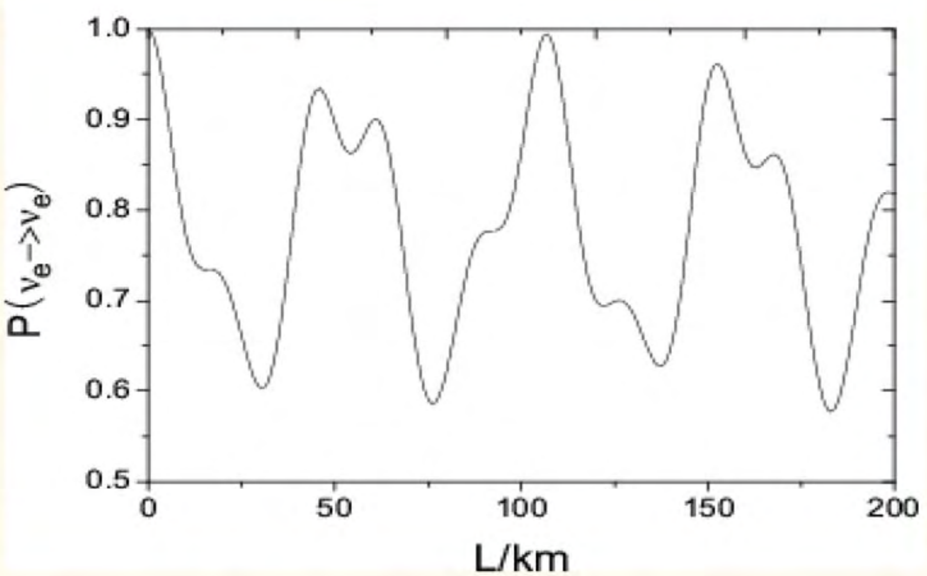
Neutrino energy equal to 10MeV

Model 3

- Given

$$a = a^0_{e\mu} \qquad b = a^0_{\mu\tau} \qquad b = xa \qquad f = a^0_{\mu\mu} + c^{0Z}_{\mu\mu} E\cos\theta$$



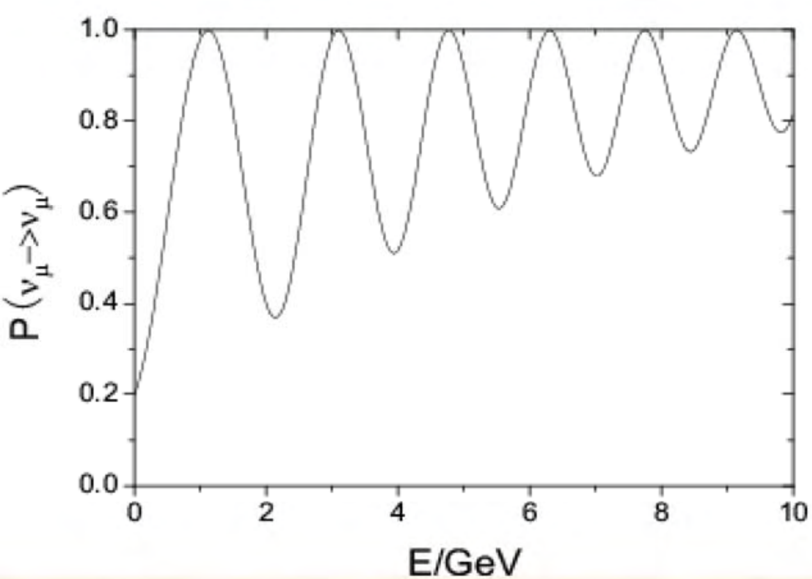


Propagation length L equal to 180km

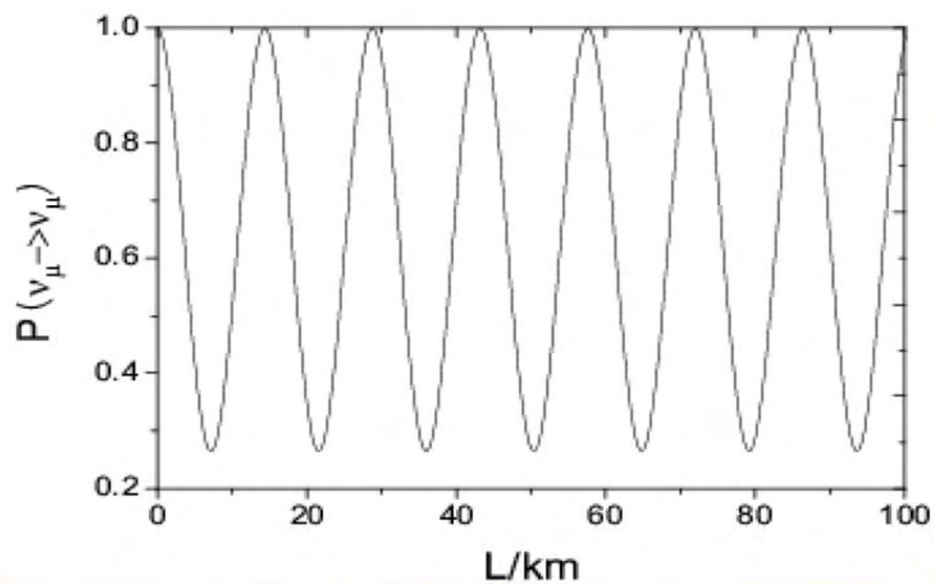
Neutrino energy equal to 10MeV

Model 3

- Given $a^{0}_{e\mu} = 1.28\times10^{-11}eV$ $a^{0}_{\mu\mu} = 3.40\times10^{-11}eV$ $c^{0Z}_{\mu\mu} = 1.04\times10^{-20}$



Propagation length L equal to 100km



Neutrino energy equal to 1GeV

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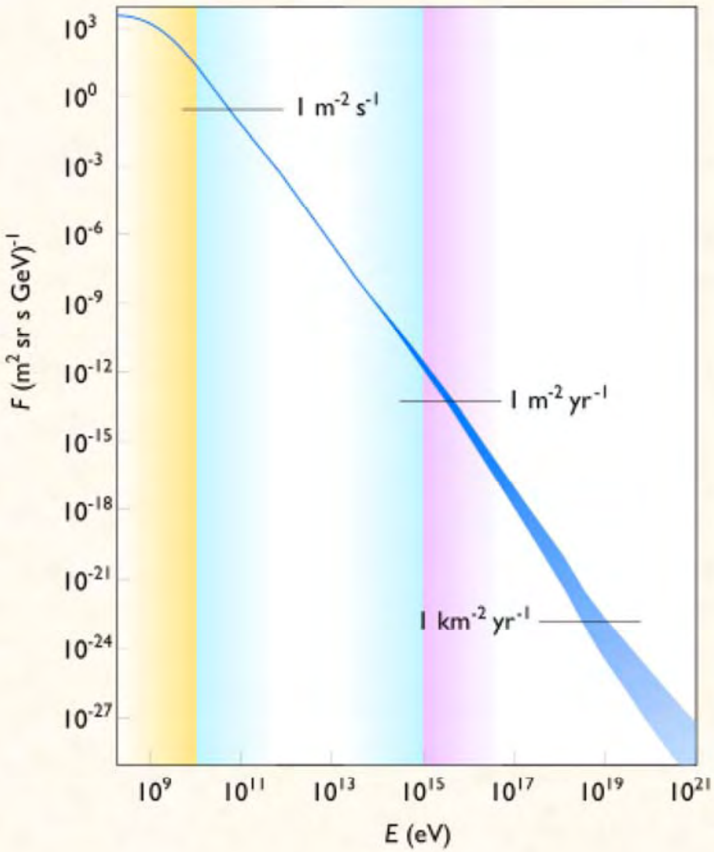
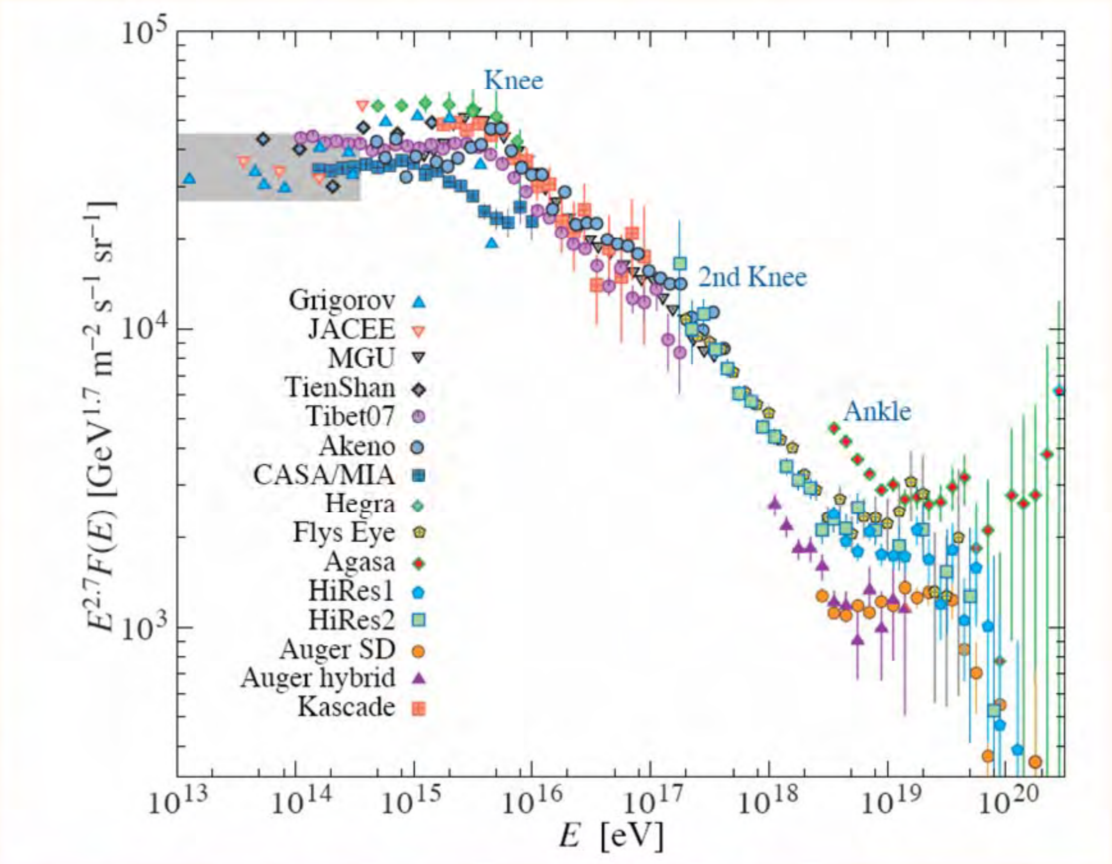
Neutrino oscillation for Lorentz violation:

- We carried out Lorentz violation contribution to neutrino oscillation by the effective field theory for LV and give out the equations of neutrino oscillation probabilities.
- In our model, neutrino oscillations do not have drastic oscillation at low energy and oscillations still exist at high energy.
- Neutrinos may have small mass and both LV and the conventional oscillation mechanisms contribute to neutrino oscillation.

S.Yang and B.-Q.Ma, IJMPA 24 (09) 5861, arXiv:0910.0897
Z.Xiao and B.-Q.Ma, IJMPA 24 (09) 1539

The energy spectrum of cosmic rays





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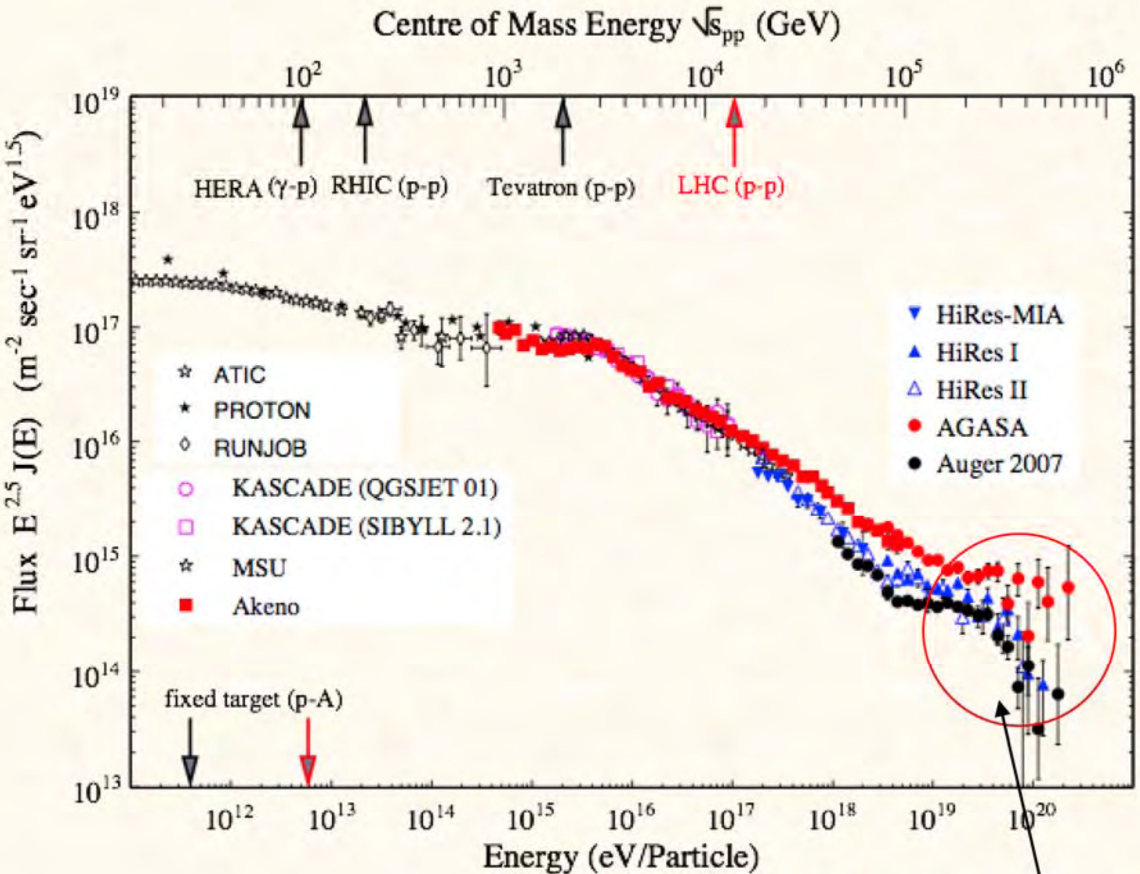
Ultra-high energy cosmic rays (UHECRS)



Ultra-high energy cosmic rays

- E>10^18-10^19 eV
- Extragalactic origin above the ankle

Energy=50J, the same as a well-hit tennis ball at 42 m/s.

Cosmic microwave background (CMB)

Discovered in 1965 by Penzias and Wilson

as evidence of relic photons from the big bang

$\varepsilon_\gamma = 6.35 \times 10^{-4}$ eV

temperature $T = 2.73\,\mathrm{K}$

photon number density $n_\gamma = 413\,\text{photon/cm}^3$

mean energy per photon

Greisen-Zatsepin-Kuzman (GZK) cutoff energy of nucleon cosmic rays

predicted in 1966

pion production $N + \gamma_{CMB} \rightarrow \pi + N$

$$E = \frac{S - m_\pi^2}{2\varepsilon_\gamma (1 - \cos\theta)}$$

threshold energy $E \approx \dfrac{2m_N m_\pi + m_\pi^2}{4\varepsilon_\gamma} = 1.10 \times 10^{20}\ \text{eV}$

$\sim 50\,\text{Mpc}$

mean free path $\lambda_N \sim 3\,\text{Mpc}$ GZK zone

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Energy limitation of other particles

- photon

$$\gamma + \gamma_{CMB} \to e^+ + e^-$$

$$4E\varepsilon_\gamma \approx (2m_e)^2 \qquad E \sim 4\times 10^{14}\ \text{eV}$$

- electron

$$e + \gamma_{CMB} \to e + \gamma$$

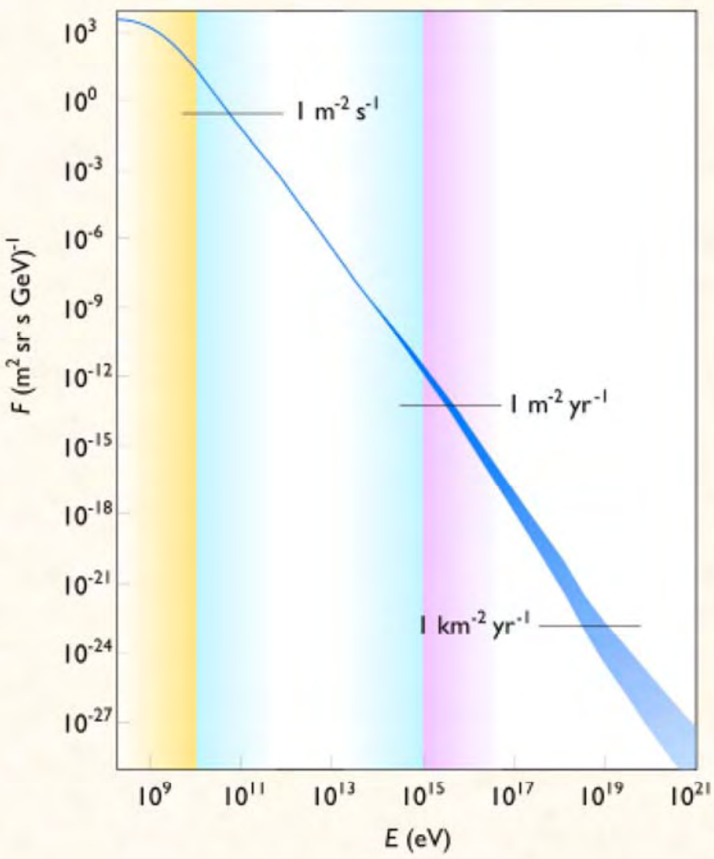
- neutrino

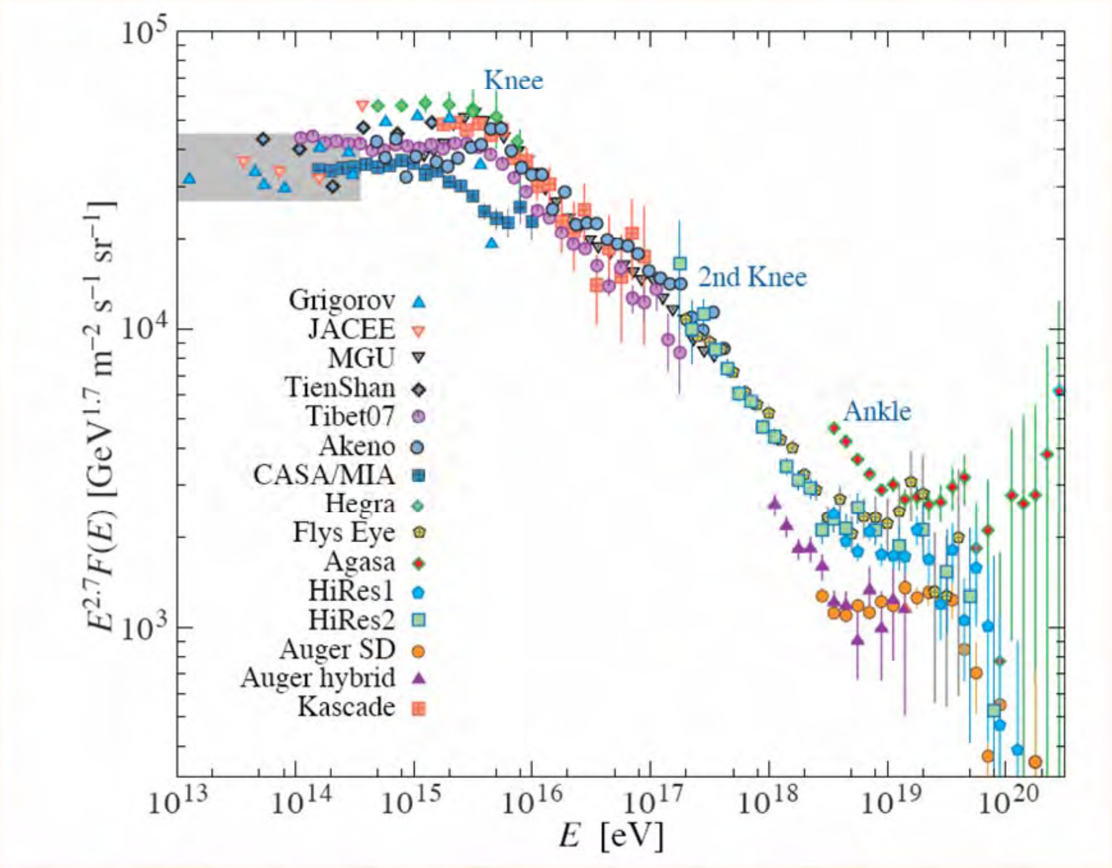
$$\nu + \bar{\nu} \to Z \to q + \bar{q} \to N + \overline{N} + \gamma + X$$

$$E = \frac{M_Z^2}{2m_\nu} \sim 4\times 10^{21}\left(\frac{1\,\text{eV}}{m_\nu}\right)\ \text{eV}$$

A way for possible detection of relic neutrinos

The energy spectrum of cosmic rays





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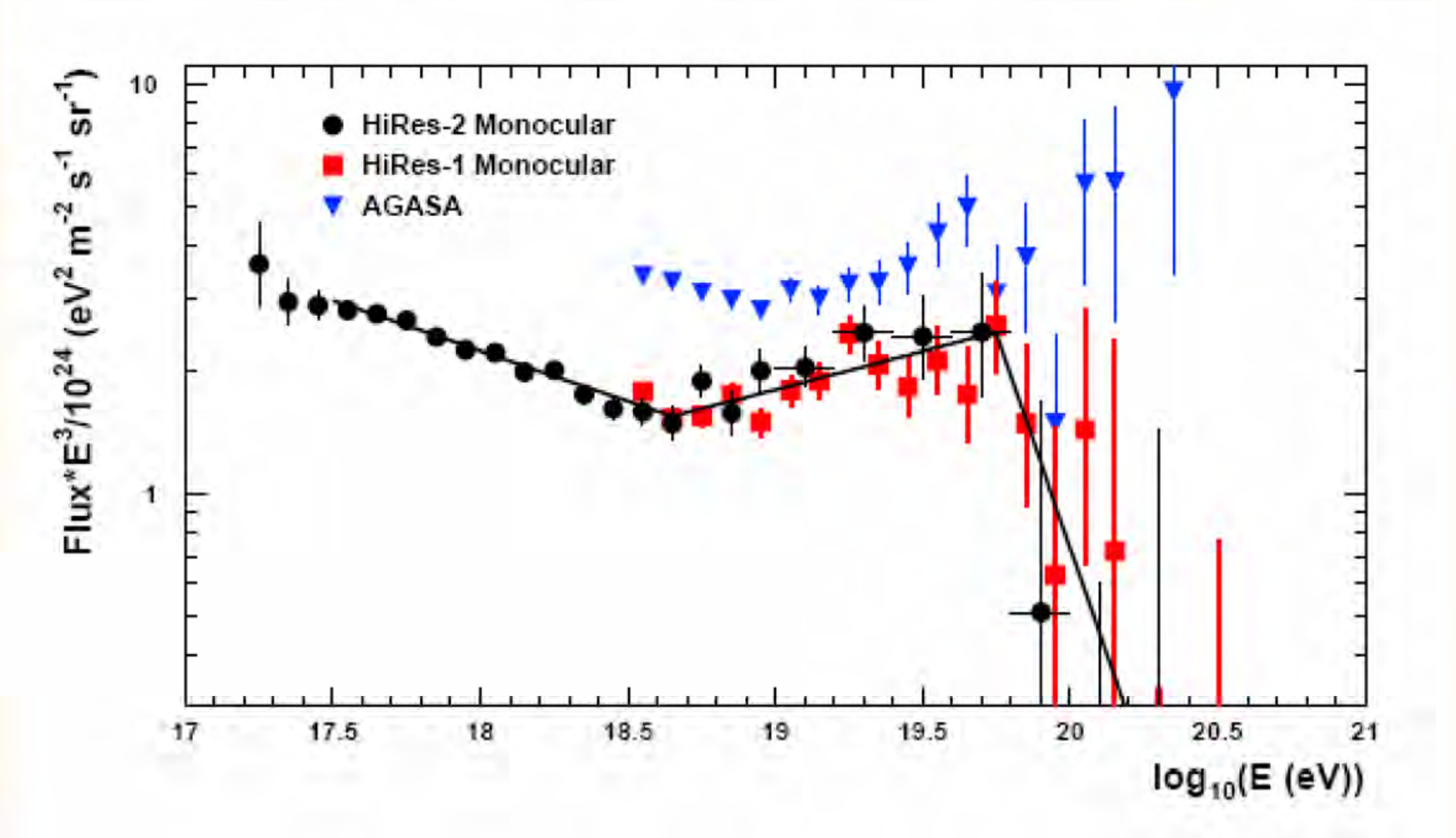
Proposed origins of super GZK events ---before 2007

- Z-bursts
- Decay of relic superheavy particles in galactic halo
- Heavy primaries: i.e., iron
- New hadrons
- Violation of Lorentz symmetry
- Photons with photon-axion mixing
- …

However, new experimental results appeared: HiRes 2007



HiRes, Phys.Rev.Lett.100:101101,2008.

Lorentz Violation & Super-GZK events

- The earlier reports on super-GZK events triggered attention on Lorentz-Violation (LV or LIV).

  S.R.Coleman and S.L.Glashow, PRD 59 (1999) 116008

- The new results of observation of GZK cut-off put strong constraints on Lorentz-Violation parameters, see, e.g.,

  Z.Xiao, B.-Q. Ma, IJMPA 24 (2009) 1539.
  X.J.Bi, Z.Cao, Y.Li, Q.Yuan , PRD 79 (2009) 083015.
  F.W.Stecker, S.T.Scully, New J.Phys.11(2009) 085003.

Lorentz Violation as a mechanism for Super-GZK events *Coleman&Glashow*

- Starting from a free field Lagrangian,

$$\mathcal{L} = \partial_\mu \Psi^* Z \partial^\mu \Psi - \Psi^* M^2 \Psi,$$

and adding a LV term $\mathcal{L} \to \mathcal{L} + \partial_i \Psi \epsilon \partial^i \Psi,$

- Modified dispersion relation with LV effect

$$p^2 = E^2 - \vec{p}^2 = m^2 + \epsilon \vec{p}^2 \ .$$

$$c_a = \sqrt{1+\epsilon} c$$

$$E_a^2 = \vec{p}_a^2 c_a^2 + m_a^2 c_a^4.$$

$$m_a = m/(1+\epsilon)$$

where c_a is the maximal attainable velocity for the ath particle

Lorentz violation & enhancement of threshold energy

- Take the nucleon-photon to Delta process as example

$$P + \gamma(\mathrm{CMB}) \rightarrow \triangle(1232) \qquad \omega + E_p \geq E_\triangle$$

- With LV effect

$$\omega + E_p \geq \sqrt{(|\vec{P}_p| - \omega)^2 c_\triangle^2 + m_\triangle^2 c_\triangle^4}\,.$$

$$E_p = \frac{\omega\sqrt{1 - 1/2\frac{K}{\omega^2}\left(1 - \frac{c_\triangle}{c_p}\right)} - \omega}{1 - \frac{c_\triangle}{c_p}} \simeq -\frac{K}{4\omega} - \frac{K^2}{32\omega^3}\left(1 - \frac{c_\triangle}{c_p}\right) + \cdots$$

$$1 - \frac{c_\triangle}{c_p} = -\frac{2w(E_p - E_{\mathrm{thre}})}{E_{\mathrm{thre}}^2}. \qquad \text{simply assume } c_\triangle = 1$$

Z.Xiao, B.-Q. Ma, IJMPA 24 (2009) 1539.

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Constraints on LV parameters

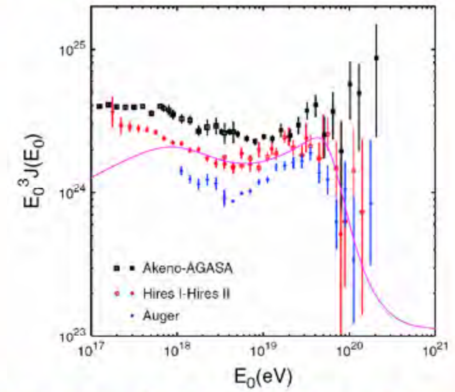
- *Z.Xiao, B.-Q. Ma, IJMPA 24 (2009) 1539.*

A rough estimate $\epsilon_p \sim 10^{-24}$

- *X.J.Bi, Z.Cao, Y.Li, Q.Yuan , PRD 79 (2009) 083015.*

An analysis with shape

$$\epsilon_p \sim 10^{-23}$$



- *F.W.Stecker, S.T.Scully, New J.Phys.11(2009) 085003.*

$$\epsilon_p \sim 10^{-23}$$

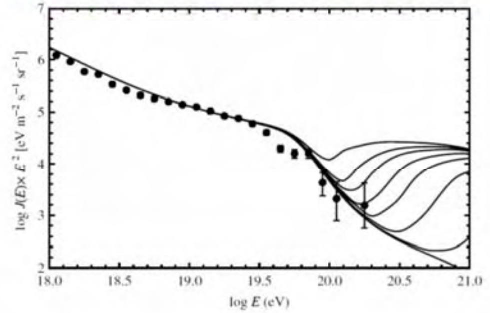


Figure 4. Comparison of the latest Auger data with calculated spectra for various values of $\delta_{\pi p}$, taking $\delta_p = 0$ (see text). From top to bottom, the curves give the predicted spectra for $\delta_{\pi p} = 1 \times 10^{-22}, 6 \times 10^{-23}, 4.5 \times 10^{-23}, 3 \times 10^{-23}, 2 \times 10^{-23}, 1 \times 10^{-23}, 3 \times 10^{-24}$ and 0 (no Lorentz violation) [44].

LV from cosmological VHE photon emissions

Z.Xiao and B.-Q.Ma, PRD 80 (09) 116005, arXiv:0909.4927

L.Shao, Z.Xiao and B.-Q.Ma, APP 33(10)312, arXiv:0911.2276

Modified photon dispersion relation from LV

$$v(E) = c_0 \left(1 - \xi \frac{E}{M_{\mathrm{P}} c^2} - \zeta \frac{E^2}{M_{\mathrm{P}}^2 c^4}\right)$$

$$\sqrt{\hbar c / G} \simeq 1.22 \times 10^{19} \ \mathrm{GeV/c^2}$$

For reviews, see, e.g.,

Jacobson et al.'06, Ann. Phys.
Kostelecky & Mewes'09, PRD
Mattingly'05, Living Rev. Rel.
Amelino-Camelia & Smonlin'09, PRD

Formulas in our analysis of LV paramter

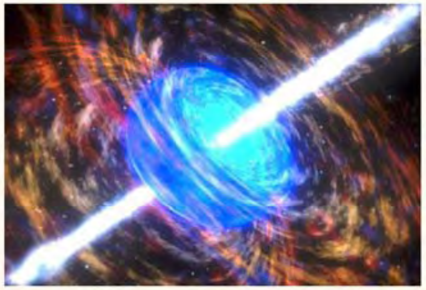
linear and quadratic energy dependence

$$v(E) = c_0 \left(1 - \frac{E}{M_{\mathrm{QG}} c^2}\right)$$

$$v(E) = c_0 \left(1 - \frac{E^2}{M_{\mathrm{QG}}^2 c^4}\right)$$

Gammy-ray Bursts (GRBs)

- the most energetic astrophysical process except the Big Bang
- 2 types [Piran'05, Rev. Mod. Phys.]
  - long GRBs: duration > 2 s; collapses of massive rapidly rotating stars
  - short GRBs: duration < 2 s; coalescence of two neutron stars or a neutron star and a black hole
- use GRBs to test LV [Amelino-Camelia et al.'98, Nature]



Time lag by LV effect

- expansion universe [Jacob & Piran'08, JCAP]

$$\Delta t_{\rm LV} = \frac{1+n}{2H_0}\left(\frac{E_{\rm h}^n - E_{\rm l}^n}{M_{\rm QG}^n c^{2n}}\right)\int_0^z \frac{(1+z')^n {\rm d}z'}{h(z')}$$

$$M_{\rm QG,L} = |\xi|^{-1} M_{\rm P} \;\; \text{and} \;\; M_{\rm QG,Q} = |\zeta|^{-1/2} M_{\rm P}$$

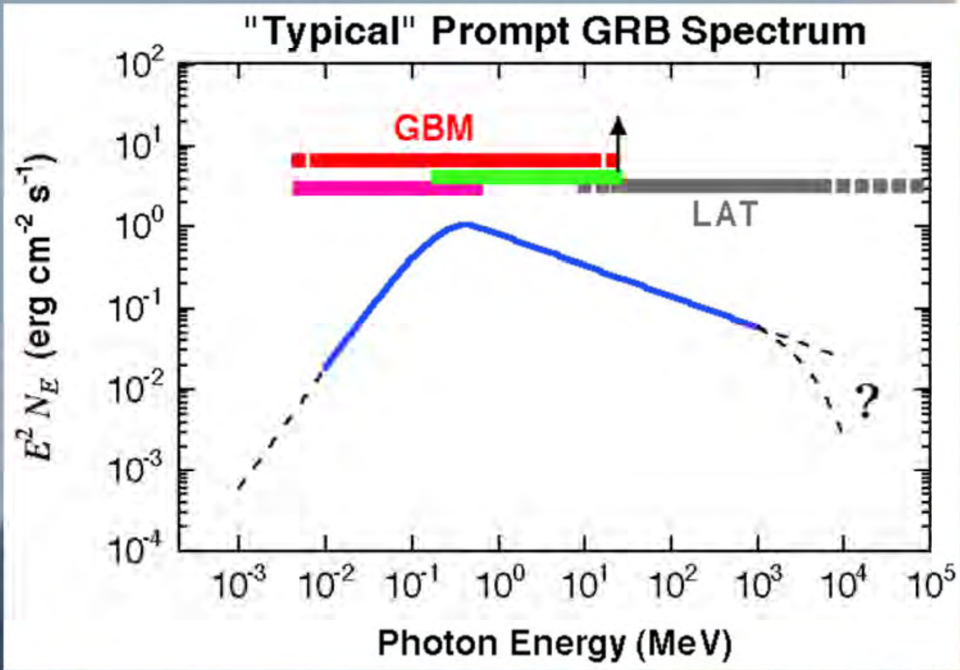
$$h(z) = \sqrt{\Omega_\Lambda + \Omega_{\rm M}(1+z)^3}$$

$$H_0 \simeq 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

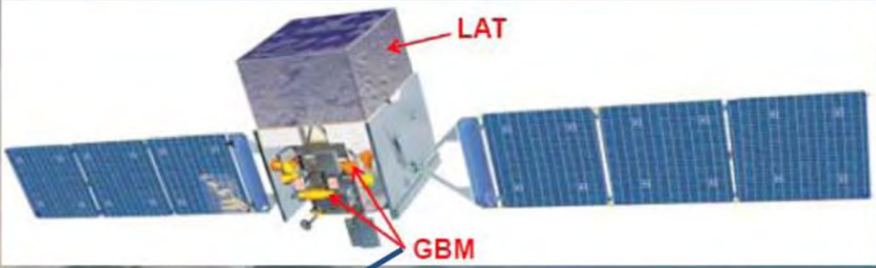
$$\Omega_\Lambda \simeq 0.73 \;\; \Omega_{\rm M} \simeq 0.27$$

June 11, 2008

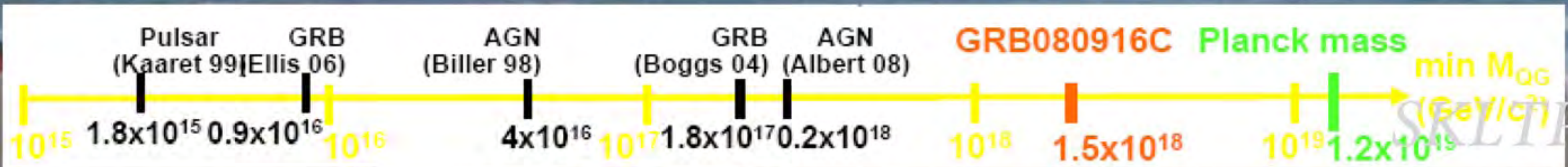
Fermi instruments



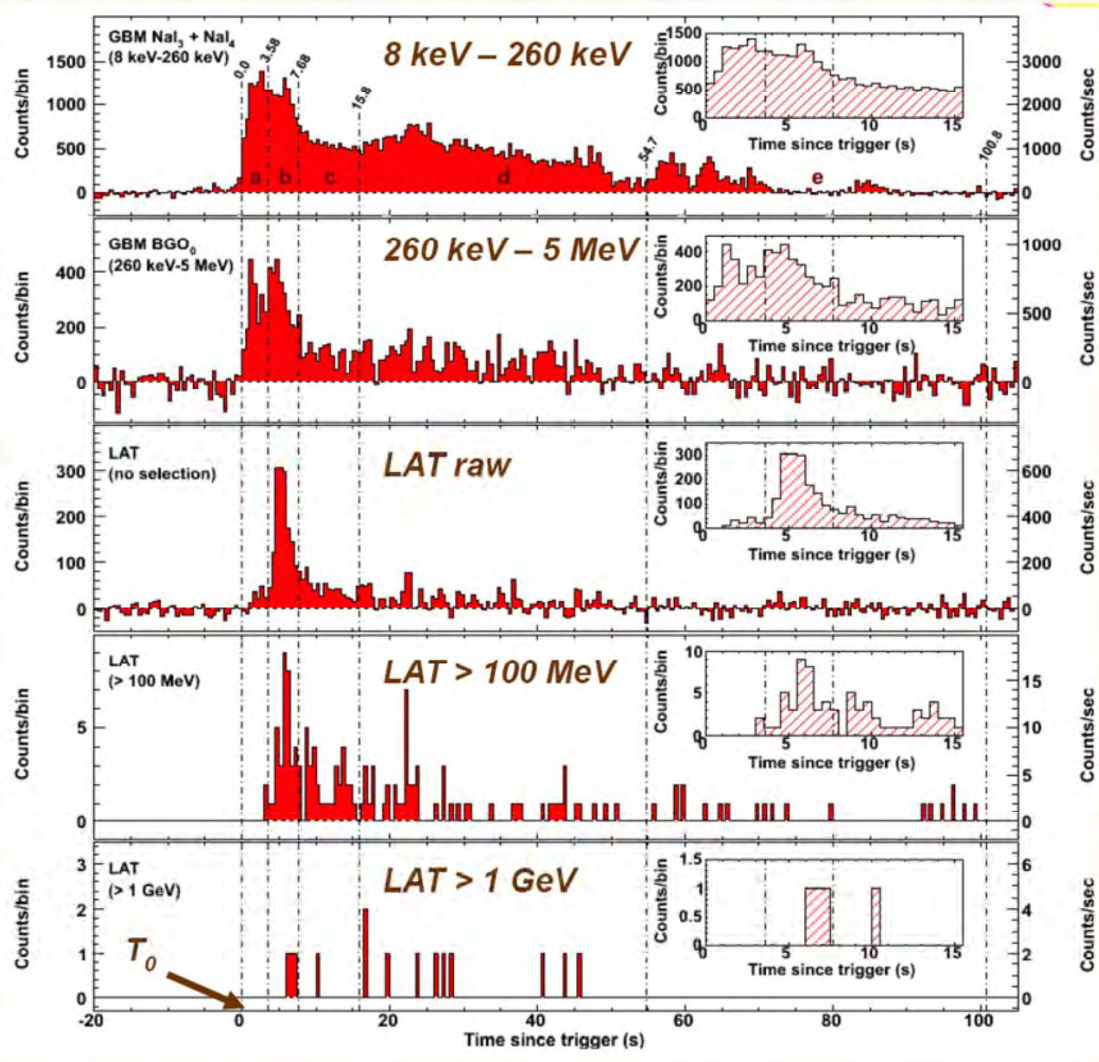
~ 300 GeV



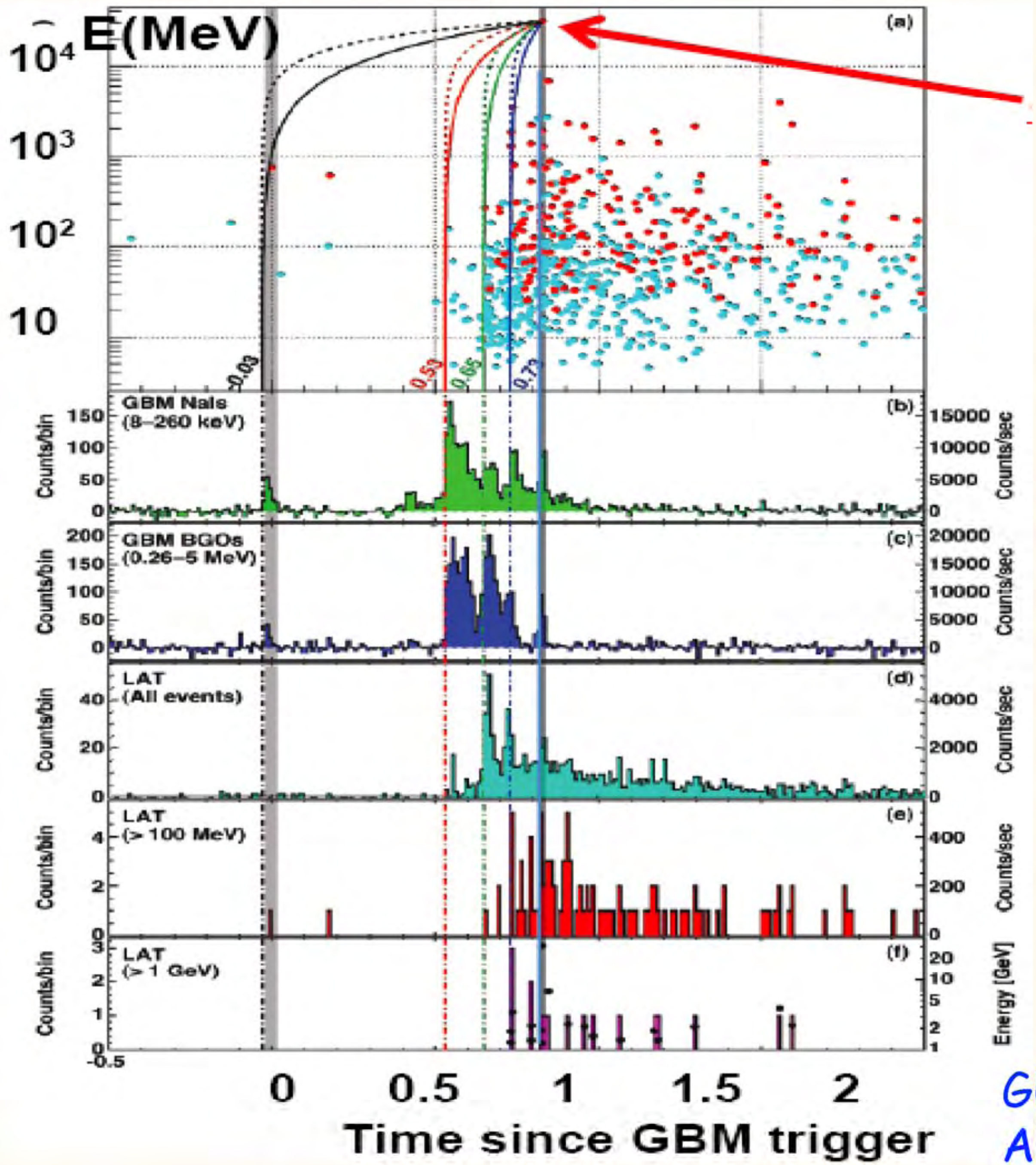
trigger photons ~ 0.1 MeV



Lag determinations



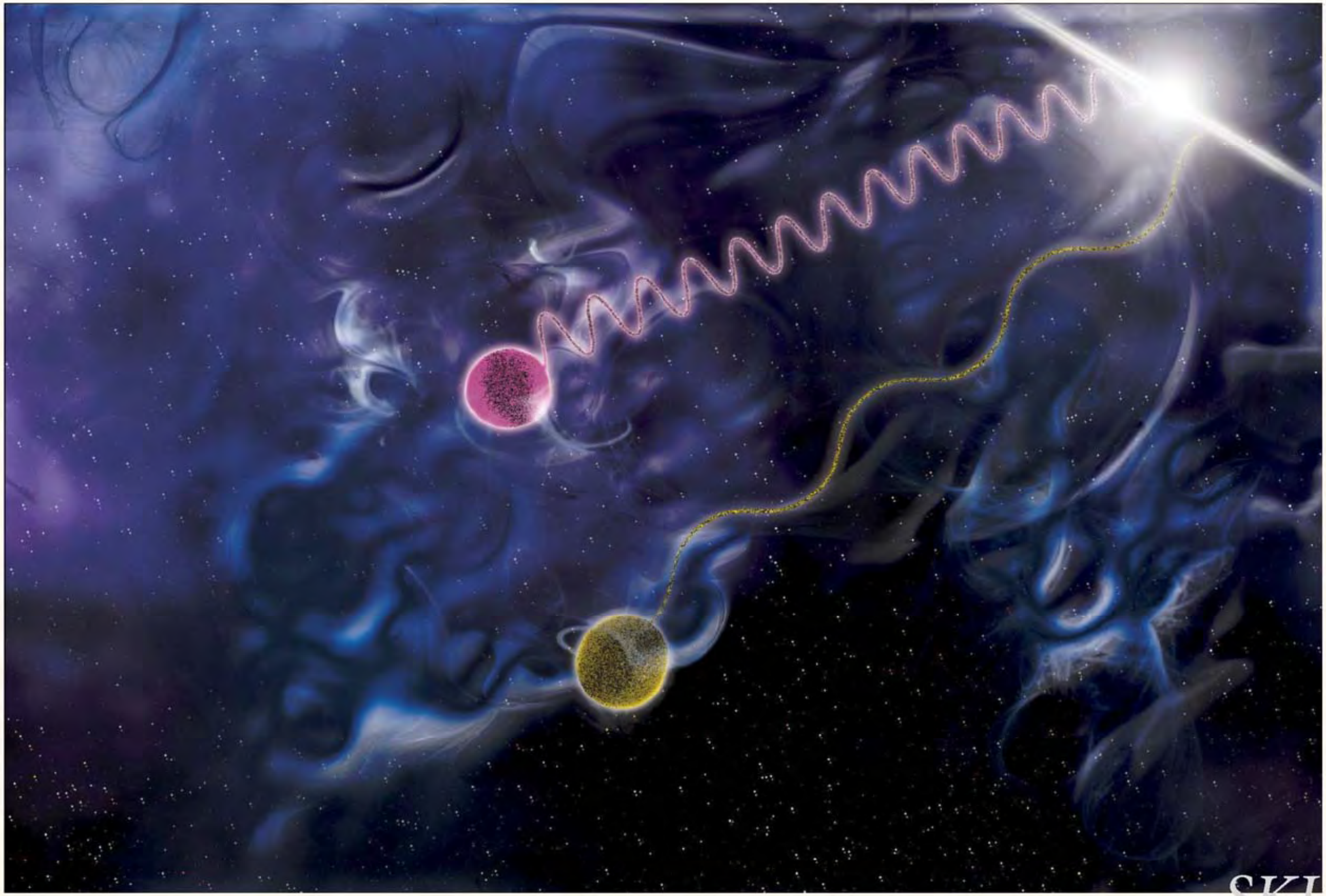
GRB080916C -- Abdo et al.'09, Science



Time lags are affected both artificially and instrumentally

GRB090510
Abdo et al.'09, Nature

Time-lag by GRB



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Four Fermi observations

the arrival of the highest energy photon to GBM trigger

GRBs	z	E (GeV)	$\Delta t_{\rm obs}$ (s)	$M_{\rm QG,L}$ (GeV/c^2)	$M_{\rm QG,Q}$ (GeV/c^2)
080916C [19]	4.35 [21]	13.22	16.54	1.5×10^{18}	9.7×10^{9}
090510 [20]	0.903 [22]	31	0.829	1.7×10^{19}	3.4×10^{10}
090902B [23]	1.822 [24]	33.4	82	3.7×10^{17}	5.9×10^{9}
090926A [25]	2.1062 [26]	19.6	26	7.8×10^{17}	6.8×10^{9}

$$\Delta t_{\rm obs} = \Delta t_{\rm LV}$$

$$M_{\rm QG,L} \sim (4.9 \pm 8.1) \times 10^{18}\ \mathrm{GeV}$$

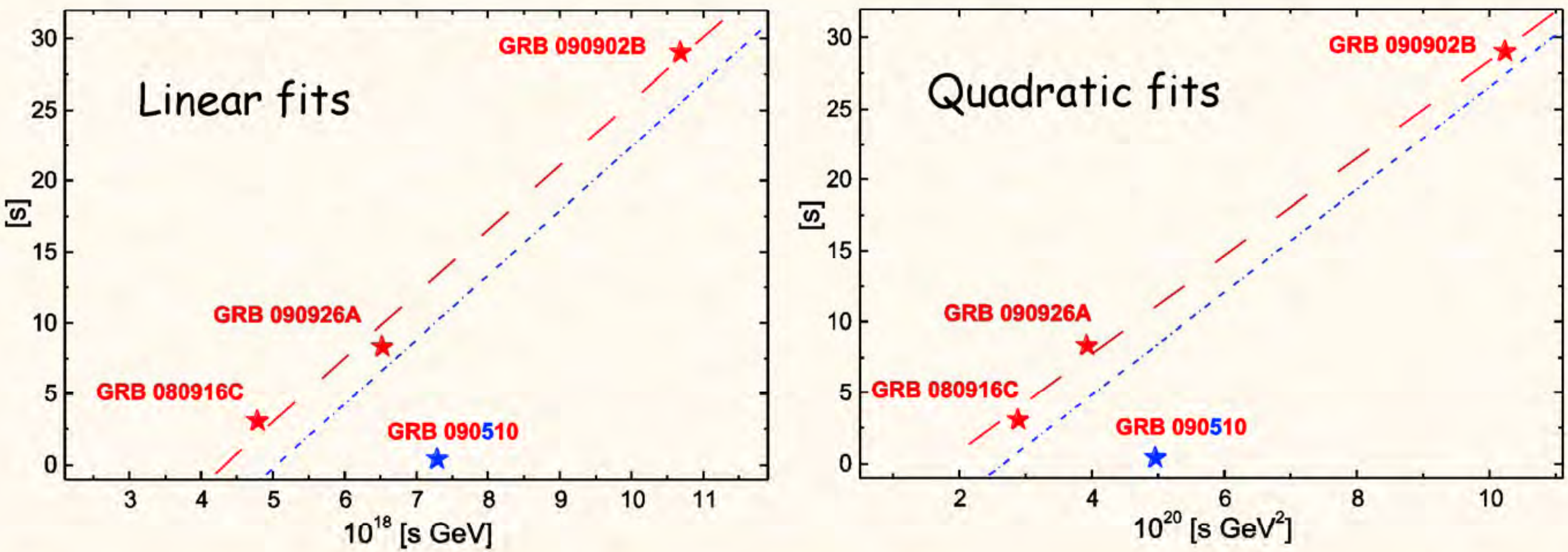
$$M_{\rm QG,Q} \sim (1.4 \pm 1.3) \times 10^{10}\ \mathrm{GeV}$$

Separation of astrophysical time lags from LV delay

- imperfect knowledge of radiation mechanism of GRBs
- a survey of GRBs at different redshifts
  - the time lag induced by LV accumulates with propagation distance
  - the **intrinsic source** induced time lag is likely to be a distance independent quantity
- A robust survey [Ellis et al.'06 & 08, Astropart. Phys.]

$$\Delta t_{\rm LV} = \frac{1+n}{2H_0}\left(\frac{E_{\rm h}^n - E_{\rm l}^n}{M_{\rm QG}^n c^{2n}}\right)\int_0^z \frac{(1+z')^n {\rm d}z'}{h(z')}$$

$$\Delta t_{\rm obs} = \Delta t_{\rm LV} + \Delta t_{\rm in}(1+z)$$



$$M_{\rm QG,L} = (2.2 \pm 0.2) \times 10^{17}\ {\rm GeV/c^2} \text{ and } M_{\rm QG,Q} = (5.4 \pm 0.2) \times 10^{9}\ {\rm GeV/c^2}$$

$$M_{\rm QG,L} = (2.2 \pm 0.9) \times 10^{17}\ {\rm GeV/c^2} \text{ and } M_{\rm QG,Q} = (5.3 \pm 0.8) \times 10^{9}\ {\rm GeV/c^2}$$

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Active galactic nuclei (AGNs)

- AGN is a compact region at the centre of a galaxy which has a much higher than normal luminosity over some or all of the electromagnetic spectrum [wikipedia]
- AGNs vs GRBs [Ellis et al.'09, PLB]
  - distance & time structure
  - energy of flares; rare & unpredictable
  - different types & distinct intrinsic time lags?

COMPOSITE

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A brief review on LV AGNs

- **Markarian 421** – no time lag > 280 s between energy bands < 1 TeV and > 2 TeV [Biller et al.'99, PRL]

$$M_{\rm QG,L} > 4.9 \times 10^{16}\ {\rm GeV/c^2}\ \text{and}\ M_{\rm QG,Q} > 1.5 \times 10^{10}\ {\rm GeV/c^2}$$

- **Markarian 501** – 4 min lag for ΔE ~ 2 TeV [Albert et al.'08, PLB]

$$M_{\rm QG,L} \sim 1.2 \times 10^{17}\ {\rm GeV/c^2}$$

- **PKS 2155-304** – ~20 s lag for ΔE ~ 1.0 TeV & ΔE^2 ~ 2.0 TeV^2 [Aharonian et al.'08, PRL]

$$M_{\rm QG,L} \sim 2.6 \times 10^{18}\ {\rm GeV/c^2} \quad M_{\rm QG,Q} \sim 9.1 \times 10^{10}\ {\rm GeV/c^2}$$

$$\boxed{\Delta t_{\rm in} = 0}$$

New Theory: the replacement of basic principle in Special Relativity

- Principle of Relativity: the equations describing the laws of physics have the same form in all admissible frames of reference.

- Principle of physical invariance :

 the equations describing the laws of physics have the same form in all admissible mathematical manifolds.

Zhou Lingli and B.-Q. Ma, MPLA 25 (2010) 2489, arXiv:1009.1331;
CPC 35 (2011) 987, arXiV: 1109.6387

A new theory of Lorentz violation

- a replacement of the common derivative operators by covariant co-derivative ones

$$\partial^\alpha \to M^{\alpha\beta}\partial_\beta, \quad D^\alpha \to M^{\alpha\beta}D_\beta,$$

- The effective minimal Standard Model

$$\begin{aligned}
\mathcal{L}_{SM} &= \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_{HG} + \mathcal{L}_{HF}, \\
\mathcal{L}_G &= -\frac{1}{4}F^{a\alpha\beta}F^a_{\alpha\beta}, \\
\mathcal{L}_F &= i\bar{\psi}\gamma^\alpha D_\alpha\psi, \\
\mathcal{L}_{HG} &= (D^\alpha\phi)^\dagger D_\alpha\phi + V(\phi),
\end{aligned}$$

- A new standard model with supplementary terms

$$\begin{aligned}
\mathcal{L}_{SMS} &= \mathcal{L}_{SM} + \mathcal{L}_{LV}, \\
\mathcal{L}_{LV} &= \mathcal{L}_{GV} + \mathcal{L}_{FV} + \mathcal{L}_{HFV}
\end{aligned}$$

Zhou Lingli and B.-Q. Ma, MPLA 25 (2010) 2489, arXiv:1009.1331 ; CPC 35 (2011) 987, arXiv: 1109.6387

The Lorentz invariance violation matrix

$$M^{\alpha\beta} = g^{\alpha\beta} + \Delta^{\alpha\beta},$$

$$\Delta^{\alpha\beta} = \begin{cases} 0 & \text{LI exact} \\ \to 0 & \text{LV small} \\ \text{otherwise} & \text{LV big} \end{cases} .$$

Zhou Lingli and B.-Q. Ma, MPLA 25 (2010) 2489

The Lorentz violation for protons from GZK cut-off

A special case is

$$\Delta^{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \xi & 0 & 0 \\ 0 & 0 & \xi & 0 \\ 0 & 0 & 0 & \xi \end{pmatrix},$$

$$E^2 = (1-\delta)\vec{p}^2 + m^2,$$
$$\delta = -\xi^2 + 2\xi.$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 10^{-23} & 0 & 0 \\ 0 & 0 & 10^{-23} & 0 \\ 0 & 0 & 0 & 10^{-23} \end{pmatrix},$$

Zhou Lingli and B.-Q. Ma, MPLA 25 (2010) 2489

Photon speed in GRAAL experiment

The GRAAL facility of the European Synchrotron Radiation Facility (ESRF) in Grenoble.

In the head-on Compton scattering of the ultra-energy electrons and the low energy photons, the energy E of the scattered photon is given by

$$E = \frac{4\gamma^2 E_0}{1 + 4\gamma E_0/m_e + \theta^2\gamma^2}, \qquad (57)$$

The maximum energy E of the Compton scattered photons is called as the Compton Edge (CE). So

$$\delta x_{\mathrm{CE}} = \frac{4AE_0}{m_e}\delta\gamma = -\frac{4AE_0}{m_e}\beta^2\gamma^3\delta c, \qquad (58)$$

where $\gamma = (1-\beta^2)^{-1/2}$ is the Lorentz factor of the incident electron.

Zhou Lingli and B.-Q. Ma, arXiv:1009.1675

Anisotropy of light speed

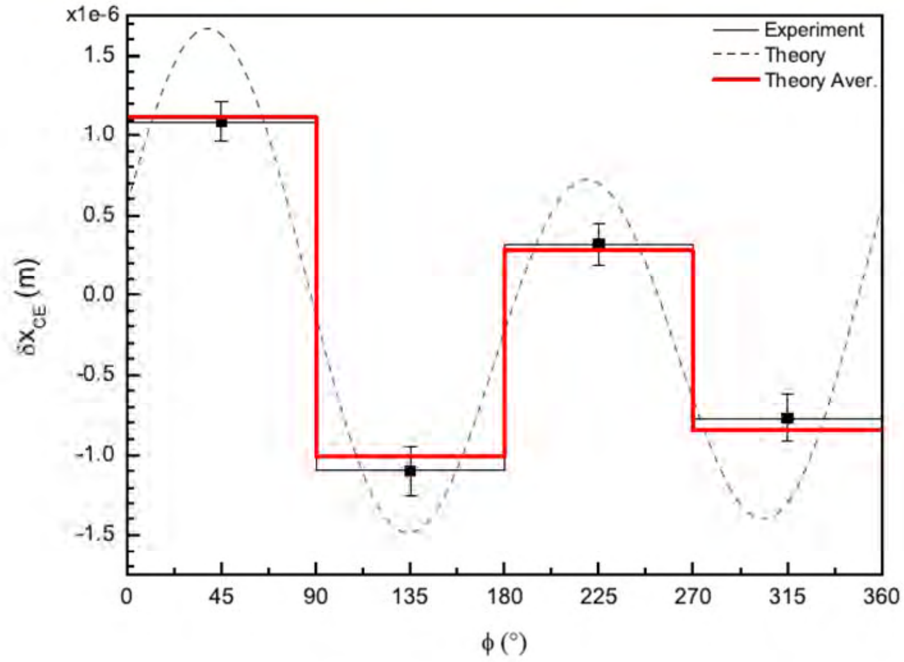

Fig. 1. δx_{CE} azimuthal distribution vs angles of the GRAAL data of the years 1998-2005 on a plane (x-y plane or $\theta = \pi/2$). $\xi = -2.89 \times 10^{-13}$, $\lambda = 6.53 \times 10^{-14}$.

Zhou Lingli and B.-Q. Ma, arXiv:1009.1675

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Anisotropy of light speed

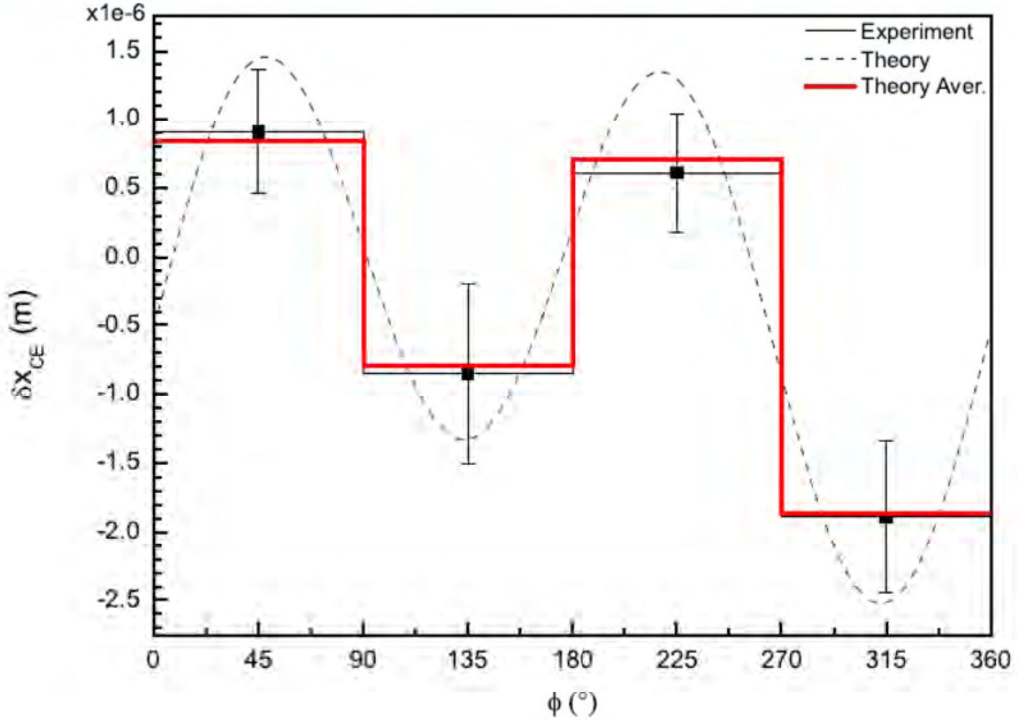

Fig. 2. $\delta x_{\rm CE}$ azimuthal distribution vs angles of the GRAAL data of the year 2008 on a plane (x-y plane or $\theta = \pi/2$). $\xi = -3.64 \times 10^{-13}$, $\lambda = 8.24 \times 10^{-14}$.

Zhou Lingli and B.-Q. Ma, arXiv:1009.1675

The Lorentz violation for photons from comparison with SME constraints

- There is no one-to-one correspondence between SMS and minimal SME.
- They could have an intersection.
- From the identity of the intersection for the two models, we get some constraints of SMS parameters from SME parameters.

$$\Delta^{\alpha\beta}_{\text{photon}} = \begin{pmatrix} 3\Delta^{33} + 10^{-17} & 10^{-5} & 10^{-5} & 10^{-6} \\ 10^{-9} & \Delta^{33} + 10^{-17} & 10^{-9} & 10^{-9} \\ 10^{-9} & 10^{-9} & \Delta^{33} + 10^{-17} & 10^{-9} \\ 10^{-9} & 10^{-8} & 10^{-8} & \Delta^{33} \end{pmatrix},$$

Zhou Lingli and B.-Q. Ma, arXiv:1110.1850

The OPERA anomaly as a signal for Lorentz violation

$$\mathcal{L}_{\mathrm{F}} = i\bar{\psi}_{\mathrm{A,L}}\gamma^{\alpha}\partial_{\alpha}\psi_{B,\mathrm{L}}\delta_{AB} + i\Delta^{\alpha\beta}_{\mathrm{L},AB}\bar{\psi}_{\mathrm{A,L}}\gamma_{\alpha}\partial_{\beta}\psi_{B,\mathrm{L}}$$
$$+ i\bar{\psi}_{\mathrm{A,R}}\gamma^{\alpha}\partial_{\alpha}\psi_{B,\mathrm{R}}\delta_{AB} + i\Delta^{\alpha\beta}_{\mathrm{R},AB}\bar{\psi}_{\mathrm{A,R}}\gamma_{\alpha}\partial_{\beta}\psi_{B,\mathrm{R}},$$

$$p^2 + g_{\alpha\mu}\Delta^{\alpha\beta}_{AA}\Delta^{\mu\nu}_{AA}p_{\beta}p_{\nu} + 2\Delta^{\alpha\beta}_{AA}p_{\alpha}p_{\beta} - m^2_A = 0.$$

$$\Delta^{\alpha\beta}_{AA} = \mathrm{diag}(\eta, \xi, \xi, \xi)$$

$$\frac{v-c}{c} = (2.48 \pm 0.28(\mathrm{stat.}) \pm 0.30(\mathrm{sys.})) \times 10^{-5},$$

$$\eta_{\nu_\mu} + \xi_{\nu_\mu} = (-2.48 \pm 0.28(\mathrm{stat.}) \pm 0.30(\mathrm{sys.})) \times 10^{-5}.$$

$|\xi_p| \leq 10^{-23}$ $|\xi_\gamma| \leq 10^{-14}$

Zhou Lingli and B.-Q. Ma, arXiv: 1109.6097

The OPERA anomaly confronts with Supernova neutrinos

$$\frac{v-c}{c} = (2.48 \pm 0.28(\text{stat.}) \pm 0.30(\text{sys.})) \times 10^{-5},$$

$$\eta_{\nu_\mu} + \xi_{\nu_\mu} = (-2.48 \pm 0.28(\text{stat.}) \pm 0.30(\text{sys.})) \times 10^{-5}.$$

$$|(v_{\nu_e} - c)/c| \le 2 \times 10^{-9}$$

$$|\eta_{\nu_e} + \xi_{\nu_e}| \le 2 \times 10^{-9}$$

we have totally $16 \times 6 + 3 = 99$ degrees of freedom, i.e., 6 Lorentz violation matrices for 3 kinds of neutrinos (when $\Delta_{AB}^{\alpha\beta}$ has the symmetric indices A and B) together with 3 neutrino masses, within the SMS framework to adjust parameters for confronting with relevant experimental observations.

Zhou Lingli and B.-Q. Ma, arXiv: 1109.6097

The OPERA anomaly in the minimal SME

$$\mathcal{L} = \frac{1}{2} i \overline{\nu}_A \gamma^\mu \overleftrightarrow{D_\mu} \nu_B \delta_{AB} + \frac{1}{2} i c^{\mu\nu}_{AB} \overline{\nu}_A \gamma^\mu \overleftrightarrow{D^\nu} \nu_B - a^\mu_{AB} \overline{\nu}_A \gamma^\mu \nu_B + \cdots ,$$

$$E = |\overrightarrow{p}| + \frac{1}{|\overrightarrow{p}|}(a^\mu p_\mu - c^{\mu\nu} p_\mu p_\nu).$$

TABLE I: Data used for fitting from Fermilab [2, 3], MINOS [4], and OPERA [1].

Fermilab	Energy (GeV)	32	44	59	69	90	120	170	195
	$v_\nu - c$ (10^{-5})	-2^{+2}_{-3}	2 ± 7	-1^{+2}_{-3}	-1^{+2}_{-3}	1^{+3}_{-4}	1 ± 7	1^{+2}_{-3}	6^{+3}_{-4}
MINOS	Energy (GeV)	3							
	$v_\nu - c$ (10^{-5})	5.1 ± 2.9							
OPERA	Energy (GeV)	13.9	42.9						
	$v_\nu - c$ (10^{-5})	2.17 ± 0.83	2.74 ± 0.80						

N. Qin and B.-Q. Ma, arXiv:1110.4443

Summary

- Researches on Lorentz violation have been active for many years.
- No convincing evidence yet, including the OPERA anomaly.
- The OPERA anomaly might provide new chance for Lorentz violation study.
- Lorentz violation is being an active frontier both theoretically and experimentally.