A Model Independent Method to Study Dark Matter induced Leptons and Photons

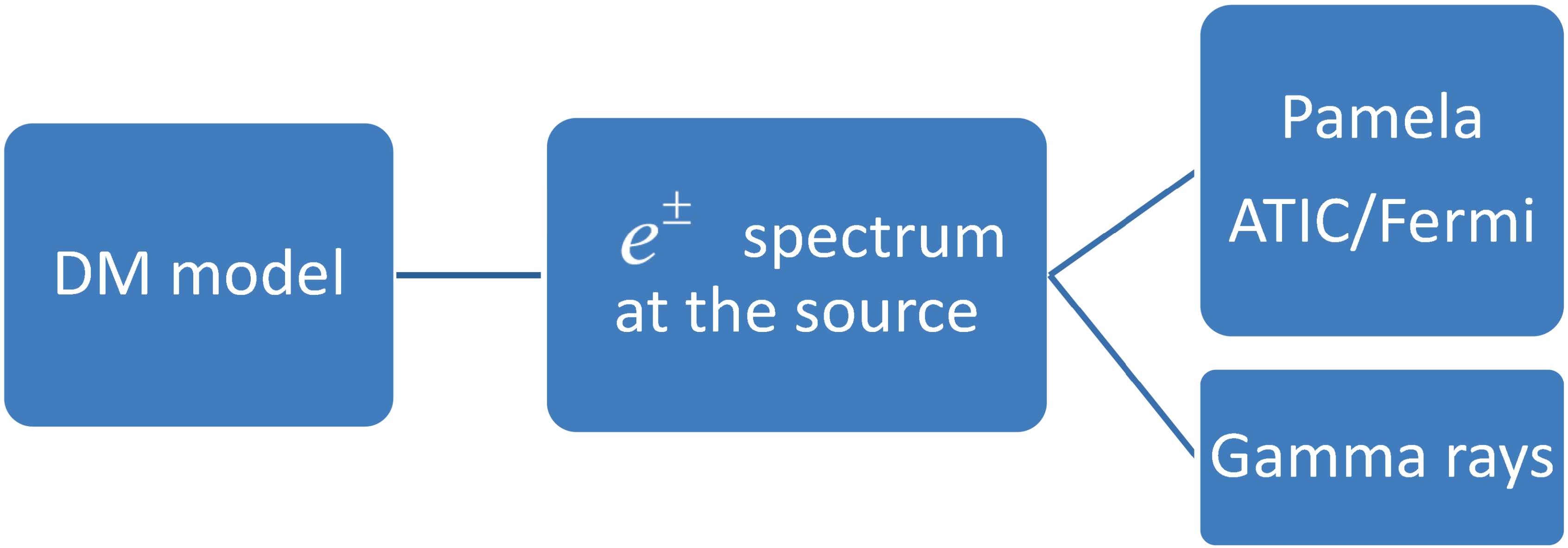
Mingxing Luo, Liucheng Wang and Guohuai Zhu

Zhejiang Institute of Modern Physics
Physics Department, Zhejiang University

Dec. 21, 2011, Beijing

arXiv:1107.3024

Motivation



- Determining $e^{\pm}$ at the source from observations at the Earth
- Relating cosmic $e^{\pm}$,γ: Independent of particle physics models

DM interpretation for Fermi-LAT $e^{\pm}$ spectrum

- M_{DM} above 1TeV
- a softer spectrum
  - cascade decay: ν,τ,SUSY particles...
  - three-body decay with a missing particle
- Decaying DM senario:

$$Q_e^{\text{DM}}(E,\vec{r}) = \rho_{\text{DM}}(\vec{r})\sum_i \frac{\Gamma_i^{\text{DM}}}{M_{\text{DM}}}\frac{dN_i^{\text{DM}}}{dE} = \rho_{\text{DM}}(\vec{r})\,X(E)$$

i for different channels, $\tau = (\sum_i \Gamma_i)^{-1} \sim 10^{26}s$

- Annihilating DM senario:

$$Q_e^{\text{DM}}(E,\vec{r}) = \rho_{\text{DM}}^2(\vec{r})\frac{B}{2}\sum_i \frac{<\sigma\nu>_i}{M_{\text{DM}}^2}\frac{dN_i^{\text{DM}}}{dE} = \rho_{\text{DM}}^2(\vec{r})\,X(E)$$

$<\sigma\nu>\sim$ relic density, $B \sim 100 - 1000$

Propagation in the Galaxy

- GALPROP code
- semi-analytical methods

The number of parameters in realistic models incorporating all of this information is large, and using the available data to perform statistical inference on the model's free parameters is a highly non-trivial task. So far, this has only been possible with semi-analytical methods where the computation is fast. But such methods necessarily require many simplified assumptions to allow the problem to be analytically tractable and to reduce the computational load, making the estimation of the confidence level of their results difficult.

—GALPROP group

$e^{\pm}$ propagation in the Galaxy

$$K(E)\cdot\nabla^2 f_e^{\mathrm{DM}}(E,\vec{r}) + \frac{\partial}{\partial E}\left[B(E) f_e^{\mathrm{DM}}(E,\vec{r})\right] + Q_e^{\mathrm{DM}}(E,\vec{r}) = 0$$

- Diffusion coefficient: related to the rigidity of the particle

$$K(E) = K_0(E/\mathrm{GeV})^{\alpha}$$

  - α, K_0: different propagation models [PRD77(2008)063527]

Model	α	K_0 in kpc^2/Myr	L in kpc
MIN	0.55	0.00595	1
MED	0.70	0.0112	4
MAX	0.46	0.0765	15

minimal, medium and maximal positron flux, B/C ratio constraint

- Energy loss: synchrotron radiation, inverse Compton scattering (excluding the galaxy disk)
  Effectively, $B(E) = E^2/(\mathrm{GeV}\cdot\tau_E)$ with $\tau_E = 10^{16}$ s

Solution of the diffusion equation

- Solution in a solid flat cylinder $R = 20$ kpc, $L = 1,\ 4,\ 15$ kpc
  Simplified caricature of the galaxy, zero $e^{\pm}$ flux on the boundary

$$f_e^{\rm DM}(E, \vec{r_\odot}) = \int_E^\infty dE' G_1(E, E') X(E')$$

$$G_1(E, E') = \frac{\tau_E}{E^2} \sum_{m,n=1}^{\infty} B_{mn} \exp\left[\lambda_{mn}\left(E^{\alpha-1} - (E')^{\alpha-1}\right)\right]$$

$$B_{mn} = \frac{2\sin(m\pi/2)}{J_1^2(\zeta_n) R^2 L} J_0\left(\frac{\zeta_n\, r_\odot}{R}\right) \int_0^R dr\, r \int_{-L}^{L} dz$$
$$\times\, \rho^{DM}(\sqrt{r^2 + z^2}) J_0\left(\frac{\zeta_n\, r}{R}\right) \sin\left[\frac{m\pi}{2L}(z + L)\right]$$

$$\lambda_{mn} = \left(\frac{\zeta_n^2}{R^2} + \frac{m^2\pi^2}{4L^2}\right) K_0\, \tau_E \frac{1}{\alpha - 1}$$

- Usual strategy: $X(E) \Rightarrow f_e$ (commonly using fit functions)
- Our story: $f_e \Rightarrow X(E)$

The Volterra integral Equation

- $e^{\pm}$ propagation

$$f_e^{\mathrm{DM}}(E, \vec{r_\odot}) = \int_E^\infty dE' G_1(E, E') X(E')$$

- The Volterra integral Equation :

$$f(x) = \int_a^x K(x-t) y(t) dt$$

- An inverse solution: Volterra integral equation
  With the help of the Laplace transform about convolution and the inverse Laplace transform, $y(x)$ can be represented as

$$y(x) = \frac{df(x)}{dx} + \int_a^x dt\, R(x-t) \frac{df(t)}{dt}.$$

  $R(x) = \boldsymbol{L}^{-1}\left[\frac{1}{p\widetilde{K}(p)} - 1\right]$, $\widetilde{K}(p) = \boldsymbol{L}\left[K(x)\right]$.
  $\boldsymbol{L} \sim$ Laplace transform
  $\boldsymbol{L}^{-1} \sim$ its inverse

Reconstructing $X(E)$ from $f_e^{\mathrm{DM}}(E,\vec{r_\odot})$ with energy$> E$

- $e^\pm$ propagation

$$f_e^{\mathrm{DM}}(E,\vec{r_\odot}) = \int_E^\infty dE' G_1(E,E') X(E')$$

$$G_1(E,E') = \frac{\tau_E}{E^2} \sum_{m,n=1}^{\infty} B_{mn} \exp\left[\lambda_{mn}\left(E^{\alpha-1} - (E')^{\alpha-1}\right)\right]$$

- An inverse solution:

$$X(E) = \frac{dg(E)}{dE} + (\alpha-1)E^{\alpha-2} \int_\infty^E dE' \frac{dg(E')}{dE'} R\left(E^{\alpha-1} - (E')^{\alpha-1}\right)$$

$$g(E) = -\frac{E^2}{\tau_E} f_e^{\mathrm{DM}}(E,\vec{r_\odot}) \Big/ \sum_{m,n=1}^{\infty} B_{mn} \qquad R(x) = \boldsymbol{L}^{-1}\left[\frac{1}{p\widetilde{K}(p)} - 1\right]$$

$$\widetilde{K}(p) = \boldsymbol{L}\left[K(x)\right] = \boldsymbol{L}\left[\sum_{m,n=1}^{\infty} B_{mn} \exp[\lambda_{mn} x] \Big/ \sum_{m,n=1}^{\infty} B_{mn}\right]$$

$$= \sum_{m,n=1}^{\infty} \frac{B_{mn}}{p-\lambda_{mn}} \Big/ \sum_{m,n=1}^{\infty} B_{mn}$$

Truncated

- $e^{\pm}$ Propagation

$$f_e^{\rm DM}(E,\vec{r_\odot}) = \int_E^\infty dE' G_1(E,E') X(E')$$

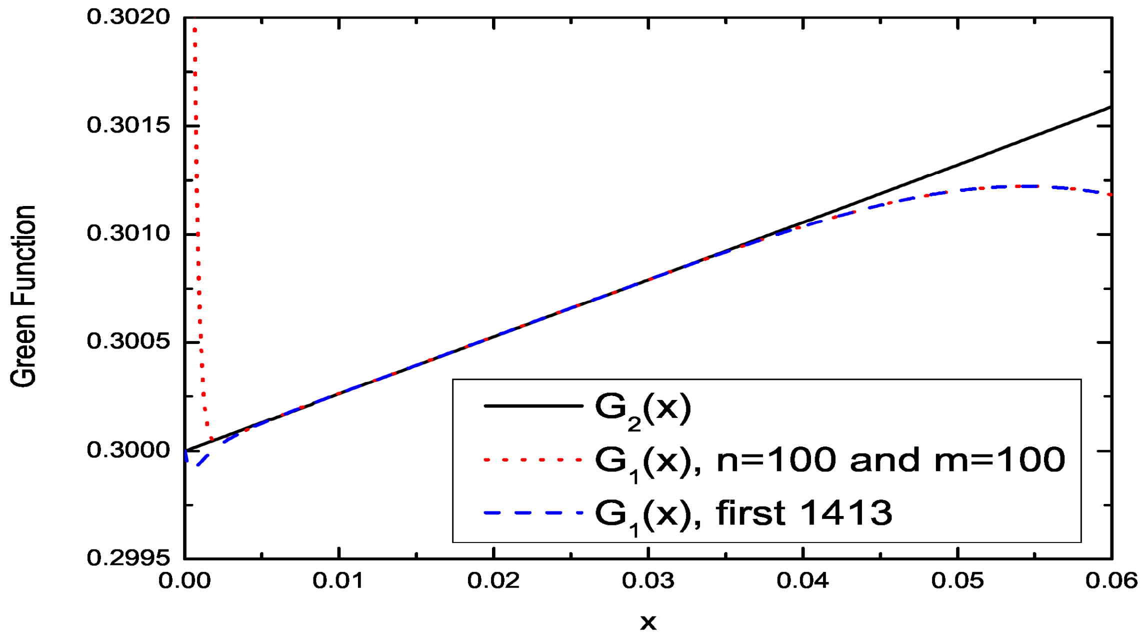
$$G_1(E,E') = \frac{\tau_E}{E^2} \sum_{m,n=1}^{\infty} B_{mn} \exp\left[\lambda_{mn}\left(E^{\alpha-1} - (E')^{\alpha-1}\right)\right]$$

- $E' > E$: Exponential suppression
- $E' \sim E$: Converges very slowly
- $E' \sim E$: Better expressed in another form, adopted

$$G_2(E,E') = \frac{\tau_E}{E^2} \exp\left[\frac{K_0\,\tau_E}{1-\alpha}\left(E^{\alpha-1} - (E')^{\alpha-1}\right)\nabla^2\right] \rho^{\rm DM}(\vec{r})\Big|_{\vec{r}=\vec{r_\odot}}$$

- Reordering G_1 from small to large $|\lambda_{mn}|$

Truncated:Continued



$$x = E^{\alpha-1} - (E')^{\alpha-1}$$

- Decaying DM scenario
- NFW DM density profile, MED propagation model
- Truncated: first 1413 terms

Inverse Laplace Transformation

$$R(x) = \boldsymbol{L}^{-1}\left[-\sum_{1413}\frac{B_{mn}\lambda_{mn}}{p-\lambda_{mn}}\Bigg/\sum_{1413}\frac{pB_{mn}}{p-\lambda_{mn}}\right]$$

$$= \sum_i \text{Res}\left[-\sum_{1413}\frac{B_{mn}\lambda_{mn}}{p_i-\lambda_{mn}}\exp(p_i x)\Bigg/\sum_{1413}\frac{p_i B_{mn}}{p_i-\lambda_{mn}}\right]$$

- Cauchy's Residue Theorem
- Defining $\psi(p) = \sum_{1413}\frac{B_{mn}}{p-\lambda_{mn}}$, singularities $\Rightarrow$ roots
- Complicated, though not impossible, to find all 1413 singularities
- Exponential suppression $\exp(p_i x)$, find singularities in the region $-200 <$ Re(p)< 0
- Still not an easy task

Inverse Laplace Transformation: Continued

- Divide $-200 < \mathrm{Re(p)} < 0$ into many small regions
- Locate the approximate position of singularities by using the Argument principle in complex analysis

$$\frac{1}{2\pi i}\oint_C \frac{\psi'(p)}{\psi(p)}\,dp = \mathrm{N} - \mathrm{P}$$

- Useful to find complex roots
- Newton's method to get the singularity position inside each region

Inverse Laplace Transformation: Errors

$$R(x) = \sum_i \mathrm{Res}\left[\frac{-\sum\limits_{1413}\frac{B_{mn}\lambda_{mn}}{p_i-\lambda_{mn}} - \sum\limits_{>1413}\frac{B_{mn}\lambda_{mn}}{p_i-\lambda_{mn}}}{\sum\limits_{1413}\frac{p_i B_{mn}}{p_i-\lambda_{mn}} + \sum\limits_{>1413}\frac{p_i B_{mn}}{p_i-\lambda_{mn}}}\exp(p_i x)\right]$$

- New singularities to $-200 <$Re(p)< 0 from $\sum\limits_{>1413}\frac{B_{mn}}{p-\lambda_{mn}}$?
  - $\sum\limits_{1413}\frac{B_{mn}}{p-\lambda_{mn}} \gg \sum\limits_{>1413}\frac{B_{mn}}{p-\lambda_{mn}}$ on the contour of $-200 <$Re(p)< 0 as the 1413-th λ_{mn} equals -2630
  - Rouche's theorem : $\sum\limits_{1413}$ and $\sum\limits_{1413} + \sum\limits_{>1413}$ have the same number of zeros
- Errors of $R(x)$ well under control

SKLTP

Astrophysical background of $e^{\pm}$

- How to get f_e from measurements? Problematic for all analysis
- conventional model 0 background[JCAP 1001(2010)009]

$$\Phi_{e^-}^{\text{bkg}}(E) = \frac{82.0\epsilon^{-0.28}}{1+0.224\epsilon^{2.93}}$$

$$\Phi_{e^+}^{\text{bkg}}(E) = \frac{38.4\epsilon^{-4.78}}{1+0.0002\epsilon^{5.63}} + 24.0\epsilon^{-3.41}$$

with $\epsilon = E/1$ GeV, $\Phi_{e^\pm} = cf_{e^\pm}/4\pi$

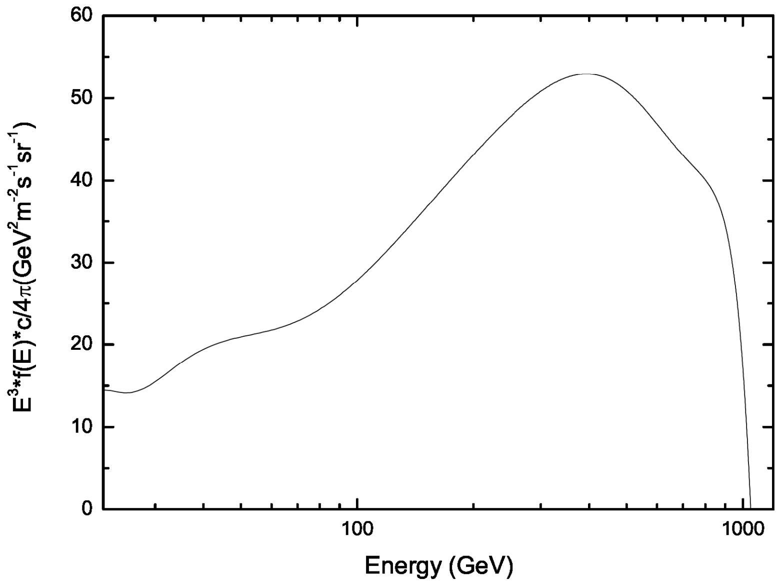
$$\Phi_{e^\pm}^{\oplus}(E_\oplus) = \frac{E_\oplus^2}{E^2}\left[\Phi_{e^+}^{\text{bkg}}(E) + \text{N}\times\Phi_{e^-}^{\text{bkg}}(E)\right]$$

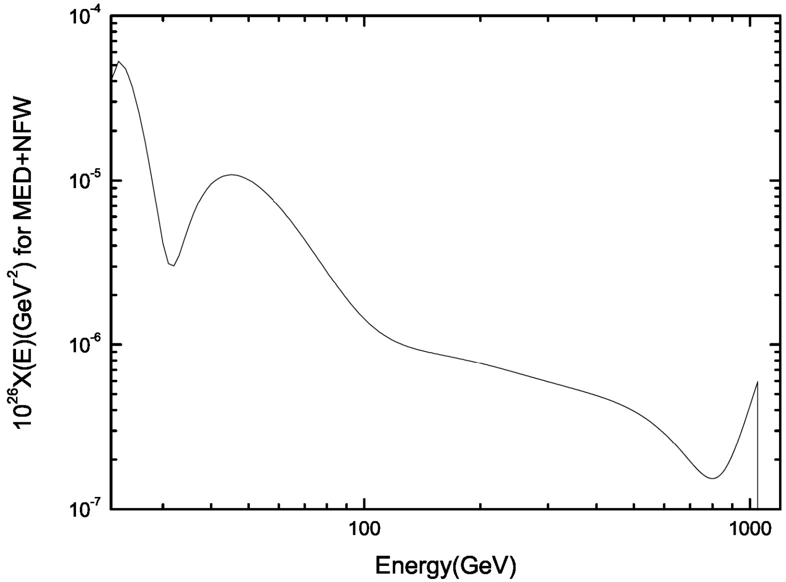
with $N = 0.8$, to leave room to the additional DM component below 100 GeV (PAMELA anomaly);

$E_\oplus = E + e\,\phi_F$ with $\phi_F = 0.55$ GV for the solar modulation effects

- Usual strategy: $\Phi_{e^\pm}^{\text{bkg}}(E) + f_e \to$ Fermi-LAT data
- Our story: Fermi-LAT data - $\Phi_{e^\pm}^{\text{bkg}}(E) \to f_e \to X_e$

Reconstruction of $X(E)$: results

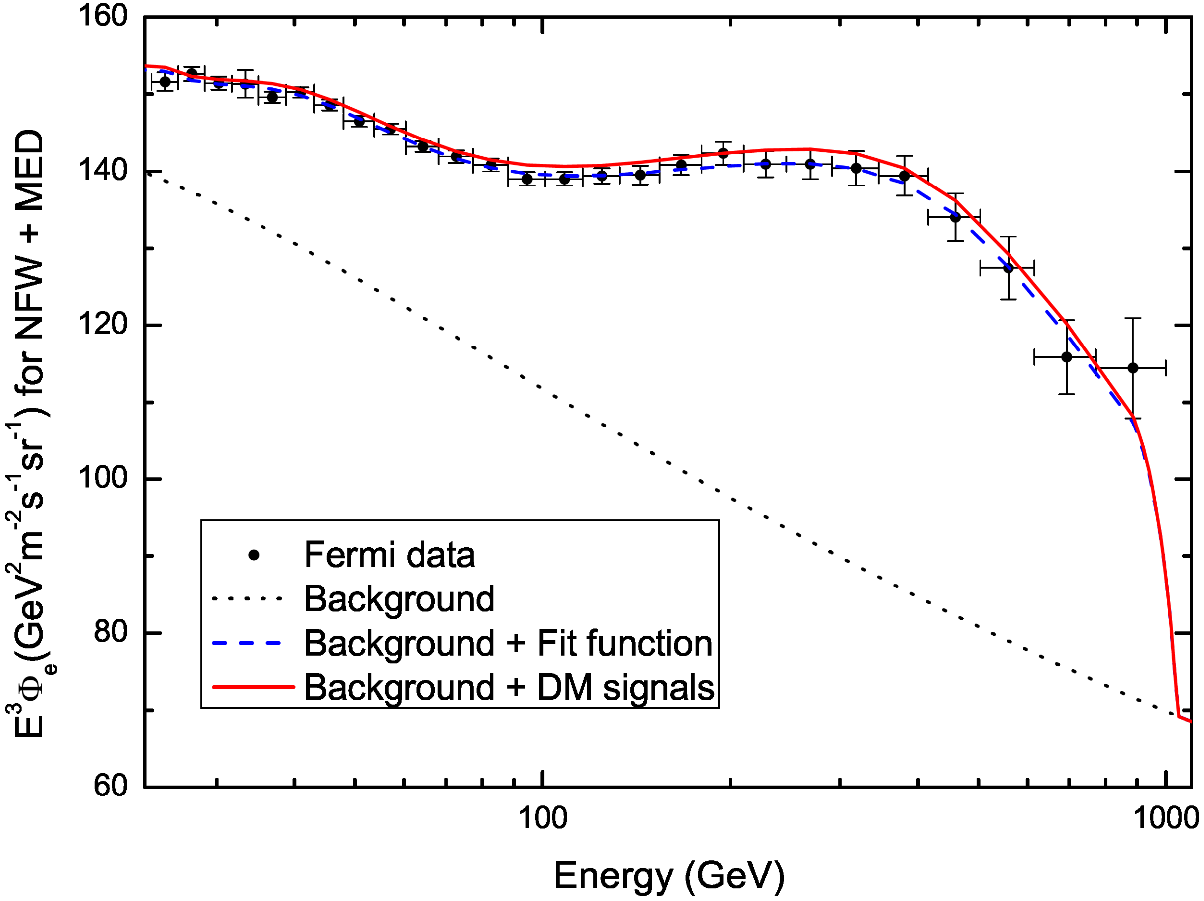




Left: $f_e^{\rm DM}(E, r_\odot)$ extracted from the Fermi-LAT spectrum of $e^\pm$ by subtracting off the background $\Phi_{e^\pm}^{\rm bkg}(E)$.
Right: $X(E)$ determined from $f_e^{\rm DM}(E, r_\odot)$, assuming MED propagation model and NFW DM density profile.

- DM model independent

Reconstruction of $X(E)$: self-consistent check



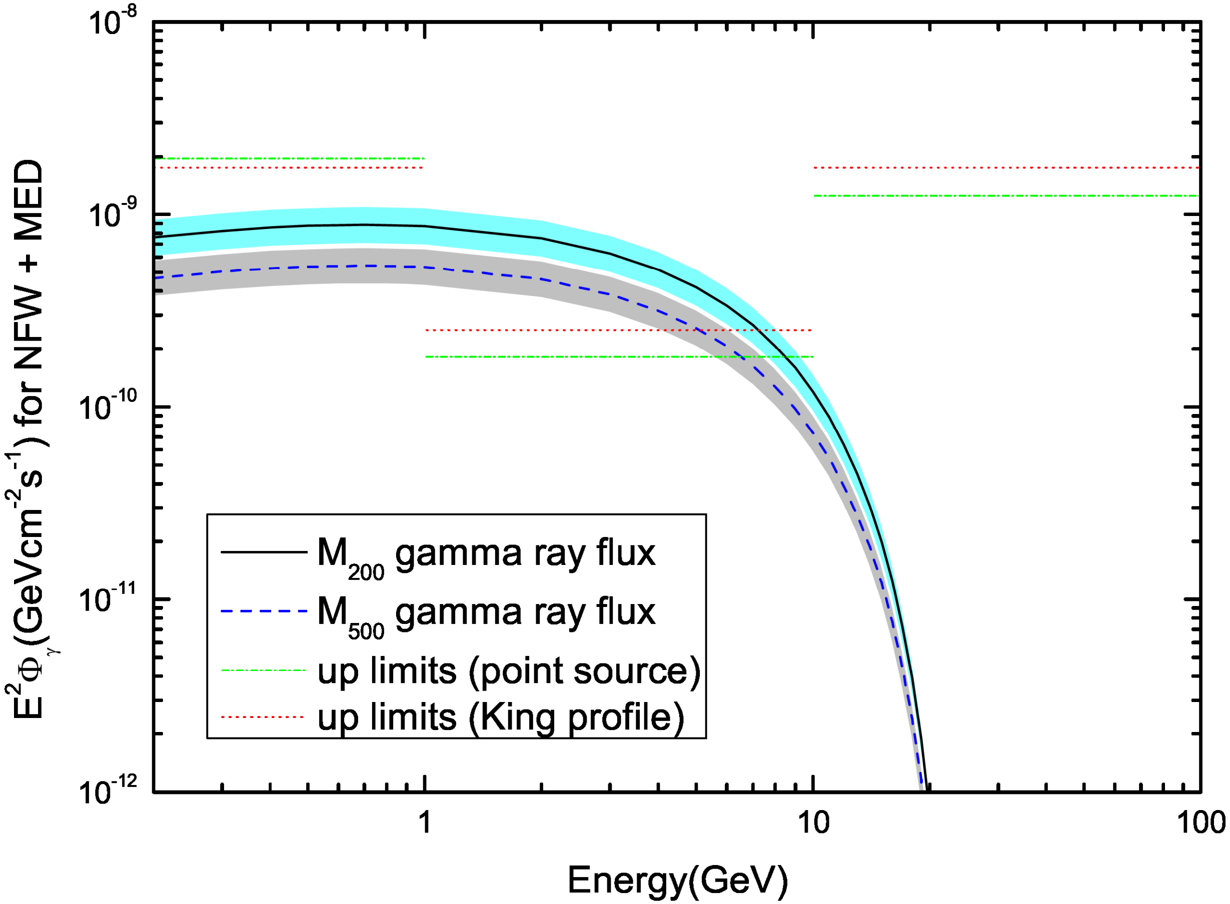
$f \Rightarrow X \Rightarrow f$: the difference between the red solid line and the blue dashed line can be viewed as a demonstration of the theoretical error in determining $X(E)$

Inverse Compton scattering from $X(E)$

- Using the $X(E)$ thus obtained to say something about photons: inside/outside the Galactic halo
- Gamma rays from nearby clusters
  - highly DM dominated and isolated at high galactic latitudes
  - high signal-to-noise ratios are anticipated for gamma-ray observations
  - ICS scattering on the starlight can be neglected
  - Model dependent studies: the Fornax cluster provides strongest constraint for decaying DM
- decaying DM: $\Phi_\gamma \sim \int d\theta \int ds \rho \sim \int dV \rho \sim M_{\text{cluster}}$
- annihilating DM: $\Phi_\gamma \sim \int d\theta \int ds \rho^2 \sim \int dV \rho^2 \sim M^{\alpha}_{\text{cluster}}$
- decaying DM: DM-model independent component

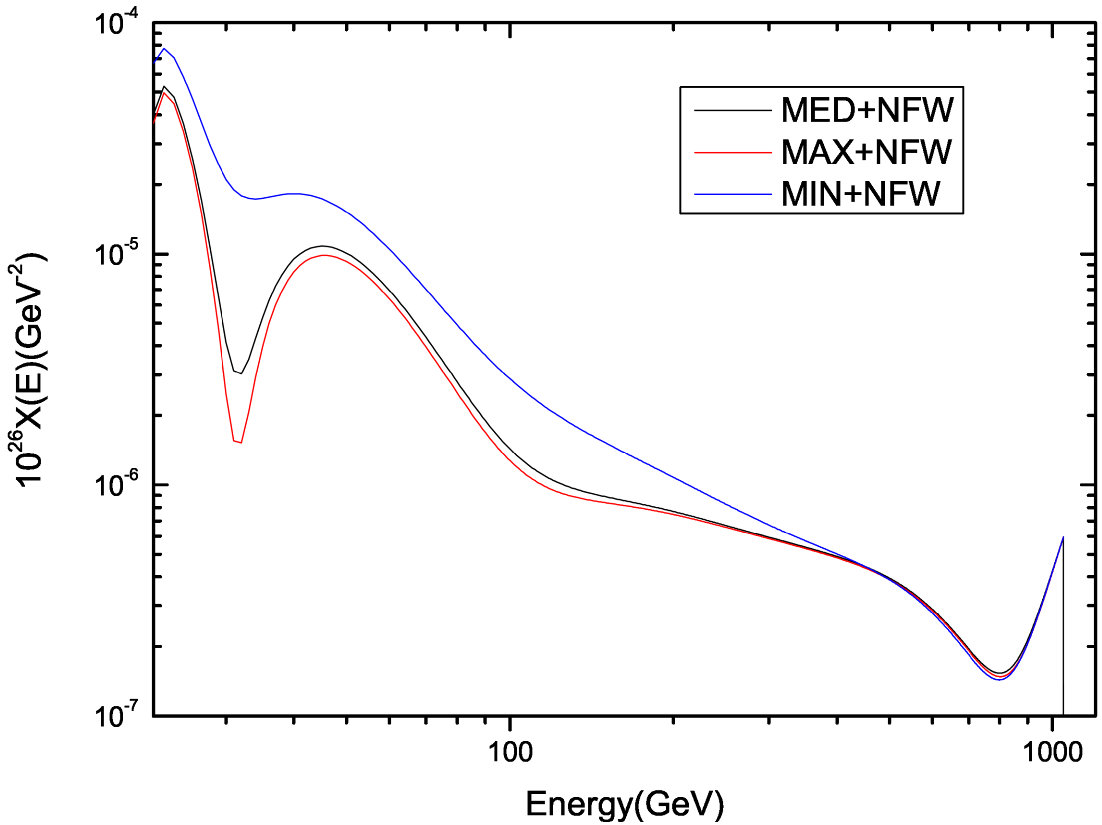
$$\Phi(E_\gamma) = \frac{M_{\text{cluster}}}{4\pi D^2} \int d\epsilon f_{CMB}(\epsilon) \int dE_e \frac{d\sigma^{ICS}(E_e, \epsilon)}{dE_\gamma} X(E_e)$$

ICS Gamma rays from the Fornax cluster



Experimental upper limits: Fermi-LAT 18-month data from ApJ, 717, L71 (2010)

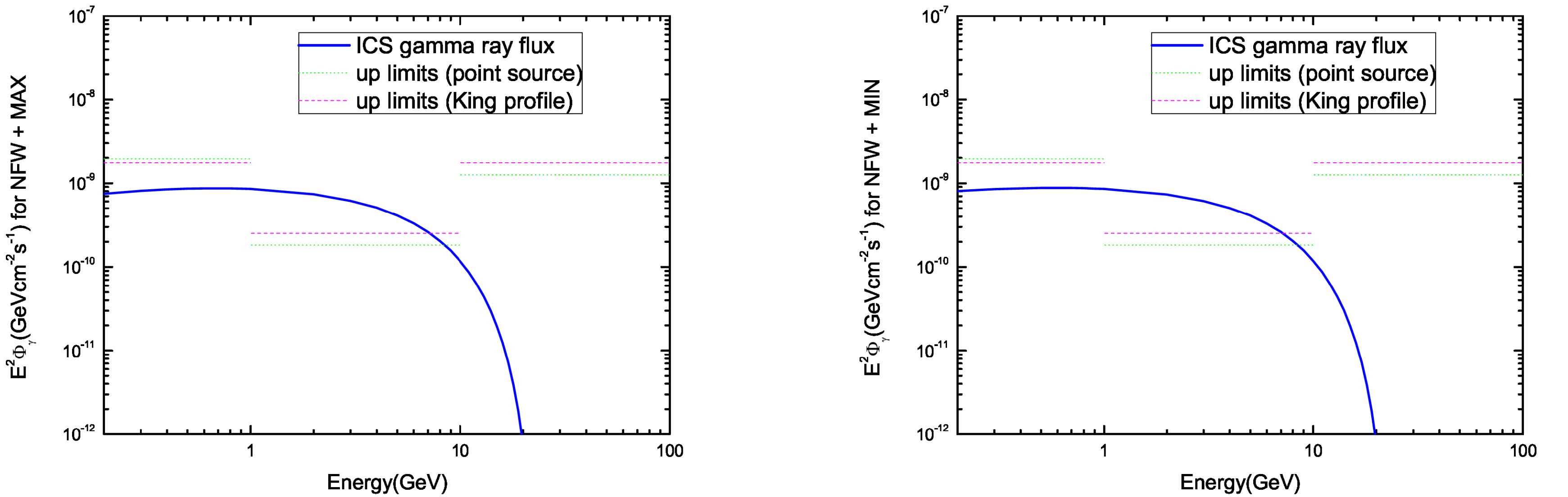
Astrophysical uncertainties of ICS: propagation models



Astrophysical uncertainties for the determination of $X(E)$ from $f_e^{\rm DM}(E, r_\odot)$

high energy leptons must come from the neighborhood of the solar system, so the propagation effects should not have significant uncertainties in such a small distance.

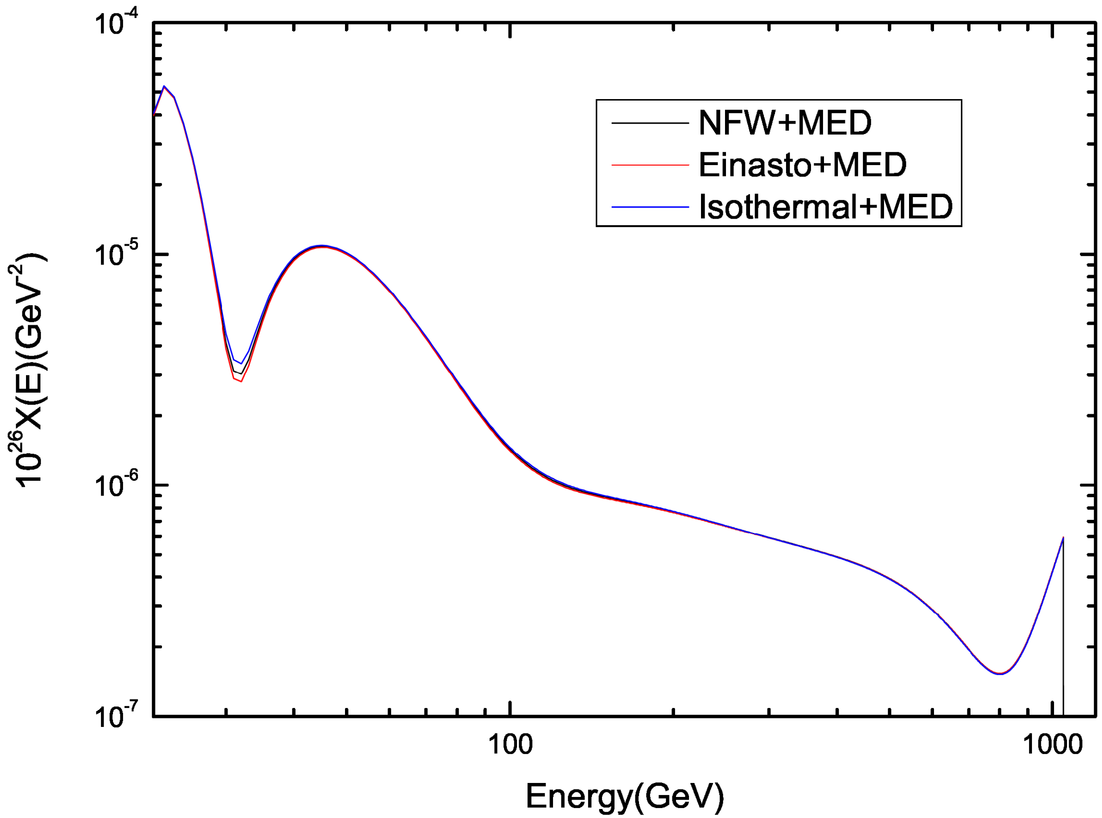
Astrophysical uncertainties of ICS: propagation models



$E_e \gtrsim m_e\sqrt{E_\gamma/\epsilon}/2 \Longrightarrow$ initial state $E_e \gtrsim$ 500 GeV for final state $E_\gamma = 1$ GeV

(if the CMB photon energy $\epsilon \sim kT$, with $T = 2.7$k)

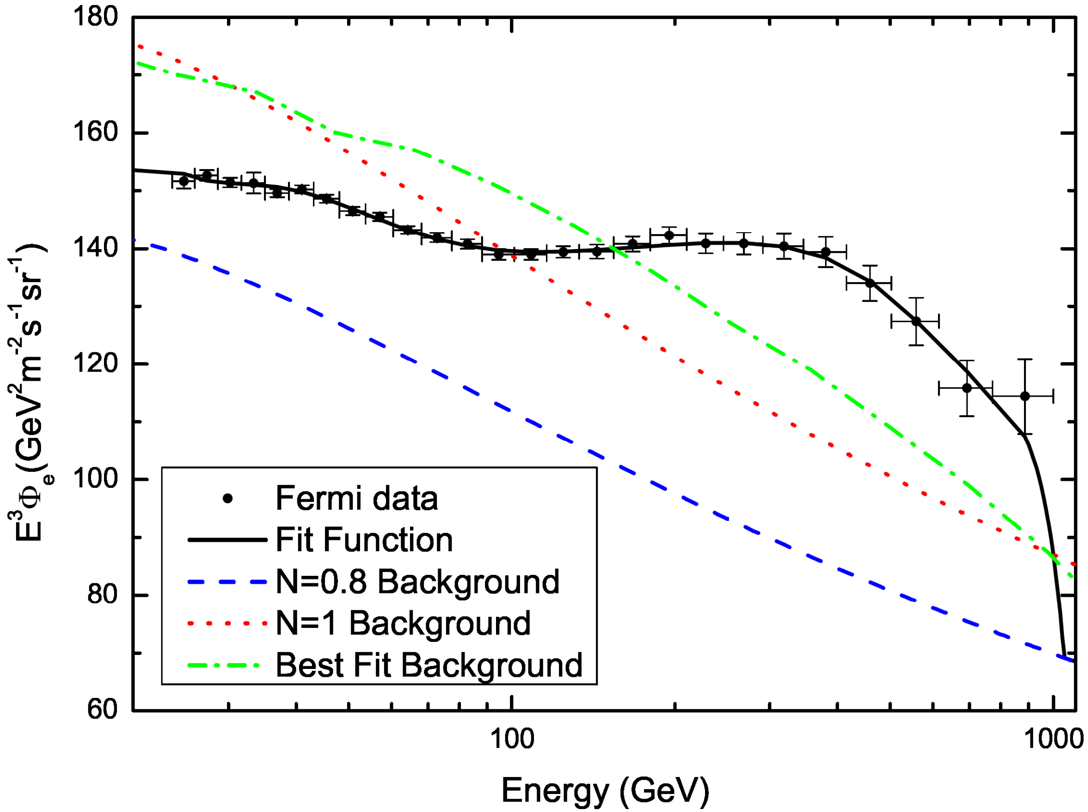
Astrophysical uncertainties of ICS: DM density

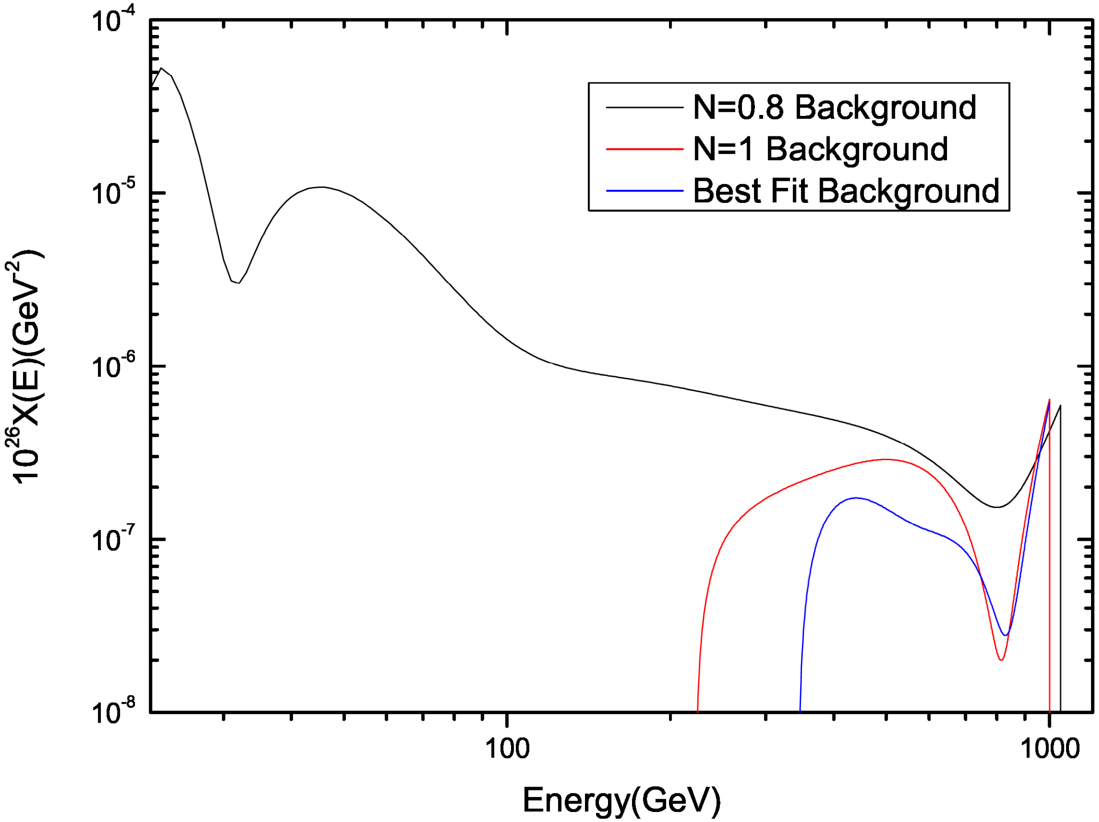


Astrophysical uncertainties for the reconstruction of $X(E)$ from $f_e^{\rm DM}(E, r_\odot)$

the energetic leptons can not propagate a long distance and the different DM profiles have very similar behavior except for the region near the Galaxy center

Astrophysical uncertainties of ICS: $e^{\pm}$ background

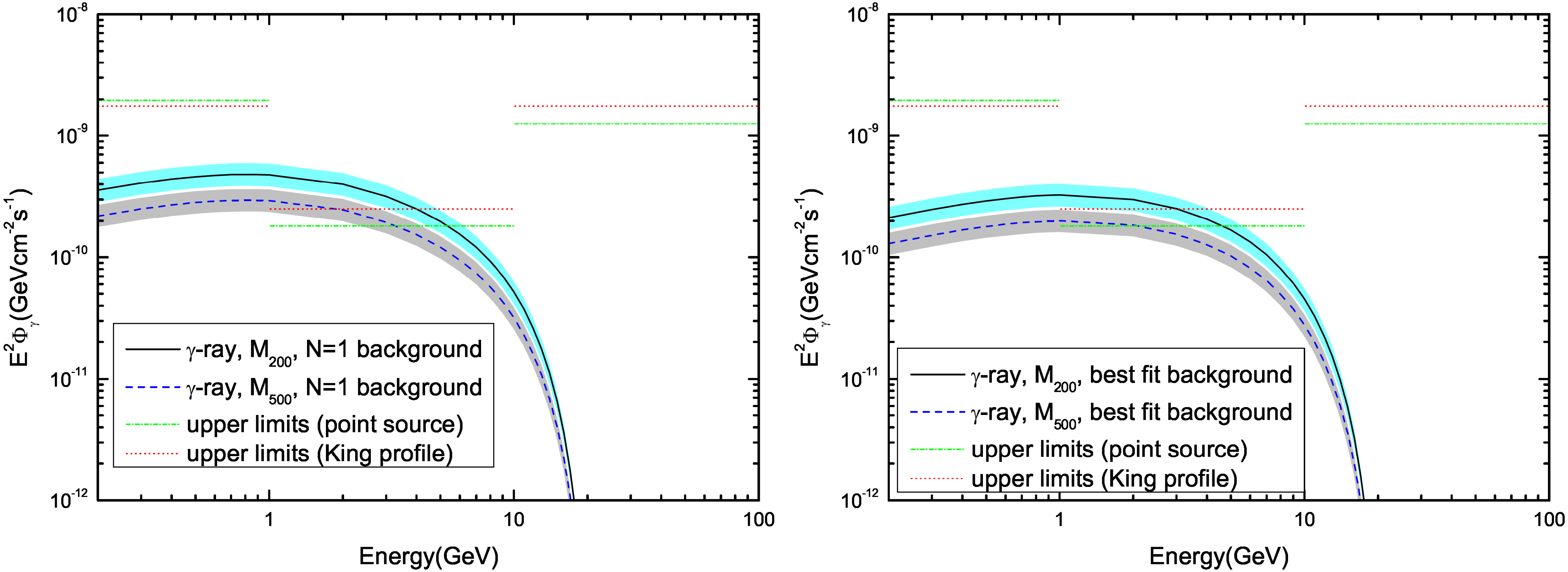




$e^{\pm}$ background uncertainties for the reconstruction of $X(E)$ from $f_e^{\mathrm{DM}}(E, r_\odot)$

- $N = 1$, to interpret low energy pre-Fermi data
- best fit background from a Bayesian analysis [ApJ, 729, 106 (2011)]

Astrophysical uncertainties of ICS: $e^{\pm}$ background



$e^{\pm}$ background uncertainties for ICS

- $N = 1$, inconsistent
- best fit background from a Bayesian analysis, survives only if a small total mass M_{500}

SKLTP

Summary1

- A new method to determine the DM induced $e^{\pm}$ flux at the source
  - Input: Subtracting astrophysical background from the experimental data
  - Output: DM induced $e^{\pm}$ source spectrum $X(E)$
  - Key point: solving a Volterra integral equation-technically challenge
  - without the need to introduce any specific DM model
  - applicable to both decaying and annihilating DM

Summary2

- Predict DM-independent component ICS gamma rays in the Fornax cluster
  - Input: $X(E)$, CMB photon, M_{cluster}
  - Output: ICS gamma rays
  - results depend on the detailed shape of the $e^{\pm}$ background
  - conventional model 0 background with $N \leq 1$, inconsistent
  - best fit background from a Bayesian analysis, survives only if a small total mass M_{500}
  - ICS gamma rays with $E_{\gamma} \gtrsim$ 1 are independent of choices of propagation model and of DM density profile.

Thank you!