Pole Counting and Pole Classification

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Outline

1. Pole counting mechanism
   - 有效力程程展开和共振分类
   - 色散关系 T 矩阵与 CDD 极点
2. 两个例子
   - X(3872)
   - $f_0(980)$ and $a_0(980)$

概要

1. Pole counting mechanism
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根据已有的散射截面或者相移数据，判断粒子是基本的还是复合的一直是困扰粒子物理学家的难题。在一些特殊的情况下，还是可以建立模型无关的方法来进行判断的。例如可以根据阈附近 s 波的极点个数来进行共振分类。

D. Morgan, Nucl. Phys. A543 (1992) 632-644

对于散射振幅总是可以写为

$$T = \frac{1}{M - ik} \tag{1}$$

共振极点对应着 $M - ik$ 的零点，而 M 可以在 k^2 处展开

$$M(k^2) = k cot\delta = -\frac{1}{a} + \frac{1}{2} r_{eff}^2 k^2 + o(k^4) \tag{2}$$

a 是散射长度，r_{eff} 为有效力程。当 r_{eff} 模上限为势宽 R 时，$M - ik = 0$ 有两个解

$$k_{1,2} = \frac{i}{r_{eff}} \pm \sqrt{-\frac{1}{r_{eff}^2} + \frac{2}{ar_{eff}}} \tag{3}$$

一般势散射 r_{eff} 比较小，所以当 $|a| >> 2|r_{eff}|$ 时，只有一个解在阈附近。

势散射在阈处只产生一个极点。例如 deuteron 是一个 np 系统阈下的分子态。结合能为 B 很小，对应着阈下的一个极点 $k = i/R$, 其中 $R = 1/(2\mu B)^{1/2}$,μ 是 np 系统约化质量。由 $M - ik = 0$ 有

$$1/a + r_{eff}/2R^2 = 1/R \tag{4}$$

故 $k = \frac{i}{R}$ 确实对应阈处的一个极点。

- 可以证明对于一般非相对论情况的单道势散射，M 矩阵满足有效势程展开

$$M = -\frac{1}{a} + \frac{1}{2} r_{eff} k^2 + o(k^4) \tag{5}$$

势程 $r_{eff} \approx R$.

- 对于双道，振幅 T 是一个 2×2 的矩阵

$$\hat{T} = [\hat{M} - i\hat{k}]^{-1} \tag{6}$$

$$T_{22} = [\chi_{22} - ik_2]^{-1} \tag{7}$$

可以证明，χ_{22} 也满足有效势程展开

$$\chi_{22} = \chi_{22}^{(0)} + 1/2 r_{eff}^{\chi} k_2^2 + 0(k_2^4) \tag{8}$$

$$r_{eff}^{(\chi)} \approx R \tag{9}$$

由此可知，双道势散射在第二个道阈附近也只有一个极点。

概要

色散关系的 N/D 方法

$$T(\nu) = N(\nu)/D(\nu) \tag{10}$$

$N(\nu)$ 代表左手割线贡献，$D(\nu)$ 代表右手割线的贡献。

$$M(\nu) = T^{-1}(\nu) - i\rho(\nu) \tag{11}$$

其中 $\nu = \frac{k^2}{m^2}, \rho(\nu) = [\nu/(\nu+1)]^{1/2}$.

从分波色散关系出发，如果只有 t 道和 u 道的粒子交换，没有额外的共振和束缚态，解 N/D 方程也会出现类似势散射的有效势程展开

$$M(\nu) = \rho cot\delta = M^{(0)} + \frac{1}{2} r_{eff} m\nu + 0(\nu^2) \tag{12}$$

其中 $r_{eff} \approx 2\sqrt{2}/\mu$,$\mu$ 是交换粒子的质量。

如果有新的粒子耦合到散射系统中，一般会出现 CDD 极点。CDD 极点用两个参数来描述：极点位置和耦合常数 g。这样原来的 S 矩阵会发生改变，$M(\nu)$ 也会发生变化

$$M(\nu) = \frac{\nu_p - \nu}{g^2} + rescattering \quad corrections \tag{13}$$

当 g 很小时，在阈附近就会有两个极点 $\sqrt{\nu} \approx \pm\sqrt{\nu_p}$.

- 对于双道情况，如果只有 t 道和 u 道，也会跟势散射双道情况一样只有一个极点在第二个阈附近。
- 当有 CDD 极点耦合进来时会增加三个参数来描述

$$D_{ij} = (s_P - s)\delta_{ij} - \frac{s - s_p}{\pi} \int_{s_i}^{\infty} \frac{ds' \rho_i(s') N_{ij}(s')}{(s' - s_P)(s' - s)} \tag{14}$$

$$N_{ij} = g_i g_j + \frac{s - s_p}{\pi} \sum_k \int_{-\infty}^{s_L} \frac{ds' Im T^L_{ik}(s') D_{kj}(s')}{(s' - s_P)(s' - s)} \tag{15}$$

其中 g_1, g_2 分别是 CDD 极点与道 1、2 的耦合参数，s_P 是 CDD 极点位置。由 $M = T^{-1} - i\rho = D/N - i\rho$ 可知，在第二个阈附近有两个极点。

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The experimental discovery of X(3872)

Belle Collaboration, Phys.Rev.Lett.91:262001,2003.
Observation of a narrow charmonium-like state in the $B^{\pm} \to K^{\pm}X$, $X \to J/\Psi\pi^+\pi^-$, $M_X = 3872.0 \pm 0.6(\mathrm{stat}) \pm 0.5(\mathrm{syst})$ MeV, very near the $M_D + M_{D^*}$ mass threshold. $\Gamma < 2.3$MeV. $\pi\pi$ produced from ρ decay.

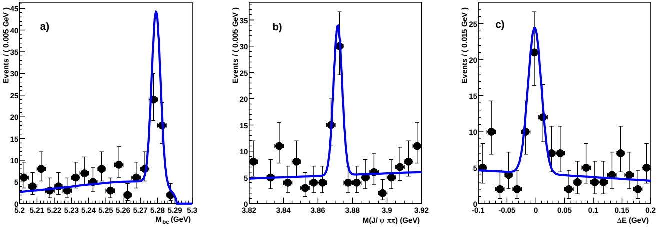

Figure: $M_{J/\Psi\pi^+\pi^-}$ invariant mass spectrum

In addition, BABAR (Phys. Rev. D77: 011102,2008) also gives new measurement on $X \to D^0 \bar{D}^{*0}$

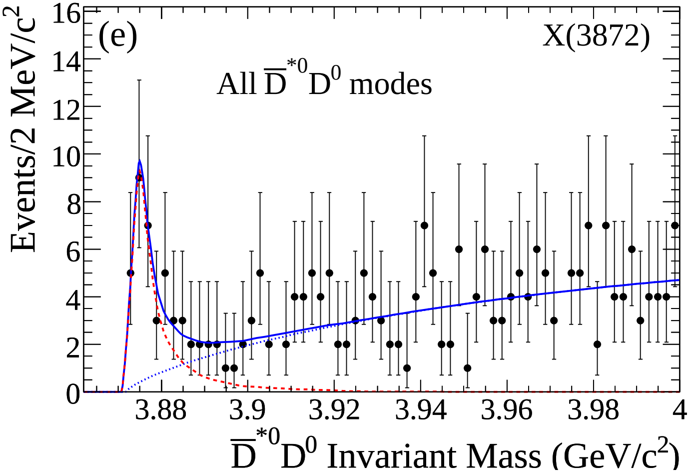

Figure: $M_{D^0\bar{D}^0\pi^0}$ invariant mass spectrum

Theoretical considerations

1. D^0D^{0*} molecular bound state: N.A. Tornqvist, Phys. Lett. B **590**, 209 (2004); F. Close and P. Page, Phys. Lett. B **578**, 119 (2004); C.Y. Wong, Phys. Rev. C **69**, 055202 (2004);E. Braaten and M. Kusunoki Phys. Rev. D **69**, 074005 (2004); M.B. Voloshin, Phys. Lett. B **579**, 316 (2004); E.S. Swanson, Phys. Lett. B **588**, 189 (2004); **598**, 197 (2004).
   (large production rate of X(3872))
2. Normal $c\bar{c}$ state: C. Meng, Y.J. Gao and K.T. Chao, arXiv: hep-ph/0506222. M. Suzuki, Phys. Rev. D **72**, 114013 (2005).
   (mass so close to $D^0\bar{D}^{*0}$ threshold, accident?)
3. Dynamical complexity: couple channel effects, cusp, etc. (D.V. Bugg, e-Print: arXiv:0802.0934 [hep-ph])
4. Virtual state: C. Hanhart, Yu. S. Kalashnikova, A. E. Kudryavtsev and A. V. Nefediev, Phys. Rev. **D76**, 034007 (2007).

Bound state, virtual state, resonances

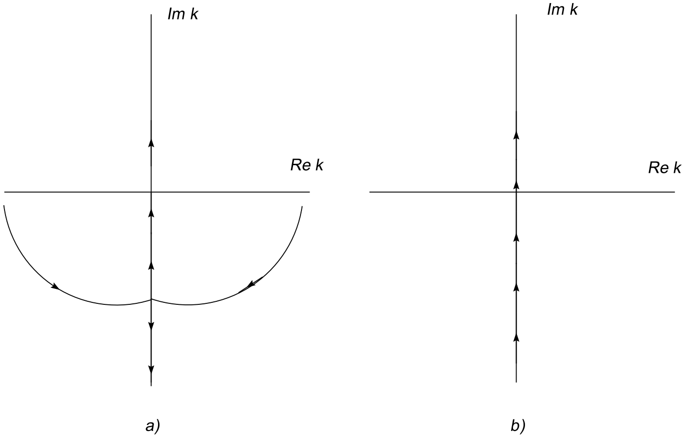

Figure: Typical behavior of pole trajectories w.r.t. coupling strength, calculated in a square well potential.

One needs to look deeper into the pole structures of the scattering amplitude involving $X(3872)$.

1. For a dynamical molecule of $D^0\bar{D}^{*0}$, there is only one pole near the threshold.
2. Two nearby poles imply that it is a $c\bar{c}$ state near the threshold.

D. Morgan's pole counting mechanism!

Previous fits to the line shape of $B^+ \to XK^+$ in the $J/\Psi\pi^+\pi^-$ and $D^0\bar{D}^0\pi^0/D^0\bar{D}^{*0}$ channel give an one-pole structure (bound state or virtual bound state).

Energy Resolution has to be taken into account

$$Br(E) = \frac{1}{\sqrt{2\pi}\sigma(E)} \int dE_x \, Br(E_X) \, e^{-\frac{(E_X - E)^2}{2\sigma(E)^2}} \; . \qquad (16)$$

In general, the energy resolution parameter σ is a function of E.
For $J/\Psi\pi^+\pi^-$ channel at Belle:

$$\sigma(E) = 3MeV \; . \qquad (17)$$

For $D^{0*}D^0$ at Belle:

$$\sigma(E) \simeq 0.176\sqrt{E - M_{D^{0*}D^0}} \; . \qquad (18)$$

(S. Olsen, private communications.)
Refit to new data!

Fit to $X \to D^{*0}\bar{D}^0$: B. Aubert et al. (BaBar Collaboration), Phys. Rev. D77: 011102,2008.

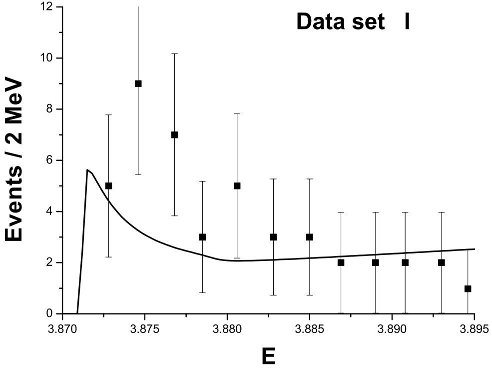

Figure:

Fit to $X \to D^0\bar{D}^0\pi^0$: G. Gokhroo et al. (Belle Collaboration), Phys. Rev. Lett. 97(2006)162002.

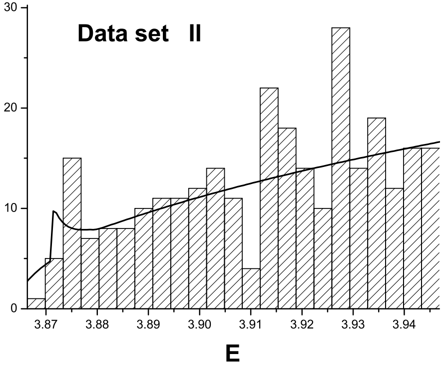

Figure:

Fit to $X \to J/\Psi\pi^+\pi^-$: B. Aubert et al. (BaBar Collaboration), Phys. Rev. **D73**(2006)011101.

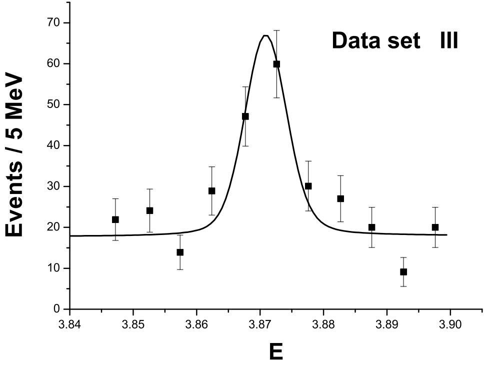

Figure:

Fit to $X \to J/\Psi\pi^+\pi^-$: Belle Collaboration, arXive:0809.1224 [hep-ex]

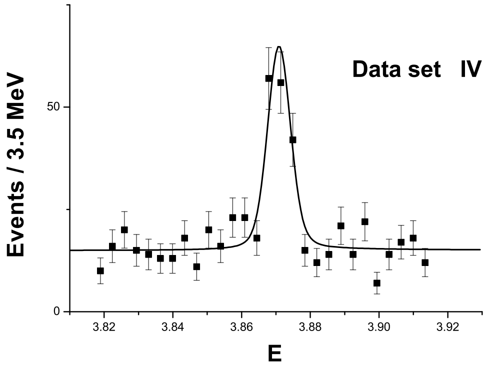

Figure:

Fit program

With one group of un-binmed data set, we use a mixed minimization procedure:

$$\chi^2_{eff} \equiv -2\sum_i \log \mathcal{L}_i + \sum_j \chi^2_j + \chi^2_R \ , \qquad (19)$$

where $i = \mathrm{II}, \mathrm{IV}$; $j = \mathrm{I}, \mathrm{III}$.
The ratio R has to be put into the fitting program as a constraint.
We set $R = \frac{Br(X \to J/\Psi\rho)}{Br(X \to J/\Psi\omega)}$ is set to 1. (variations tested!)

Flatté formalism

Essentially the same as Hanhardt et al., except the modification in the Breit-Wigner propagator,

$$D(E) = E - E_f + \frac{i}{2}(g_1 k_1 + g_2 k_2 + \Gamma(E) + \Gamma_c)\ , \qquad (20)$$

Here subscript 1: $D^0\bar{D}^{0*}$ channel; 2: D^+D^{*-} channel; $\Gamma(E)$ for $J/\Psi\rho$, $J/\Psi\omega$; Γ_c: all other channels. For more details, we refer to the papers.

Numerical Results

$\mathcal{B} = \mathrm{Br}(B \to KX)$ parameters $\sim$ a few times 10^{-4}!
$\mathcal{B}$ has to be confined within reasonably small value!

$\mathcal{B} = 3 \times 10^{-4}$	$g_X(GeV)$	E_f(MeV)	$f_\rho \times 10^3$	$f_\omega \times 10^2$	Γ_c
$\chi^2 = 4090$	4.20	-6.89	1.46	1.01	2.02 ± 1.6
$\chi^2 = 4092$	5.57	-10.3	0.74	0.53	$-$

Table: Pole positions: $E_X^{III} = M - i\Gamma/2 = -4.82 - 1.58i$MeV,
$E_X^{II} = M - i\Gamma/2 = -0.20 - 0.40i$MeV (with Γ_c);
$E_X^{III} = M - i\Gamma/2 = -7.66 - 0.12i$MeV,
$E_X^{II} = M - i\Gamma/2 = -0.02 - 0.01i$MeV (w/o Γ_c)

Third sheet pole runs away when $\mathcal{B}$ large!

Conclusions:

1. First evidence in support of the existence of two nearby poles, if $\mathrm{Br}(B \to KX)$ reasonably small.
2. Above qualitative picture stable against variations of fit scheme.
3. Strong indication that X(3872)$=c\bar{c}$ confining states heavily renormalized by $D^0\bar{D}^{*0}$ continuum.

Results supported by Kalashnikova, Nefediev, Phys. Rev. D80: 074004, 2009, in a different approach, and by other groups.

O. Zhang, C. Meng, HQZ, Phys.Lett. B680 (2009) 453-458; AIP Conf.Proc. 1257 (2010) 457.

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SU(3) chiral perturbative amplitudes

L. Y. Dai, X.G. Wang, H.Q. Zheng, arXiv:1108.1451 [hep-ph]

As an exercise, we have recalculated all the 1–loop amplitudes $\pi\pi \to \pi\pi$, $\pi\pi \to \bar{K}K$, $\bar{K}K \to \bar{K}K$, $K\pi \to K\eta$ and $\pi\eta \to \bar{K}K$ and confirm previous results found in the literature.

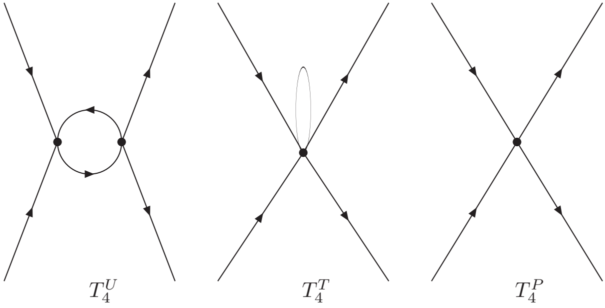


Figure: Feynman diagrams of $\mathcal{O}(p^4)$

Partial wave expansion and matrix Padé unitarization

In two-channel case,

$$\begin{aligned}
\mathrm{Im}\, T_{11} &= T_{11}\rho_1 T_{11}^*\theta(s-4m_\pi^2) + T_{12}\rho_2 T_{21}^*\theta(s-4m_K^2)\ , \\
\mathrm{Im}\, T_{12} &= T_{11}\rho_1 T_{12}^*\theta(s-4m_\pi^2) + T_{12}\rho_2 T_{22}^*\theta(s-4m_K^2)\ , \\
\mathrm{Im}\, T_{22} &= T_{21}\rho_1 T_{12}^*\theta(s-4m_\pi^2) + T_{22}\rho_2 T_{22}^*\theta(s-4m_K^2)\ , \qquad (21)
\end{aligned}$$

The [1,1] matrix Padé approximant reads,

$$T = T^{(2)}\cdot[T^{(2)} - T^{(4)}]^{-1}\cdot T^{(2)}\ . \qquad (22)$$

However, it will cause the difficulty that the left hand cut $(-\infty, 4m_K^2 - 4m_\pi^2]$ will appear not only in T_{22}, but also in the other two amplitudes as well. Hence, Eq. (21) is satisfied exactly only above $\bar{K}K$ threshold, although the deviation from unitarity may be numerically small in some cases.

Fit results

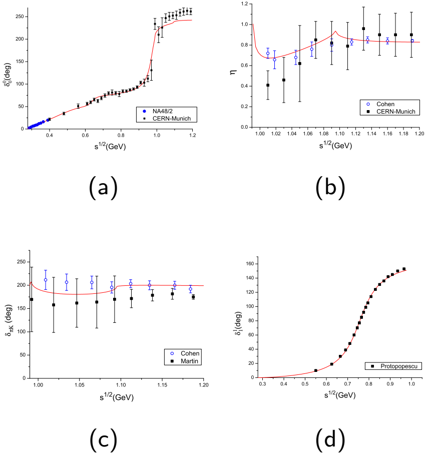


Figure: From Fit I: a) $I = 0$ S wave $\pi\pi$ phase shift; b) inelasticity; c) $\delta_{\pi K}$; d) I,J=1,1 channel $\pi\pi$ phase shift.

Pole counting: $f_0(980)$ and $a_0(980)$

L.Y Dai, X.G. Wang, H.Q. Zheng , arXiv:1108.1451 [hep-ph]

Resonance	II	III	IV
σ	$0.457 - i0.242$		
$f_0(980)$	$0.974 - i0.025$		
$a_0(980)$		$0.640 - i0.002$	$1.131 - i0.079$
$\rho(770)$	$0.740 - i0.069$	$0.782 - i0.056$	
$(I, J) = (2, 0)$	$0.045 m_\pi^2$		
$\kappa(800)$	$0.673 - i0.254$		
$K^*(892)$	$0.895 - i0.026$	$0.921 - i0.021$	

Table: Resonance pole positions on $\sqrt{s}$ plane in unit of GeV, and virtual pole position on s plane.

N_c trajectories

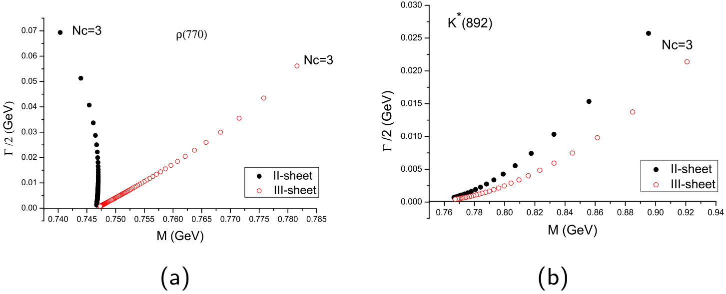
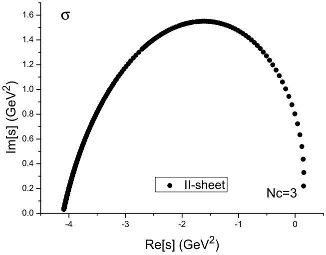
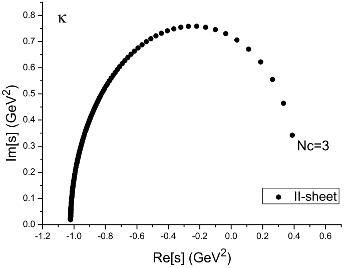


Figure: $\rho(770)$ and $K^*(892)$

Standard couple channel Breit–Wigner resonances!



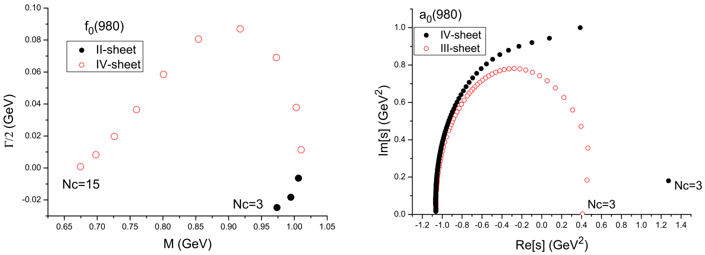
(a)



(b)

Figure: $f_0(600)$ and $\kappa(800)$

Single channel resonances.



(a) (b)

Conclusions: $a_0(980)$ more likely a Breit–Wigner resonance
$f_0(980)$ more like a molecular state.

Figure: $f_0(980)$ and $a_0(980)$

Thank you patience!