η_c mixing effects on charmonium and B meson decays

Hsiang-nan Li

Presented at XS Workshop

May 08, 2012

collaborated with Y.D. Tsai and Q. Zhao

Motivation

- How to understand large
  $Br(B \to \eta' K) \approx 70 \times 10^{-6}$
- How to understand deviation of
  $A_{CP}^{mix}(B \to \eta' K_S) = 0.59 \pm 0.07$ from
  $A_{CP}^{mix}(B \to J/\Psi K_S) \approx 0.687$
- Charm content of η'? Halperin and Zhitnitsky 97, Petrov 97
- Gluonic content of η' does not help. Charng, Kurimoto and Li 06
- Extend $\eta - \eta' - G$ to tetramixing $\eta - \eta' - G - \eta_C$

Mixing angles

- Define singlet and octet states, and unmixed glue and heavy-flavor states
- 3 mixing angles

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |G\rangle \\ |\eta_c\rangle \end{pmatrix} = U_{34}(\theta)U_{14}(\phi_G)U_{12}(\phi_Q) \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \\ |g\rangle \\ |\eta_Q\rangle \end{pmatrix}$$

Rotational matrices

- U_{34} means 3-4 plane fixed during rotation

$$U_{34}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad U_{14}(\phi_G) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi_G & \sin\phi_G & 0 \\ 0 & -\sin\phi_G & \cos\phi_G & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$U_{12}(\phi_Q) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\phi_Q & \sin\phi_Q \\ 0 & 0 & -\sin\phi_Q & \cos\phi_Q \end{pmatrix}.$$

- Flavor states

$$\begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \\ |g\rangle \\ |\eta_Q\rangle \end{pmatrix} = U_{34}(\theta_i) \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \\ |g\rangle \\ |\eta_Q\rangle \end{pmatrix}$$

Parameterization of mixing matrix

$$U(\theta, \phi_G, \phi_Q) \;=$$

$$\begin{pmatrix} c\theta c\theta_i - s\theta c\phi_G s\theta_i & -c\theta s\theta_i - s\theta c\phi_G c\theta_i & -s\theta s\phi_G c\phi_Q & -s\theta s\phi_G s\phi_Q \\ s\theta c\theta_i + c\theta c\phi_G s\theta_i & -s\theta s\theta_i + c\theta c\phi_G c\theta_i & c\theta s\phi_G c\phi_Q & c\theta s\phi_G s\phi_Q \\ -s\phi_G s\theta_i & -s\phi_G c\theta_i & c\phi_G c\phi_Q & c\phi_G s\phi_Q \\ 0 & 0 & -s\phi_Q & c\phi_Q \end{pmatrix}$$

$c\theta \equiv \cos\theta,\ s\theta \equiv \sin\theta$

Mixing angles

- Still uncertain, depending on data inputs and fitting procedures
- $\theta = -11^o \ (-11^o \le \theta \le -17^o)$ Kou 99, corresponding to sizable gluonic content
- $\phi_G = 12^o$, Escribano 08 (wide range, central values varies between 10 and 30 degrees).
- $\phi_Q = ?$, unknown.

Determination of ϕ_Q

- Ratio of decay widths

$$\frac{\Gamma(^1S_0 \to gg)}{\Gamma(^1S_0 \to \gamma\gamma)} \simeq \frac{2\alpha_s^2}{9e_c^4\alpha_e^2} = \frac{9}{8}\left(\frac{\alpha_s}{\alpha_e}\right)^2$$

- PDG2010 data $\Gamma(^1S_0 \to gg) \simeq \Gamma_{\rm tot} = 28.6 \pm 2.2 \ \mathrm{MeV}$
  $\Gamma(^1S_0 \to \gamma\gamma) = 6.7^{+0.9}_{-0.8} \ \mathrm{keV}$ gave $\alpha_S = 0.41 - 0.49$, larger than $\alpha_S(m_C) = 0.24 - 0.26$
- Include NLO correction

$$\frac{\Gamma(^1S_0 \to gg)}{\Gamma(^1S_0 \to \gamma\gamma)} \simeq \frac{9}{8}\left(\frac{\alpha_s}{\alpha_e}\right)^2 \frac{(1+4.8\frac{\alpha_s}{\pi})}{(1-3.4\frac{\alpha_s}{\pi})}$$

- $\alpha_S = 0.28 - 0.33$ still larger

Glueball-η_C mixing

- If η_C has gluonic content

$$|\eta_c\rangle = -s\phi_Q|g\rangle + c\phi_Q|\eta_Q\rangle$$

$$\begin{aligned}\langle gg|\hat{V}|\eta_c\rangle &= -s\phi_Q\langle gg|\hat{V}|g\rangle + c\phi_Q\langle gg|\hat{V}|\eta_Q\rangle \\ &= \left(-\frac{\pi s\phi_Q}{\alpha_s} + c\phi_Q\right)\langle gg|\hat{V}|\eta_Q\rangle,\end{aligned}$$

$$\frac{\Gamma(\eta_c \to gg)}{\Gamma(\eta_c \to \gamma\gamma)} = \frac{9}{8}\frac{(-\pi t\phi_Q + \alpha_s)^2}{\alpha_e^2}\frac{(1+4.8\frac{\alpha_s}{\pi})}{(1-3.4\frac{\alpha_s}{\pi})}$$

- $\alpha_s = 0.25$, if $\phi_Q = 11^o$

Mixing matrix

- Given the three angles, the $\eta - \eta' - G - \eta_C$ mixing matrix

$$U = \begin{pmatrix} 0.720 & -0.693 & 0.039 & 0.008 \\ 0.673 & 0.711 & 0.200 & 0.039 \\ -0.170 & -0.120 & 0.960 & 0.186 \\ 0 & 0 & -0.191 & 0.982 \end{pmatrix}$$

Decay constants

- For flavor states

$$\langle 0|\bar{q}\gamma^{\mu}\gamma_5 q|\eta_q(P)\rangle = -\frac{i}{\sqrt{2}} f_q P^{\mu},$$
$$\langle 0|\bar{s}\gamma^{\mu}\gamma_5 s|\eta_s(P)\rangle = -i f_s P^{\mu},$$
$$\langle 0|\bar{c}\gamma^{\mu}\gamma_5 c|\eta_Q(P)\rangle = -i f_c P^{\mu},$$

- For physical states

$$\langle 0|\bar{q}\gamma^{\mu}\gamma_5 q|\eta(P),\eta'(P),G(P),\eta_c(P)\rangle = -\frac{i}{\sqrt{2}} f^q_{\eta,\eta',G,\eta_c} P^{\mu},$$
$$\langle 0|\bar{s}\gamma^{\mu}\gamma_5 s|\eta(P),\eta'(P),G(P),\eta_c(P)\rangle = -i f^s_{\eta,\eta',G,\eta_c} P^{\mu},$$
$$\langle 0|\bar{c}\gamma^{\mu}\gamma_5 c|\eta(P),\eta'(P),G(P),\eta_c(P)\rangle = -i f^c_{\eta,\eta',G,\eta_c} P^{\mu},$$

Feldmann-Kroll-Stech scheme

- Neglecting OZI-suppressed contributions

$$\begin{pmatrix} f_\eta^q & f_\eta^s & f_\eta^c \\ f_{\eta'}^q & f_{\eta'}^s & f_{\eta'}^c \\ f_G^q & f_G^s & f_G^c \\ f_{\eta_c}^q & f_{\eta_c}^s & f_{\eta_c}^c \end{pmatrix} = U \begin{pmatrix} f_q & 0 & 0 \\ 0 & f_s & 0 \\ 0 & 0 & 0 \\ 0 & 0 & f_c \end{pmatrix}$$

$$\begin{pmatrix} f_q & f_q^s & f_q^c \\ f_s^q & f_s & f_s^c \\ f_g^q & f_g^s & f_g^c \\ f_c^q & f_c^s & f_c \end{pmatrix}$$

Decay constants

- $f_q = f_\pi,\ f_s / f_q = 1.2 - 1.3$, Feldmann, Kroll, and Stech 98
- $f_c = 487.4$ MeV, Peng and Ma 11

$$\begin{pmatrix} f_\eta^q & f_\eta^s & f_\eta^c \\ f_{\eta'}^q & f_{\eta'}^s & f_{\eta'}^c \\ f_G^q & f_G^s & f_G^c \\ f_{\eta_c}^q & f_{\eta_c}^s & f_{\eta_c}^c \end{pmatrix} = \begin{pmatrix} 113 & -90.8 & 3.69 \\ 106 & 93.1 & 19.0 \\ -26.7 & -15.7 & 91.0 \\ 0 & 0 & 478 \end{pmatrix}$$

- $f_{\eta'}^c = 19$ MeV larger than literature, 2.4 MeV from $Br(J/\psi \to \gamma\eta')/Br(J/\psi \to \gamma\eta_c)$ without gluonic content. Ali, Greub 98;Ahmady, Kou 99

The $B \to \eta^{(\prime)} K$ decays

- larger charm content leads to more significant effects on $B \to \eta^{(\prime)} K$
- For CP asymmetries

$$\lambda_{CP} = \eta_f e^{-2i\beta} \frac{\langle f|H_{eff}|\overline{B^0}\rangle}{\langle f|H_{eff}|B^0\rangle}$$

$$A_{CP}^{dir} = \frac{|\lambda|^2 - 1}{1 + |\lambda|^2}, \quad A_{CP}^{mix} = \frac{2Im\lambda}{1 + |\lambda|^2}$$

Without and with η_C mixing

$$\lambda_{CP} = \eta_f e^{-2i\beta} \frac{V_{ub}V_{us}^* T_{\eta' K} - V_{tb}V_{ts}^* P_{\eta' K}}{V_{ub}^* V_{us} T_{\eta' K} - V_{tb}^* V_{ts} P_{\eta' K}}$$

$$\lambda_{CP} = \eta_f e^{-i2\beta} \frac{V_{ub}V_{us}^* T_{\eta' K} - V_{tb}V_{ts}^* P_{\eta' K} + c\theta s\phi_G s\phi_Q V_{cb}V_{cs}^* T_{\eta_c K}}{V_{ub}^* V_{us} T_{\eta' K} - V_{tb}^* V_{ts} P_{\eta' K} + c\theta s\phi_G s\phi_Q V_{cb}^* V_{cs} T_{\eta_c K}}$$

- Amplitudes from NLO PQCD, Xiao, Zhang, Liu, and Guo 08; Liu, Zhang and Xiao 09

Modified values of $\eta' K_S$

- Br: 50X10(-6) -> 60X10(-6)
- Data: (66.1 +- 3.1)X10(-6)
- Significant effect on Br
- Direct CP asymmetry: 0.024 -> 0.023
- Data: 0.05 +- 0.05
- Mixing-induced CP asymmetry: 0.667 -> 0.664
- Data: 0.59 +- 0.07
- Negligible effect on CP asymmetries

Modified values of ηK_S

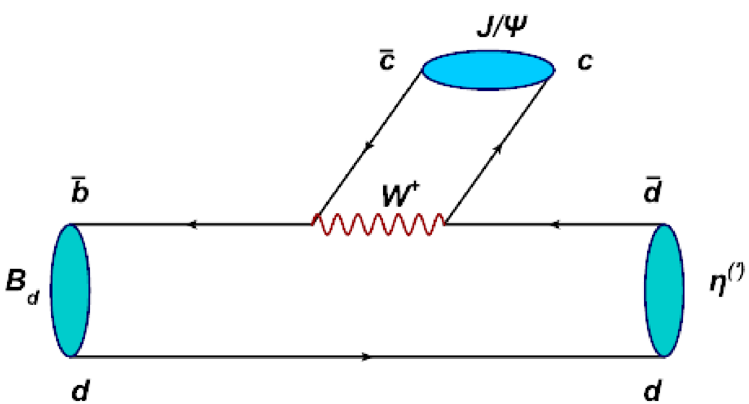
- Br: 2.1X10(-6) -> 2.3X10(-6)
- Data: (1.12 +0.30 - 0.28)X10(-6)
- Direct CP asymmetry:- 0.128 -> -0.123
- Data: --
- Mixing-induced CP asymmetry: 0.659 -> 0.644
- Data: --
- Negligible effect on both Br and CP
- Puzzle due to large $Br(B \to \eta' K)$ resolved

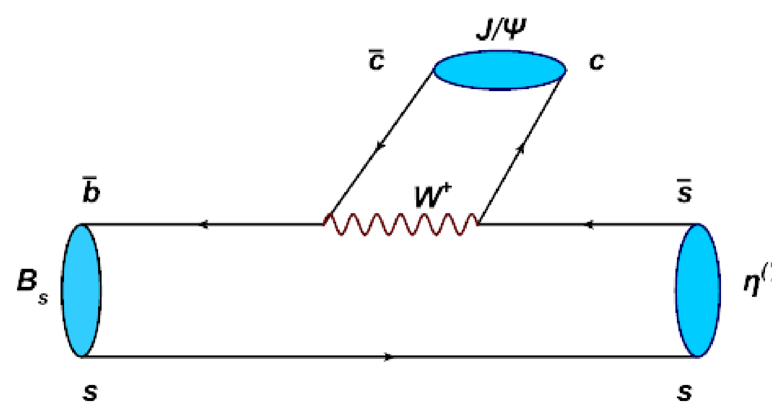
Bs decays (Li, Liu, Xiao 12)

- Recent Belle Data

$$\mathrm{BR}(B_d \to J/\psi\eta)_{\mathrm{Exp.}} = 12.3^{+1.9}_{-1.8} \times 10^{-6} ,$$
$$\mathrm{BR}(B_d \to J/\psi\eta')_{\mathrm{Exp.}} < 7.4 \times 10^{-6} , \quad (90\% \ \mathrm{C.L.})$$

$$\mathrm{BR}(B_s \to J/\psi\eta)_{\mathrm{Exp.}} = 5.10 \pm 0.50(stat.) \pm 0.25(syst.)^{+1.14}_{-0.79}(N_{B_s^{(*)}\bar{B}_s^{(*)}}) \times 10^{-4}$$
$$\mathrm{BR}(B_s \to J/\psi\eta')_{\mathrm{Exp.}} = 3.71 \pm 0.61(stat.) \pm 0.18(syst.)^{+0.83}_{-0.57}(N_{B_s^{(*)}\bar{B}_s^{(*)}}) \times 10^{-4}$$





Anomaly in Bs decays?

- Conventional FKS scheme

$$\mathrm{R_d} \equiv \frac{\mathrm{BR}(B_d \to J/\psi \eta')}{\mathrm{BR}(B_d \to J/\psi \eta)} \approx \tan^2\phi < 1$$

$$\mathrm{R_s} \equiv \frac{\mathrm{BR}(B_s \to J/\psi \eta')}{\mathrm{BR}(B_s \to J/\psi \eta)} \approx \cot^2\phi > 1$$

- In $\eta - \eta' - G$ mixing

$$\mathrm{R_d^{Th}} \approx \left(\frac{\sin\phi - \cos\theta \sin\theta_i \Delta_G}{\cos\phi + \sin\theta \sin\theta_i \Delta_G}\right)^2, \quad \Delta_G = 1 - \cos\phi_G$$

$$\mathrm{R_s^{Th}} \approx \left(\frac{\cos\phi - \cos\theta \cos\theta_i \Delta_G}{-\sin\phi + \sin\theta \cos\theta_i \Delta_G}\right)^2$$

Large glueball contents

- With $\phi_G = 33^o$ (Escribano, 2010)

$$\mathrm{BR}(B_d \to J/\psi\eta) = 11.2^{+2.8}_{-2.1}(\omega_B)^{+1.4}_{-1.3}(f_{J/\psi})^{+1.0}_{-0.8}(a_2) \times 10^{-6}$$
$$\mathrm{BR}(B_d \to J/\psi\eta') = 7.3^{+1.8}_{-1.4}(\omega_B)^{+0.9}_{-0.9}(f_{J/\psi})^{+0.6}_{-0.6}(a_2) \times 10^{-6}$$

$$\mathrm{BR}(B_s \to J/\psi\eta) = 5.14^{+1.45}_{-1.10}(\omega_B)^{+0.64}_{-0.59}(f_{J/\psi})^{+0.58}_{-0.50}(a_2) \times 10^{-4}$$
$$\mathrm{BR}(B_s \to J/\psi\eta') = 4.08^{+1.15}_{-0.87}(\omega_B)^{+0.51}_{-0.47}(f_{J/\psi})^{+0.46}_{-0.40}(a_2) \times 10^{-4}$$

- For $\eta' K_S$ in tetramixing
- Br: 60X10(-6) -> 70X10(-6)
- Data: (66.1 +- 3.1)X10(-6)

Summary

- Proposed a more complete picture for pseudoscalar meson mixing.
- Many puzzles related to η', η_C resolved. Interesting phenomenology!
- Charm content more important than gluonic content in η'
- Theoretical estimates for charm content of η' in literature are also small. Ali, Chay, Greub, Ko 98; Franz, Polyakov, Goeke, 00
- They are for extrinsic charm through two-gluon channel, not intrinsic charm through mixing