High Spin Topologically Massive Gravity

Bin Chen

School of Physics, Peking University

Based on the works with Jiang Long and Jun-bao Wu, 1106.5141,
with Jiang Long, 1110.5113
with jiang Long and Jian-dong Zhang, 1204.3282

Chongqing, 7th May, 2012

High spin field theory I

- There has been a long history on high spin(HS) field theory, since Fierz-Pauli (1939);
- Free HS theory well-defined in flat and curved spacetime; Frondal,
- Interacting HS field theory only well-defined in a spacetime with cosmological constant, positive or negative, to have a gauge invariant theory; Vasiliev
- Remarkable feature: in $D \geq 4$, once we include one massless field with spin higher than two into the interaction, we must include an infinite tower of massless fields with various higher spins and also other compensator fields;
- The interactions involves an infinite number of derivatives;
- Though it looks intractable, HS field theory has drawn much attention in the past decade, for its close relation with string theory and AdS/CFT correspondence;

HS and string theory

- It has been known for a long time string theory has a rich symmetry;
- The massless HS fields appear as the excitations of tensionless string: $m^2 \propto \frac{1}{\alpha'} \to 0$;
- In AdS/CFT correspondence, what's the dual to the free field limit of SYM theory?
- Naively, one may expect that it's the tensionless string in AdS;
- However, the degrees of freedom on both sides in $\mathrm{AdS}_5/\mathrm{CFT}_4$ do not match;
- Nevertheless, it was conjectured by Polyakov and Klebanov in 2002 that the singlet sector of three dim. O(N) vector model in the large N limit is dual to HS theory in AdS_4;
- It is very interesting, but will not be the topic I am going to talk about;

HS in AdS_3

- Strictly speaking, the concept of spin in 3D is ill-defined, as the Poincare group does not allow massless repr. of arbitrary spin;
- Nevertheless, we can still consider symmetric tensor of rank s as "spin"-s field, as widely used in the community;
- HS in AdS_3 is simpler:
  1. Extra compensator fields vanish in 3D;
  2. Moreover, finite truncation to spin n is possible;
  3. Especially, the action of HS in AdS_3 could be rewritten as a Chern-Simons gravity;
- No local physical d.o.f. in 3D gravity;
- This is also true for high spin fluctuations in 3-dim;
- However, there could be boundary d.o.f. $==>$ BTZ black hole;

AdS_3 gravity as CS theory

For pure AdS_3 gravity, it could be written as a Chern-Simons theory: Achucarro and Townsend 1986;E. Witten 1988

1. Combine the dreibein and the spin connection into two SL(2,R) gauge potentials:

$$A = (\omega^a_\mu + \frac{1}{l} e^a_\mu) J_a dx^\mu, \qquad \tilde{A} = (\omega^a_\mu - \frac{1}{l} e^a_\mu) J_a dx^\mu.$$

2. Einstein action + C.C. term

$$S_{EH} + S_\Lambda = S_{CS}[A] - S_{CS}[\tilde{A}] \tag{2.1}$$

where with $k = \frac{l}{4G}$

$$S_{CS}[A] = \frac{k}{4\pi} \int \mathrm{Tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A); \tag{2.2}$$

3. In CS formulation, the asymptotic symmetry of AdS_3 gravity could be analyzed as well, leading to the same conclusion as Brown-Henneaux (1986).

Spin-3 AdS$_3$ gravity Campoleoni et.al. 1008.4744

- To account for spin-3 field, SL(2,R) $--->$ SL(3,R);
- The $SL(3,R)$ group has the generators $J_a, T_{ab} (a,b = 1,2,3)$ with T_{ab} being symmetric and traceless;
- They satisfy the following commutation relations:

$$[J_a, J_b] = \epsilon_{abc} J^c, \qquad [J_a, T_{bc}] = \epsilon^d{}_{a(b} T_{c)d},$$

$$[T_{ab}, T_{cd}] = \sigma(\eta_{a(c}\epsilon_{d)be} + \eta_{b(c}\epsilon_{d)ae}) J^e.$$

- We combine the vielbein-like fields and the connections of spin-2 and spin-3 into two gauge potentials $A, \tilde{A}$

$$\begin{aligned} A &= ((\omega^a_\mu + \frac{1}{l} e^a_\mu) J_a + (\omega^{ab}_\mu + \frac{1}{l} e^{ab}_\mu) T_{ab}) dx^\mu, \\ \tilde{A} &= ((\omega^a_\mu - \frac{1}{l} e^a_\mu) J_a + (\omega^{ab}_\mu - \frac{1}{l} e^{ab}_\mu) T_{ab}) dx^\mu; \end{aligned}$$

- Here e^{ab}_μ is the frame-like field for spin-3 field, and ω^{ab}_μ is corresponding spin-connection.

Spin-3 AdS_3 gravity II

- Then the CS action gives a theory for spin-3 field coupled to gravity with a negative cosmological constant.
- Starting from CS theory, the asymptotic symmetry has been studied;
- It was found that with generalized Brown-Henneaux b.c., spin-3 gravity in AdS_3 has W_3 asym. symmetry algebra, with the same central charge $c_L = c_R = 3l/2G$.
- It has been conjectured that for spin-n HS gravity in AdS_3, its asymp. symmetry algebra is W_n algebra with the same central charges;
- In the $n \to \infty$ limit, there is another approach starting from HS algebra directly; M. Henneaux and S.J. Rey 1008.4579
- Many interesting issues have been discussed: HS/CFT correspondence, BH with hair ....

3D Topologically massive gravity

- To have local gravitational degree of freedom, one may add higher-derivative terms;
- A simple choice is to add a gravitational Chern-Simons term, which is parity breaking and topological: S.Deser et.al. 1982

$$I_{CS} = \frac{1}{2\mu} \int d^3x \sqrt{-g} \varepsilon^{\lambda\mu\nu} \Gamma^\rho_{\lambda\sigma} \left(\partial_\mu \Gamma^\sigma_{\rho\nu} + \frac{2}{3} \Gamma^\sigma_{\mu\tau} \Gamma^\tau_{\nu\rho} \right) \qquad (3.1)$$

- It leads to a new massive, propagating d.o.f;
- However, 3D TMG in AdS_3 is not well-defined for generic value μl, either because of the instability or negative energy for black hole;
- W. Li, W. Song and A. Strominger in 2008 found that at the critical point $\mu l = \pm 1$, 3D TMG in AdS_3 could be well-defined;
- Both local mode and left-moving graviton are just pure gauge;
- The only physical d.o.f. is the right-moving boundary graviton;
- Conjecture: chiral gravity is holographically dual to a 2D chiral CFT by imposing self-consistent Brown-Henneaux B.C.;

Motivation

- How HS fields coupled to TMG?Any kind of higher-spin Chern-Simons term?Damour and Deser in 1987
- If this is possible, do they change the nature of chiral gravity?
- What's the asym. symmetry?
- Some kind of generalization of chiral gravity conjecture?
- In our work, we mainly tried to answer the first two questions.

1st order formulation of TMG

In first order formalism, TMG with a negative cosmological constant $\Lambda = -l^{-2}$ is described by the action Deser et.al.(1991),S.Carlip (1991)

$$S_{\text{TMG}} = \frac{1}{8\pi G}\int \left(e^a \wedge R_a + \frac{1}{6l^2}\epsilon_{abc}e^a \wedge e^b \wedge e^c\right) - \frac{1}{16\pi G\mu}\int \left(\mathcal{L}_{\text{CS}} + \beta^a \wedge T_a\right),$$

where

$$\mathcal{L}_{\text{CS}} = \omega^a \wedge d\omega_a + \frac{1}{3}\epsilon_{abc}\omega^a \wedge \omega^b \wedge \omega^c. \qquad (4.1)$$

The field β^a is just a Lagrangian multiplier, imposing the torsion free condition such that the above action is equivalent to the action in terms of Christoffel symbol. It would be illuminating to rewrite the above action in a form relating to Chern-Simons gravity with gauge group $SL(2,R) \times SL(2,R)$:

$$S_{\text{TMG}} = \left(1 - \frac{1}{\mu l}\right) S_{CS}[A] - \left(1 + \frac{1}{\mu l}\right) S_{CS}[\tilde{A}] - \frac{k}{4\pi\mu l}\int \left(\tilde{\beta}^a \wedge T_a\right).$$

High spin TMG BC and J. Long, 1110.5113

We start from the following action

$$S_{TMG} = (1-\frac{1}{\mu l})S_{CS}[A]-(1+\frac{1}{\mu l})S_{CS}[\bar{A}]-\frac{k}{4\pi\mu}\int tr(\beta\wedge(F-\bar{F})). \tag{4.2}$$

where we have the gauge curvature

$$F = dA + A\wedge A, \qquad \bar{F} = d\bar{A} + \bar{A}\wedge\bar{A}. \tag{4.3}$$

and one-form Lagrangian multiplier β. The gauge field A, $\bar{A}$ and the Lagrangian multiplier β are in the adjoint representation of the corresponding group, which is chosen to be $SL(n,R)\times SL(n,R)$ to describe the high spin fields from spin 2 to n.

Remarks

- when $\beta = 0$ and $\mu \to \infty$, it reduces to the action of the well-known high spin AdS_3 gravity which describes a tower of higher spin fields from spin 3 to spin n coupled to gravity;
- when $\beta \neq 0$, the last term is a Lagrangian multiplier. The imposed condition $F = \bar{F}$ looks strange, but it is nothing but torsion-free condition;
- The action describes all the high spin fields coupled to topological massive gravity: there are not only topologically Chern-Simons term for graviton, but also the similar parity-breaking Chern-Simons terms for higher spin fields.
- The equations of motion in a concise form:

$$(1 - \frac{1}{\mu})F - \frac{1}{2\mu}(d\beta + \beta \wedge A + A \wedge \beta) = 0 \tag{4.4}$$

$$(1 + \frac{1}{\mu})\bar{F} - \frac{1}{2\mu}(d\beta + \beta \wedge \bar{A} + \bar{A} \wedge \beta) = 0 \tag{4.5}$$

$$F = \bar{F}. \tag{4.6}$$

Physical fluctuations

- Spin-s gauge fields:

$$h_{\nu,\nu_1\cdots\nu_{s-1}} = \bar{e}_{\nu_1}^{\ a_1}\cdots\bar{e}_{\nu_{s-1}}^{\ a_{s-1}} e_{\nu a_1\cdots a_{s-1}} \tag{4.7}$$

- After a Lorentz transformation, it could be changed into a symmetric field:

$$\Phi_{\nu\nu_1\cdots\nu_{s-1}} = \frac{1}{s}\bar{e}_{(\nu_1}^{\ a_1}\cdots\bar{e}_{\nu_{s-1}}^{\ a_{s-1}} e_{\nu)a_1\cdots a_{s-1}} \tag{4.8}$$

- Φ has a gauge symmetry

$$\delta_\xi\Phi_{\nu\nu_1\cdots\nu_{s-1}} = \nabla_{(\nu}\xi_{\nu_1\cdots\nu_{s-1})}. \tag{4.9}$$

where $\xi_{\nu_1\cdots\nu_{s-1}} = \bar{e}_{\nu_1}^{\ a_1}\cdots\bar{e}_{\nu_{s-1}}^{\ a_{s-1}}\tilde{\Lambda}_{a_1\cdots a_{s-1}}$ is symmetric and traceless;

- More precisely, we have

$$\begin{aligned} h_{\nu,\nu_1\cdots\nu_{s-1}} &= \Phi_{\nu\nu_1\cdots\nu_{s-1}} + \Theta_{\nu,\nu_1\cdots\nu_{s-1}} + \frac{s-2}{2(s-1)}g_{\nu(\nu_1}\Phi'_{\nu_2\cdots\nu_{s-1})} \\ &\quad -\frac{1}{s-1}g_{(\nu_1\nu_2}\Phi'_{|\nu|\nu_3\cdots\nu_{s-1})}. \end{aligned} \tag{4.10}$$

Fluctuations: I

1. In AdS_3 background, the equation of fluctuations

$$\begin{aligned} G(da + a \wedge A + A \wedge a) &= d\beta + \beta \wedge A + A \wedge \beta \\ \bar{G}(d\bar{a} + \bar{a} \wedge \bar{A} + \bar{A} \wedge \bar{a}) &= d\beta + \beta \wedge \bar{A} + \bar{A} \wedge \beta \\ da + a \wedge A + A \wedge a &= d\bar{a} + \bar{a} \wedge \bar{A} + \bar{A} \wedge \bar{a} \end{aligned}$$

   where we have defined $G = 2(\mu - 1), \bar{G} = 2(\mu + 1)$.
2. This linearized equations describe free fluctuations of spin 2 to spin n;
3. When $\beta = 0$, we use the previous decomposition of $h_{\nu\nu_1\cdots\nu_{s-1}}$ and choose a gauge that $\Theta_{\nu\nu_1\cdots\nu_{s-1}} = 0$, then we can derive the Fronsdal equation

$$\begin{aligned} \mathcal{F}_{\nu_1\cdots\nu_s} \equiv \Box\Phi_{\nu_1\cdots\nu_s} - \nabla_{(\nu_1|}\nabla^\sigma\Phi_{\sigma|\nu_2\cdots\nu_s)} + \frac{1}{2}\nabla_{(\nu_1}\nabla_{\nu_2}\Phi'_{\nu_3\cdots\nu_s)} \\ -(s^2 - 3s)\Phi_{\nu_1\cdots\nu_s} - 2g_{(\nu_1\nu_2}\Phi'_{\nu_3\cdots\nu_s)} = 0 \end{aligned}$$

Fluctuations: II

1. When $\beta \neq 0$, in principle we will get a third order differential equation for each spin s field;
2. The calculation is quite tedious;
3. For arbitrary spin $s \geq 2$, we finally obtain the equations of the physical fields

$$\mathcal{F}_{a_1 \cdots a_s} + \frac{1}{\mu s(s-1)} \epsilon_{(a_1|}{}^{bc} \nabla_b \mathcal{F}_{c|a_2 \cdots a_s)} = 0. \tag{4.11}$$

4. To discuss the equations of motion of the free fluctuations, we only need to know the commutation relation between SL(2,R) generators and high spin generators. This allows us to obtain the equations of motion for arbitrary spin up to n in our formulation.

Fluctuations: III

- For every spin ≥ 3, the fluctuations could be decomposed into traceless and trace parts, each satisfying a third order differential equation;
- We can read massless left-moving, right-moving modes and a massive mode, for generic value of μl;
- At the critical point $\mu l = 1$, the massive mode are degenerate with the left-moving mode, both of which become pure gauge, and the only physical degree of freedom is massless right-moving boundary mode;
- Is the theory chiral at the critical point?

Classical solutions in HSTMG BC jiang Long and Jian-dong Zhang, 1204.3282

- Obviously, if $\beta = 0$, all the classical solutions in HS AdS_3 gravity are the solutions of HSTMG;
- In HSTMG, the equations of motion are much more complicated than pure HS AdS gravity;
- Other solutions:
  1. AdS-pp wave: its high spin cousin has been found by us;
  2. Warped AdS spacetime: we managed to find these solutions in the first order formulation of TMG;

Open questions

- We proposed a new action to describe high spin TMG. This action is more tractable;
- Gauge symmetry at non-linearized level?
- Other gauge group?
- Chiral nature?
- Asymptotic symmetry?
- Black hole solution?
- HS fields in warped spacetime?.

Thank you!