厚膜上各种自旋粒子的局域化和质量谱

刘玉孝(兰州大学)

中国科学院理论物理研究所、重庆邮电大学数理学院

重庆

2012.5.9

Content

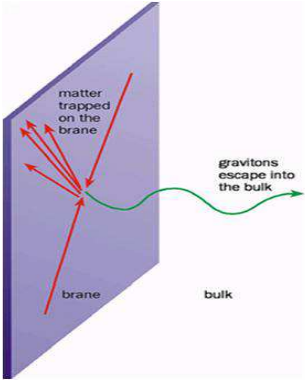
- Introduction
- Kaluza-Klein Reduction
  - Spin 0 scalar field
  - Spin 1 vector field
  - Spin 1/2 fermion field
  - Spin 2 gravity field
- Localization and Mass Spectrum on Various Branes
  - On dS brane
  - On AdS brane
  - On flat brane in f(R) gravity
  - On flat brane in critical gravity
  - On two-scalar branes
- Conclusion

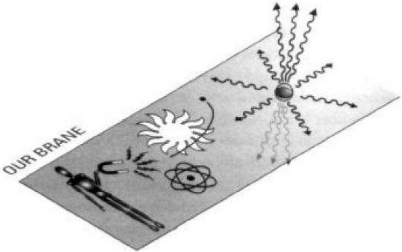
Introduction—History

Brief history of extra dimensions and braneworlds

- 1920's, Kaluza and Klein, KK Theory
- 1980's, Akama, Rubakov, Domain Wall Braneworld
- 1998, Arkani-Hamed, Dimopoulos, and Dvali, Large Extra Dimension (ADD Braneworld Scenario)
- 1999, Randall and Sundrum(RS), Warped Extra Dimension (RS thin Braneworld Scenario)
- 1999, DeWolfe, Freedman, Gubser, and Karch, Thick Braneworld Scenario

Introduction—Picture of Braneworlds

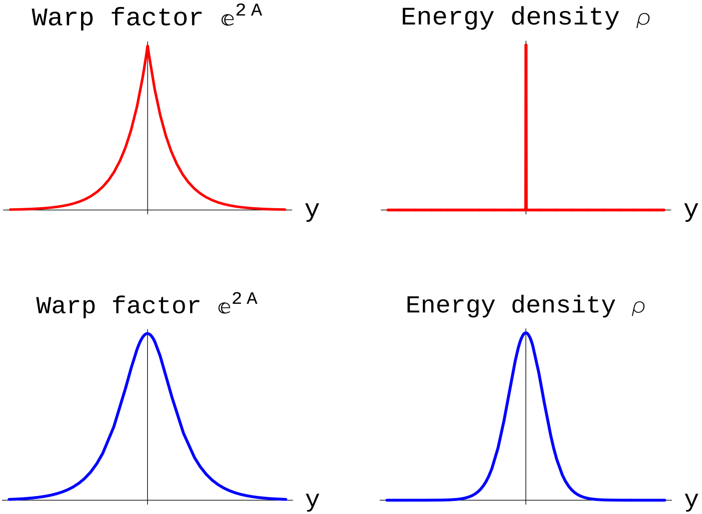




- 4D space-time is seen as a brane (hypersurface) embedded in higher-dimensional space-time
- Matter and gauge fields are confined on the brane, only the gravity can propagate in the bulk

Introduction—Thin Brane & Thick Brane

$ds^2 = e^{2A(y)}\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu - dy^2$, $e^{2A(y)}$ is the warp factor.



Content

- Introduction
- Kaluza-Klein Reduction
  - Spin 0 scalar field
  - Spin 1 vector field
  - Spin 1/2 fermion field
  - Spin 2 gravity field
- Localization and Mass Spectrum on Various Branes
  - On dS brane
  - On AdS brane
  - On flat brane in f(R) gravity
  - On flat brane in critical gravity
  - On two-scalar branes
- Conclusion

Kaluza-Klein Reduction

Whether various spin bulk fields can be localized on thick branes?

Method:

- ♣ Action of a 5D massless field $\Rightarrow$
- ♣ Equation of motion $\Rightarrow$
- ♣ KK decomposition of the 5D field $\Rightarrow$
- ♣ $\left.\begin{array}{l}\text{Schrödinger equation of KK modes}\\ \text{Orthonormality condition of KK modes}\end{array}\right\} \Rightarrow$
- ♣ Schrödinger potentials + spectrums $\Rightarrow$
- ♣ Effective action of a 4D massless or massive field

The action of a massless real scalar

$$S_0 = -\frac{1}{2}\int d^5x\sqrt{-g}\, g^{MN}\partial_M\Phi\partial_N\Phi. \tag{1}$$

The equation of motion (EOM)

$$\frac{1}{\sqrt{-\hat{g}}}\partial_\mu(\sqrt{-\hat{g}}\hat{g}^{\mu\nu}\partial_\nu\Phi) + e^{-3A}\partial_z\left(e^{3A}\partial_z\Phi\right) = 0. \tag{2}$$

By decomposing

$$\Phi(x,z) = \sum_n \phi_n(x)\chi_n(z)e^{3A/2}, \tag{3}$$

and imposing the orthonormality condition

$$\int dz\, \chi_m(z)\chi_n(z) = \delta_{mn}, \tag{4}$$

we get the Schrödinger equation of KK modes

$$\begin{aligned} \left[-\partial_z^2 + V_0(z)\right] \chi_n(z) = \mu_n^2 \chi_n(z), \\ V_0(z) = \tfrac{3}{2}A'' + \tfrac{9}{4}A'^2, \end{aligned} \tag{5}$$

and the effective action

$$S_0 = -\frac{1}{2}\sum_n \int d^4x \sqrt{-\hat{g}} \left(\hat{g}^{\mu\nu} \partial_\mu \phi_n \partial_\nu \phi_n + \mu_n^2 \phi_n^2 \right).$$

The action of $U(1)$ vector fields

$$S_1 = -\frac{1}{4}\int d^5x\sqrt{-g}\, g^{MN}g^{RS}F_{MR}F_{NS}, \tag{6}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$. The EOM is

$$\begin{aligned} \frac{1}{\sqrt{-\hat{g}}}\partial_\nu(\sqrt{-\hat{g}}\,\hat{g}^{\nu\rho}\hat{g}^{\mu\lambda}F_{\rho\lambda}) + \hat{g}^{\mu\lambda}e^{-A}\partial_z\left(e^A F_{4\lambda}\right) = 0, \\ \partial_\mu(\sqrt{-\hat{g}}\,\hat{g}^{\mu\nu}F_{\nu 4}) = 0. \end{aligned} \tag{7}$$

Supposing $A_4 = 0$, and making the decomposition

$$A_\mu(x,z) = \sum_n a_\mu^{(n)}(x)\rho_n(z)e^{A/2}, \tag{8}$$

with the orthonormality condition

$$\int dz\, \rho_m\rho_n = \delta_{mn}, \tag{9}$$

we get the equation of the KK modes $\rho_n(z)$

$$\left[-\partial_z^2 + V_1(z)\right]\rho(z) = \mu_n^2\rho_n(z),$$
$$V_1(z) = \frac{1}{2}A'' + \frac{1}{4}A'^2, \quad (10)$$

and the 4D effective action

$$S_1 = \sum_n \int d^4x\sqrt{-\hat{g}}\left(-\frac{1}{4}f_{\mu\nu}^{(n)}f^{(n)\mu\nu} - \frac{1}{2}\mu_n^2\hat{g}^{\mu\nu}a_\mu^{(n)}a_\nu^{(n)}\right), \quad (11)$$

where $f_{\mu\nu}^{(n)} = \partial_\mu a_\nu^{(n)}(x) - \partial_\nu a_\mu^{(n)}(x)$.

Kaluza-Klein Reduction—Spin 1/2 Fermions

The Dirac action is

$$S_{\frac{1}{2}} = \int d^5x\sqrt{-g}\bar{\Psi}\left\{\Gamma^M(\partial_M+\omega_M)-\eta F(\phi)\right\}\Psi,$$

where $F(\phi)=\phi$ in this report, $\{\Gamma^M,\Gamma^N\}=2g^{MN}$**,**

$$\Gamma^M = e^M{}_{\bar{M}}\Gamma^{\bar{M}} = \mathbf{e}^{-A}(\hat{e}^{\mu}_{\bar{\nu}}\gamma^{\bar{\nu}},\gamma^5) = \mathbf{e}^{-A}(\gamma^\mu,\gamma^5). \tag{12}$$

The non-vanishing components of ω_M are

$$\omega_\mu = \frac{1}{2}(\partial_z A)\gamma_\mu\gamma_5 + \hat{\omega}_\mu. \tag{13}$$

The Dirac equation is

$$\left\{\gamma^\mu(\partial_\mu+\hat{\omega}_\mu)+\gamma^5\,(\partial_z+2\partial_z A)-\eta\,\mathbf{e}^A F(\phi)\right\}\Psi=0. \tag{14}$$

Making the chiral decomposition

$$\Psi(x,z) = \mathbf{e}^{-2A}\Big[\sum\nolimits_n \psi_{Ln}(x)\alpha_{Ln}(z) + \sum\nolimits_n \psi_{Rn}(x)\alpha_{Rn}(z)\Big], \quad (15)$$

where $\psi_{Ln}(x) = -\gamma^5\psi_{Ln}(x)$ **and** $\psi_{Rn}(x) = \gamma^5\psi_{Rn}(x)$ **, and imposing the orthonormality condition**

$$\int \alpha_m^L \alpha_n^R dz = \delta^{LR}\delta_{mn}, \quad (16)$$

we get the equations for left and right chiral fermions

$$\left[-\partial_z^2 + V_{L,R}(z)\right]\alpha_n^{L,R} = \mu_n^2 \alpha_n^{L,R}, \quad (17)$$

$$V_{L,R} = \left[\eta \mathbf{e}^A F(\phi)\right]^2 \mp \partial_z\left[\eta \mathbf{e}^A F(\phi)\right], \quad (18)$$

and effective actions of 4D fermions.

$$S_{1/2} = \sum_n \int d^4x\sqrt{-\hat{g}}\;\bar{\psi}_n[\gamma^\mu(\partial_\mu + \hat{\omega}_\mu) - \mu_n]\psi_n. \quad (19)$$

The tensor fluctuation of the metric

The tensor perturbation of the background metric is

$$ds^2 = e^{2A(z)}[(\eta_{\mu\nu} + \bar{h}_{\mu\nu}(x,z))dx^\mu dx^\nu + dz^2], \tag{20}$$

where $\bar{h}_{\mu\nu}$ satisfies the transverse traceless (TT) condition [DeWolfe, Freedman, Gubser and Karch, PRD62(2000)046008]:

$$\bar{h}^\mu{}_\mu = 0 = \partial^\nu \bar{h}_{\mu\nu}. \tag{21}$$

The equation for $\bar{h}_{\mu\nu}$ is [PRD62(2000)046008]

$$\left[\partial_z^2 + 3A'\partial_z + \Box^{(4)}\right]\bar{h}_{\mu\nu} = 0 \quad \textbf{for GR,} \tag{22}$$

and [Liu, Zhong, Zhao and Li, JHEP1106(2011)135], [Zhong, Liu and Yang, PLB699(2011)398]

$$\left[\partial_z^2 + \left(3A' + \frac{\partial_z f_R}{f_R}\right)\partial_z + \Box^{(4)}\right]\bar{h}_{\mu\nu} = 0 \quad \textbf{for f(R) gravity.} \tag{23}$$

The tensor fluctuation of the metric

By performing the following decomposition

$$\bar{h}_{\mu\nu}(x,z) = e^{-\frac{3}{2}A} e^{ikx} \varepsilon_{\mu\nu} \bar{h}(z), \quad (k^2 = -m^2) \tag{24}$$

where $\varepsilon_{\mu\nu}$ **satisfies the TT condition:** $\varepsilon^{\mu}{}_{\mu} = \partial^{\nu}\varepsilon_{\mu\nu} = 0$**, we obtain the equation for the KK modes** $\bar{h}(z)$

$$\left[-\partial_z^2 + V_2(z) \right] \bar{h}(z) = m^2 \bar{h}(z), \tag{25}$$

where

$$V_2 = \frac{3}{2}A'' + \frac{9}{4}A'^2 \quad \text{for GR,} \tag{26}$$

and [JHEP1106(2011)135], [PLB699(2011)398], [Liu, Lu and Wang, JHEP1202(2012)083]

$$V_2 = \frac{3}{2}A'' + \frac{9}{4}A'^2 + \frac{3}{2}A'\frac{f_R'}{f_R} - \frac{1}{4}\frac{f_R'^2}{f_R^2} + \frac{1}{2}\frac{f_R''}{f_R} \quad \text{for f(R).} \tag{27}$$

Content

- Introduction
- Kaluza-Klein Reduction
  - Spin 0 scalar field
  - Spin 1 vector field
  - Spin 1/2 fermion field
  - Spin 2 gravity field
- Localization and Mass Spectrum on Various Branes
  - On dS brane
  - On AdS brane
  - On flat brane in f(R) gravity
  - On flat brane in critical gravity
  - On two-scalar branes
- Conclusion

For the 5D action

$$S = \int d^5dx\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{MN}\nabla_M\phi\nabla_N\phi - V(\phi)\right] \qquad (28)$$

with the potential $V(\phi) = V_0\big(\cos\frac{\phi}{\phi_0}\big)^{2(1-\delta)}$, and the line-element

$$ds^2 = \mathrm{e}^{2A(z)}\big(-dt^2 + e^{2\beta t}dx^i dx^i + dz^2\big), \qquad (29)$$

a dS thick domain wall was found in [Gass and Mukherjee, PRD60(1999)065011], [Wang, PRD66(2002)024024]:

$$\begin{aligned} e^{2A(z)} &= \cosh^{-2\delta}\left(\frac{\beta z}{\delta}\right), \\ \phi(z) &= \phi_0 \arctan\left(\sinh\frac{\beta z}{\delta}\right). \end{aligned} \quad \left(0 < \delta \leq \frac{1}{2}, \beta > 0\right) \qquad (30)$$

For the 5D action (28) with the potential $V(\phi)=-\frac{3(1+3\delta)H^2}{2\delta}\cosh^{2(1-\delta)}\left(\frac{\phi}{\phi_0}\right)$, and the line-element

$$ds^2=\mathrm{e}^{2A(z)}\left[\mathrm{e}^{2Hx_3}(-dt^2+dx_1^2+dx_2^2)+dx_3^2+dz^2\right], \qquad (31)$$

an AdS thick domain wall was found in [Wang, PRD66(2002)024024]:

$$\begin{aligned} e^{2A(z)} &= \cos^{-2\delta}\left(\left|\frac{Hz}{\delta}\right|\right), \\ \phi(z) &= \phi_0 \sinh^{-1}\left(\tan\left|\frac{Hz}{\delta}\right|\right). \end{aligned} \quad (\delta>1,\ \textbf{or}\ \delta<0) \qquad (32)$$

The range of the extra dimension is $-z_b\le z\le z_b$ with $z_b=\left|\frac{\pi\delta}{2H}\right|$.

Flat Vrane in $f(R)$ Gravity

For the 5D $f(R)$ action

$$S = \int d^5x\sqrt{-g}\left(\frac{1}{2\kappa_5^2}f(R) - \frac{1}{2}\partial^M\phi\partial_M\phi - V(\phi)\right), \qquad (33)$$

with $f(R) = R + \gamma R^2$ and $V(\phi) = \lambda(\phi^2 - v^2)^2 + \Lambda_5$, and the line-element

$$ds^2 = \mathrm{e}^{2A(z)}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right), \qquad (34)$$

an $f(R)$ domain wall was obtained in [Liu, Zhong, Zhao and Li, JHEP 1106(2011)135]**:**

$$e^{2A(z)} = \frac{1}{1+\frac{2}{3}\lambda v^2 z^2}, \qquad (35)$$

$$\phi(z) = \pm\frac{\sqrt{2\lambda}v^2 z}{\sqrt{3+2\lambda v^2 z^2}}. \qquad (36)$$

For the action of the $n(\geq 5)$-dimensional critical gravity

[Lu and Pope, PRL106(2011)181302], [Deser, Liu, Lu, et al, PRD83(2011)061502]

$$S_{\mathbf{g}} = \frac{1}{2\kappa^2}\int d^n x\sqrt{-g}\Big[R-(n-2)\Lambda_0+\alpha R^2+\beta R_{MN}R^{MN}\Big], \quad (37)$$

$$S_{\mathbf{m}} = \int d^n x\sqrt{-g}\left[-\frac{1}{2}\partial^M\phi\partial_M\phi - V(\phi)\right], \quad (38)$$

with the critical condition $4(n-1)\alpha + n\beta = 0$ and $V(\phi) = b(\phi^2 - v_0^2)^2$, and the line-element

$$ds^2 = \mathrm{e}^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad (39)$$

a flat domain wall was obtained in [Liu, Wang, Wu and Zhong, arXiv:1201.5922]:

$$e^{2A(y)} = \big[\cosh(ky)\big]^{-\frac{2}{n-2}\kappa^2 v_0^2}, \quad (40)$$

$$\phi(y) = v_0 \tanh(ky). \quad (41)$$

Two-scalar thick branes

For the 5D action

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2\kappa_5^2}R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\pi)^2 - V(\phi,\pi)\right] \tag{42}$$

with the potential

$$V(\phi) = \mathbf{e}^{-2\sqrt{b/3}\,\pi}\left[\frac{1}{2}(\frac{\partial W}{\partial\phi})^2 - \frac{4-b}{6}W^2\right], \tag{43}$$

$$W(\phi) = va\phi\left(1 - \frac{\phi^2}{3v^2}\right), \tag{44}$$

and the line-element

$$ds^2 = \mathbf{e}^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + \mathbf{e}^{2B(y)}dy^2, \tag{45}$$

Two-scalar thick branes (cont.)

a two-scalar thick domain wall was given in [Fu, Liu and Guo, PRD84(2011)044036]**:**

$$\phi(y) = v\tanh(ay), \quad (46)$$

$$A(y) = -\frac{v^2}{9}\left(\ln\cosh^2(ay) + \frac{1}{2}\tanh^2(ay)\right), \quad (47)$$

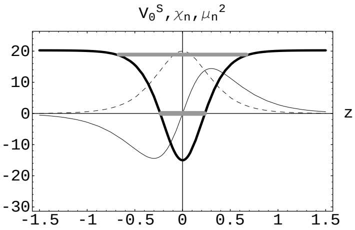
$$\pi(y) = \sqrt{3b}\,A(y), \quad (48)$$

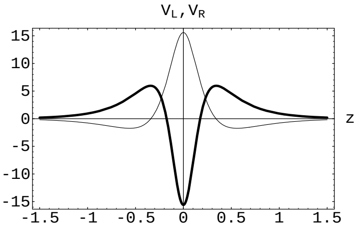
$$B(y) = b\,A(y), \quad (49)$$

Note that the physical length of the extra dimension is finite, although $-\infty \leq y \leq \infty$.

1. dS brane [Liu, Zhao, Wei and Duan, JCAP02(2009)003]

- The scalar and gravity zero modes and first exited KK modes can be localized on the dS brane.
- The vector zero mode can be localized on the dS brane.
- The fermion zero modes can not be localized on the brane for Yukawa coupling $\eta\phi\bar{\psi}\psi$.

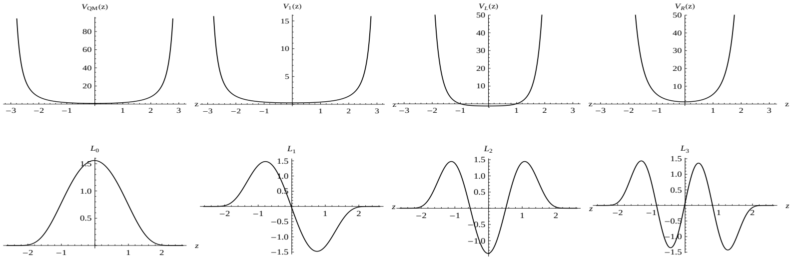




2. AdS brane

[Liu, Guo, Fu and Li, PRD84(2011)044033]

- A series of massive scalar, vector and gravity modes are localized on the AdS brane.
- The zero mode and a series of massive of left-hand fermion are localized on the AdS brane.
- A series of massive of right-hand fermion are localized on the AdS brane.



3. The flat brane in $f(R)$ gravity(see Yuan Zhong's report for details)

[Liu, Zhong, Zhao and Li, JHEP1106(2011)135], [Zhong, Liu and Yang, PLB699(2011)398]

- The scalar and gravity zero modes are localized on the brane .
- The vector zero mode can not be localized on the brane.
- The zero mode of left-hand fermion is localized on the brane if $\eta > \eta_0$.

4. The flat brane in critical gravity

[Liu, Wang, Wu and Zhong, arXiv:1201.5922]

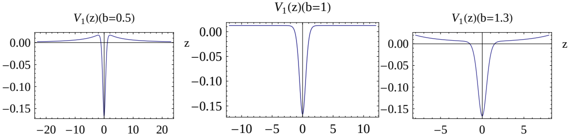
- Is the 4D graviton localized on the brane? Still unknown.
- The scalar (vector) zero mode is (not) localized on the brane.
- The zero mode of left-hand fermion is localized on the brane if $\eta > \eta_0$.

Localization and Mass Spectrum (cont.)

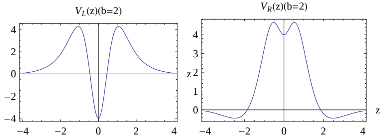
5. Two-scalar thick branes (see Chun-E Fu's report for details)

[Fu, Liu and Guo, PRD84(2011)044036]

- **The scalar, vector, and gravity zero modes can be localized on the dS brane.**
  The spectrum is depended on the parameter b.



- **The zero mode of left-hand fermion is localized on the brane if $\eta > 0$.**



Conclusion

For $f(R)$ gravity theory:

- Scalar and gravity are easy to be localized on the branes, but the trapping of vector is hard.
- In order localized fermions on the branes, we need to introduce the interaction between fermion and scalar. For the usual Yukawa coupling $\eta\phi\bar{\Psi}\Psi$, one of the left and right handed fermion zero modes could localized on the branes.
- The localization of gravity zero mode would lead to the the familiar 4D Newton potential on the branes, the massive KK modes would given a small correction.
- For a few brane models, there is a mass gap in the mass spectrum.
- For a few brane models, the mass spectrum is discrete, like the one in KK theory.

Thanks!