Brane charge and the effective dimension

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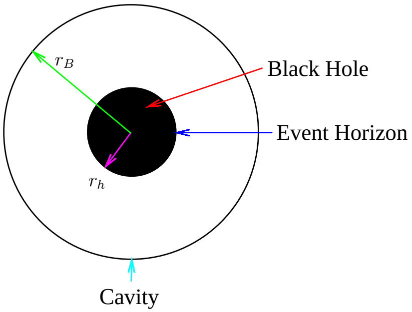
Introduction



Black hole (r_h) placed in a cavity (r_B) with fixed T and V.

Chargeless case (Schwarzschild black hole)

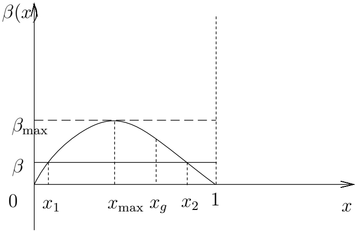


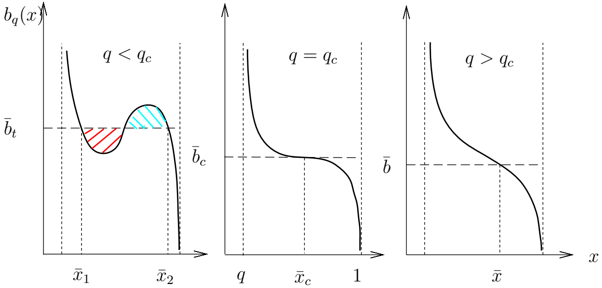
Figure 1: The typical behavior of $\beta(x)$ vs x $(x \equiv r_h/r)$.

Charged case (Reissner-Nordström)

There exists a critical charge $q_c = \sqrt{5} - 2(\bar{x}_c = 5 - 2\sqrt{5}, \bar{b}_c = 0.429)$ and we actually have three cases to consider:

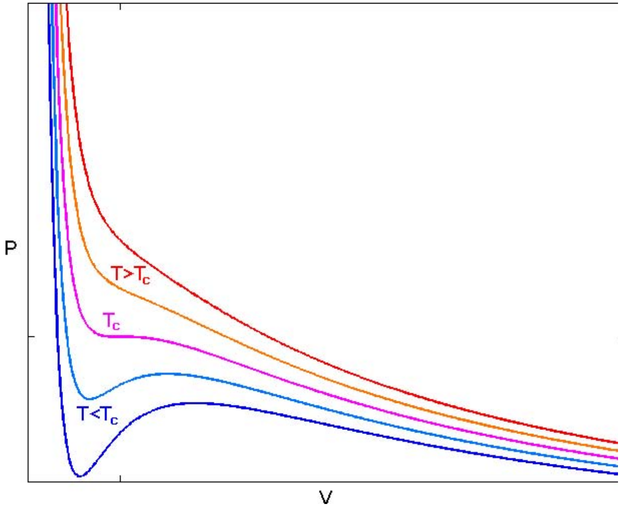
- $q < q_c$, there exists a unique temperature $T_t(q)$ for each given q at which there exists a first order phase transition between a small and large black holes. We have a line of this first-order phase transition, depending on $q < q_c$ and ending at a second-order phase transition point at $q = q_c$;
- $q = q_c$, this is a second-order critical point at which there exists no distinction between small and large black holes. The critical exponent can be read from $c_v \sim (T - T_c)^{-2/3}$ as $2/3$;
- $q > q_c$ for each given temperature T there exists a unique global stable black hole with size $r_+ = r_B \bar{x}$.

Charged case(Reissner-Nordström)



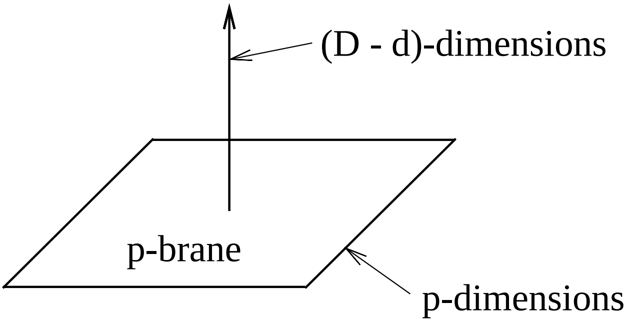
The typical behaviors of $b(x)$ vs x for $q < q_c, q = q_c, q > q_c$.

Van der Waals isotherm



Charged p-brane

The spatial dimensions transverse to the p-brane is $D - d = \tilde{d} + 2$ and note $1 \leq \tilde{d} \leq 7$.



Phase structure and transition

For each given $\tilde{d} > 2$, the phase structure here is the same as the charged black hole though the detail is different. There exists a critical charge q_c, depending on $\tilde{d}$, and we have also the three cases for each given $\tilde{d} > 2$,

Critical quantities and exponents

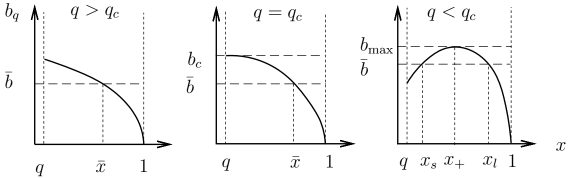
The relevant quantities at the critical point can be calculated explicitly for each allowed value of $\tilde{d}$ as:

$\tilde{d}$	q_c	x_c	b_c
2	0.333333	0.333333	0.288675
3	0.141626	0.292656	0.199253
4	0.090672	0.238800	0.159921
5	0.064944	0.202012	0.134632
6	0.049599	0.175176	0.116698
7	0.039529	0.154691	0.103210

The critical exponents α of $c_v \sim (T - T_c)^{-\alpha}$ can be calculated straightforward and take a universal value of $2/3$, independent of $\tilde{d}$.

Phase structure

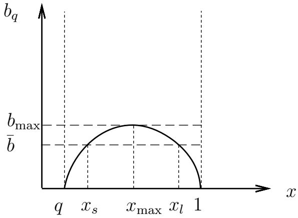
The $\tilde{d} = 2$ case ($q_c = 1/3$):



The typical behaviors of $b_q(x)$ vs x for $\tilde{d} = 2$.

Phase structure

The $\tilde{d} = 1$ case:



The typical behavior of $b_q(x)$ vs x for $\tilde{d} = 1$.

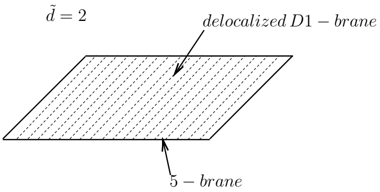
Phase structure

Summary

- The $\tilde{d} > 2$ branes have the similar phase structure as the charged black hole in canonical ensemble, for example, both have van der Waals-Maxwell like phase structure when $q < q_c$. Note the reduced critical quantities are completely determined by $\tilde{d}$.
- While the $\tilde{d} = 1$ brane phase structure resembles that of chargeless black hole instead.
- The $\tilde{d} = 2$ case serves as a boardline between $\tilde{d} > 2$ and $\tilde{d} = 1$ in that its phase structure resembles that of $\tilde{d} = 1$ case with no first order phase transition line ending on a critical point while it has also critical quantities (q_c, x_c, b_c).

D1/D5 (F/NS5)

The D1/D5 (F/NS5) system:



D1/D5 (F/NS5)

The EOS

$$\bar{b} = b_{q1,q5}(\bar{x}), \tag{2.1}$$

$$b_{q1,q5}(x) = \frac{1}{2}x^{1/2}\left(\frac{1-x}{1-\frac{q_5^2}{x}}\right)^{1/2}\left[\frac{1+\sqrt{1+4q_1^2\frac{\frac{1-x}{1-q_5^2/x}}{(1-\frac{1-x}{1-q_5^2/x})^2}}}{2}\right]^{1/2}. \tag{2.2}$$

Note

$$q_5 < x < 1. \tag{2.3}$$

D1/D5 (F/NS5)

If replaced x by f via,

$$x = \frac{1 - f + \sqrt{(1-f)^2 + 4q_5^2 f}}{2}, \tag{2.4}$$

$$\begin{aligned} b_{q_1,q_5}(f) &= \frac{1}{2}\left(\frac{f}{1-f}\right)^{1/2}\left[\frac{1 - f + \sqrt{(1-f)^2 + 4q_1^2 f}}{2}\right]^{1/2} \\ &\times \left[\frac{1 - f + \sqrt{(1-f)^2 + 4q_5^2 f}}{2}\right]^{1/2}, \end{aligned} \tag{2.5}$$

where

$$0 < f < 1. \tag{2.6}$$

D1/D5 (F/NS5)

- If either q_1 or q_5 is zero, the corresponding system behaves like the case of simple black 5-brane. In other words, the delocalized charged 1-branes along 4-spatial directions behave like a charged 5-brane in phase structure.
- When both q_1 and q_5 are non-zero, this system appears to mimics a system of simple branes with $3 > \tilde{d} > 2$.
- So the presence of q_1 effectively increases the transverse dimensions of D5 (NS5) branes.

Critical quantities and exponents

The relevant quantities at each critical point defined by $2/3 \leq f \leq 1$ can be calculated explicitly as:

f	q_{1c}	q_{5c}	x_c	b_c
0.70	0.072062	0.2380771	0.399352	0.26945
0.74	0.041510	0.273234	0.398600	0.274055
0.78	0.023976	0.295128	0.392911	0.278068
0.82	0.013113	0.310007	0.384797	0.28147
0.86	0.006433	0.320244	0.37512	0.284247
0.90	0.001431	0.329431	0.358501	0.287193
0.94	0.000676	0.331195	0.352504	0.287832
0.98	0.000041	0.333107	0.33991	0.28858

D1/D5 (F/NS5)

Now there exists a critical line of second-order transition determined by

$$q_{1c}^2 = \frac{(1+f)^2[(2-f)^2 \pm \sqrt{(2-f)(3f-2)}]^2}{4f(10-5f+f^2)^2} - \frac{(1-f)^2}{4f},$$
$$q_{5c}^2 = \frac{(1+f)^2[(2-f)^2 \mp \sqrt{(2-f)(3f-2)}]^2}{4f(10-5f+f^2)^2} - \frac{(1-f)^2}{4f} \quad (2.7)$$

where

$$2/3 < f < 1. \quad (2.8)$$

We can have either

$$\sqrt{3/2}/8(\approx 0.153093) < q_{5c} < 1/3, \;\; 0 < q_{1c} < \sqrt{3/2}/8, \quad (2.9)$$

or

$$\sqrt{3/2}/8 < q_{1c} < 1/3, \;\; 0 < q_{5c} < \sqrt{3/2}/8. \quad (2.10)$$

Discussions

- The charge q_1 of delocalized D1 (F) effectively increases the transverse dimensions of D5 (NS5) in the thermodynamical sense and so we find a way of generating spatial dimension using charge which appears to be new.
- The generated dimension can be continuous or discrete, depending on that the charge is continuous or quantized.
- So we find the familiar phenomena as in usual critical phenomenon where the spatial dimension can be continuous.

THANK YOU!