

Data Analysis in Direct Searches for Inelastic Dark Matter

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Motivation

Model-independent data analysis

- Reconstruction of the WIMP velocity distribution

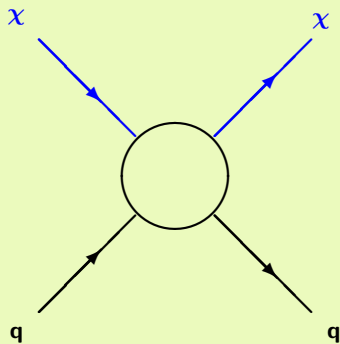
- Determinations of the WIMP mass and the mass splitting

- Reconstruction of the recoil spectrum

Summary and outlook

Motivation

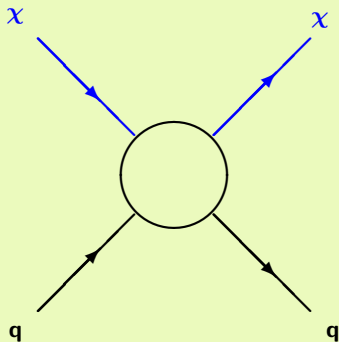
- WIMP-nucleus scattering



Elastically

Motivation

- WIMP-nucleus scattering

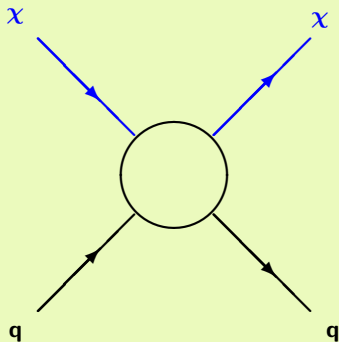


Elastically

- DAMA: 11-year annual modulation observation

Motivation

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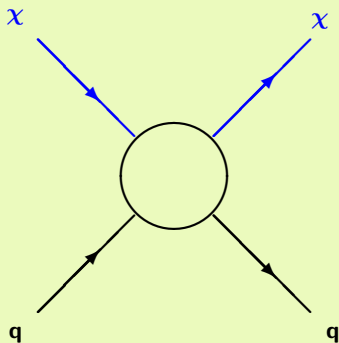


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- CoGeNT: a modulated component of unknown origin

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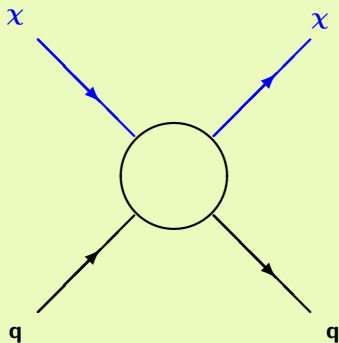


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- CRESST-II: more observed nuclear recoil events than expected backgrounds

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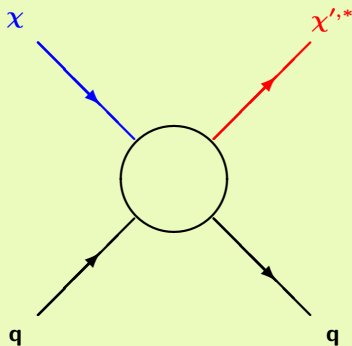


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- CDMS-II, XENON100, ZEPLIN-II: non-observations

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- WIMP-nucleus scattering



Inelastically

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- **CoGeNT**: a modulated component of unknown origin
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Motivation

- Differential event rate for elastic WIMP-nucleus scattering

$$\frac{dR}{dQ} = \mathcal{A} F^2(Q) \int_{v_{\min}(Q)}^{v_{\max}} \left[\frac{f_1(v)}{v} \right] dv$$

Here

$$v_{\min}(Q) = \alpha \sqrt{Q}$$

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy Q in the detector.

$$\mathcal{A} \equiv \frac{\rho_0 \sigma_0}{2m_\chi m_{r,N}^2} \quad \alpha \equiv \sqrt{\frac{m_N}{2m_{r,N}^2}} \quad m_{r,N} = \frac{m_\chi m_N}{m_\chi + m_N}$$

ρ_0 : WIMP density near the Earth

σ_0 : total cross section ignoring the form factor suppression

$F(Q)$: elastic nuclear form factor

$f_1(v)$: one-dimensional velocity distribution of halo WIMPs



Motivation

- Differential event rate for **inelastic** WIMP-nucleus scattering

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Reconstruction of the WIMP velocity distribution - inelastic scattering

- Normalized one-dimensional WIMP velocity distribution function

$$f_1(v) = \mathcal{N} \left\{ \left(\alpha\sqrt{Q} + \frac{\alpha_\delta}{\sqrt{Q}} \right) \left(\alpha\sqrt{Q} - \frac{\alpha_\delta}{\sqrt{Q}} \right)^{-1} \left\{ -2Q \cdot \frac{d}{dQ} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] \right\} \right\}$$

with

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and

$$\mathcal{N} = 2 \left\{ \int_0^\infty \left[\frac{1}{Q} \left(\alpha\sqrt{Q} - \frac{\alpha_\delta}{\sqrt{Q}} \right) \right] \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ \right\}^{-1}$$



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Determinations of the WIMP mass and the mass splitting

- Threshold (minimal required) velocity of incident inelastic WIMPs

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- Determining m_χ and δ (by combining two targets)

$$m_\chi = \frac{Q_{v_{\text{thre}}, Y} m_Y - Q_{v_{\text{thre}}, X} m_X}{Q_{v_{\text{thre}}, X} - Q_{v_{\text{thre}}, Y}}$$

$$\delta = \frac{Q_{v_{\text{thre}}, X} Q_{v_{\text{thre}}, Y} (m_Y - m_X)}{Q_{v_{\text{thre}}, Y} m_Y - Q_{v_{\text{thre}}, X} m_X}$$



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Reconstruction of the recoil spectrum - elastic scattering

- Integrals over two simplest theoretical velocity distributions

$$\left(\frac{dR}{dQ}\right)_{\text{Gau}} \propto F^2(Q) e^{-\alpha^2 Q/v_0^2}$$

$$\left(\frac{dR}{dQ}\right)_{\text{sh}} \propto F^2(Q) \left[\text{erf}\left(\frac{\alpha\sqrt{Q}+v_e}{v_0}\right) - \text{erf}\left(\frac{\alpha\sqrt{Q}-v_e}{v_0}\right) \right]$$

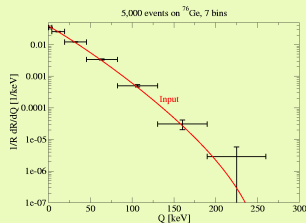
- **Exponential** ansatz for reconstructing the measured recoil spectrum (in the n th Q -bin)

$$\left(\frac{dR}{dQ}\right)_{\text{expt}, Q \simeq Q_n} \equiv r_n e^{k_n(Q-Q_{s,n})} \quad r_n \equiv \frac{N_n}{b_n}$$

- **Logarithmic slope** and **shifted point** (in the n th Q -bin)

$$\overline{Q - Q_n}|_n \equiv \frac{1}{N_n} \sum_{i=1}^{N_n} (Q_{n,i} - Q_n) = \left(\frac{b_n}{2}\right) \coth\left(\frac{k_n b_n}{2}\right) - \frac{1}{k_n}$$

$$Q_{s,n} = Q_n + \frac{1}{k_n} \ln \left[\frac{\sinh(k_n b_n/2)}{k_n b_n/2} \right]$$



[M. Drees and CLS, JCAP 0706, 011]

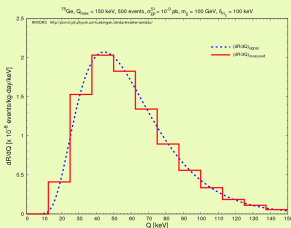
Reconstruction of the recoil spectrum - inelastic scattering

- Integrals over two simplest theoretical velocity distributions

$$\left(\frac{dR}{dQ}\right)_{\text{in, Gau}} \propto F^2(Q) e^{-(\alpha^2 Q + \alpha_\delta^2/Q)/v_0^2}$$

$$\left(\frac{dR}{dQ}\right)_{\text{in, sh}} \propto F^2(Q) \left[\text{erf}\left(\frac{\alpha\sqrt{Q} + \alpha_\delta/\sqrt{Q} + v_e}{v_0}\right) - \text{erf}\left(\frac{\alpha\sqrt{Q} + \alpha_\delta/\sqrt{Q} - v_e}{v_0}\right) \right]$$

- Typical recoil spectrum



- Ansatz for reconstructing the measured recoil spectrum

$$\left(\frac{dR}{dQ}\right)_{\text{in, expt}} = r_0 e^{-kQ - k'/Q}$$



Reconstruction of the recoil spectrum - inelastic scattering

- Moments of the measured recoil energies

$$\int_{x_1}^{x_2} \frac{1}{\sqrt{x}} e^{-ax-b/x} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} \left\{ e^{2\sqrt{a}\sqrt{b}} \left[\operatorname{erf} \left(\sqrt{a}\sqrt{x_2} + \frac{\sqrt{b}}{\sqrt{x_2}} \right) - \operatorname{erf} \left(\sqrt{a}\sqrt{x_1} + \frac{\sqrt{b}}{\sqrt{x_1}} \right) \right] + e^{-2\sqrt{a}\sqrt{b}} \left[\operatorname{erf} \left(\sqrt{a}\sqrt{x_2} - \frac{\sqrt{b}}{\sqrt{x_2}} \right) - \operatorname{erf} \left(\sqrt{a}\sqrt{x_1} - \frac{\sqrt{b}}{\sqrt{x_1}} \right) \right] \right\}$$

$$\begin{aligned} & \int_{x_1}^{x_2} \sqrt{x} e^{-ax-b/x} dx \\ &= \frac{1}{2} \sqrt{\frac{\pi}{a}} \left\{ \frac{1}{2a} \left\{ e^{2\sqrt{a}\sqrt{b}} \left[\operatorname{erf} \left(\sqrt{a}\sqrt{x_2} + \frac{\sqrt{b}}{\sqrt{x_2}} \right) - \operatorname{erf} \left(\sqrt{a}\sqrt{x_1} + \frac{\sqrt{b}}{\sqrt{x_1}} \right) \right] + e^{-2\sqrt{a}\sqrt{b}} \left[\operatorname{erf} \left(\sqrt{a}\sqrt{x_2} - \frac{\sqrt{b}}{\sqrt{x_2}} \right) - \operatorname{erf} \left(\sqrt{a}\sqrt{x_1} - \frac{\sqrt{b}}{\sqrt{x_1}} \right) \right] \right\} \right. \\ & \quad \left. - \sqrt{\frac{b}{a}} \left\{ e^{2\sqrt{a}\sqrt{b}} \left[\operatorname{erf} \left(\sqrt{a}\sqrt{x_2} + \frac{\sqrt{b}}{\sqrt{x_2}} \right) - \operatorname{erf} \left(\sqrt{a}\sqrt{x_1} + \frac{\sqrt{b}}{\sqrt{x_1}} \right) \right] - e^{-2\sqrt{a}\sqrt{b}} \left[\operatorname{erf} \left(\sqrt{a}\sqrt{x_2} - \frac{\sqrt{b}}{\sqrt{x_2}} \right) - \operatorname{erf} \left(\sqrt{a}\sqrt{x_1} - \frac{\sqrt{b}}{\sqrt{x_1}} \right) \right] \right\} \right\} \\ & \quad - \frac{1}{a} \left(\sqrt{x_2} e^{-ax_2-b/x_2} - \sqrt{x_1} e^{-ax_1-b/x_1} \right) \end{aligned}$$



Reconstruction of the recoil spectrum - inelastic scattering

- Moments of the measured recoil energies

$$\langle 1/\sqrt{x} \rangle = \int_0^{\infty} \frac{1}{\sqrt{x}} e^{-ax-b/x} dx = \sqrt{\frac{\pi}{a}} e^{-2\sqrt{a}\sqrt{b}}$$

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- Solving k and k'

$$k = \frac{1}{2} \left(\frac{\langle Q^{-1/2} \rangle \langle Q^{-3/2} \rangle}{\langle Q^{1/2} \rangle \langle Q^{-3/2} \rangle - \langle Q^{-1/2} \rangle^2} \right) \quad k' = \frac{1}{2} \left(\frac{\langle Q^{-1/2} \rangle \langle Q^{-3/2} \rangle}{\langle Q^{-1/2} \rangle \langle Q^{-5/2} \rangle - \langle Q^{-3/2} \rangle^2} \right)$$

$$\langle Q^\lambda \rangle \equiv \frac{\int_0^{\infty} Q^\lambda (dR/dQ)_{\text{in, expt}} dQ}{\int_0^{\infty} (dR/dQ)_{\text{in, expt}} dQ} \rightarrow \frac{1}{N_{\text{tot}}} \sum_a Q_a^\lambda$$



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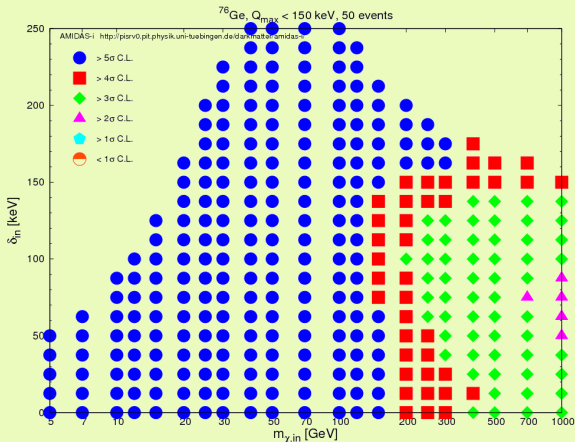
- Equation for solving $Q_{v_{\text{thre}}}$

$$\left(k - \frac{k'}{Q_{v_{\text{thre}}}^2} \right) + \frac{2}{F(Q_{v_{\text{thre}}})} \left(\frac{dF}{dQ} \right)_{Q=Q_{v_{\text{thre}}}} = 0$$

Reconstruction of the recoil spectrum - inelastic scattering

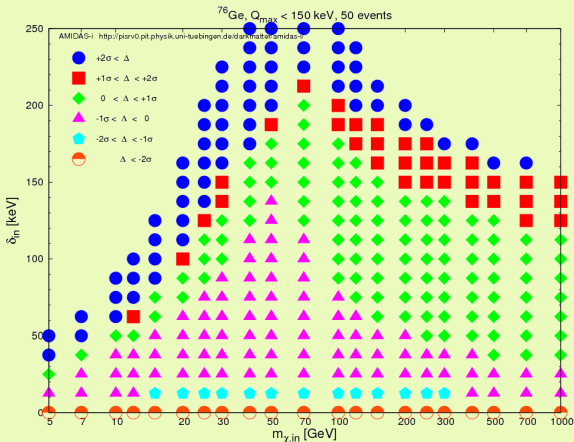
- Checking whether $Q_{\text{vthre,rec}} > 0$

(^{76}Ge , 0 – 150 keV, 50 events)



Reconstruction of the recoil spectrum - inelastic scattering

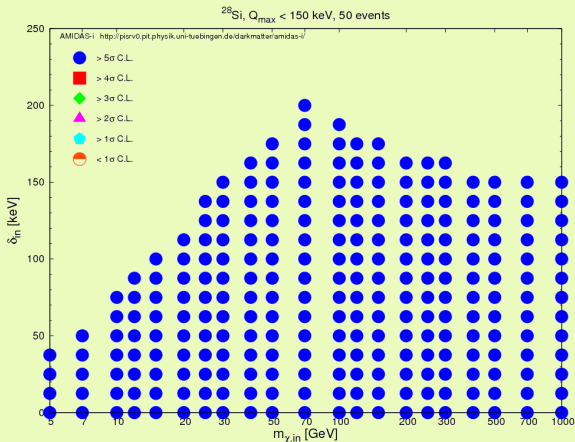
- $Q_{V_{\text{thre,th}}} - Q_{V_{\text{thre,rec}}}$ in unit of $\sigma(Q_{V_{\text{thre,rec}}})$
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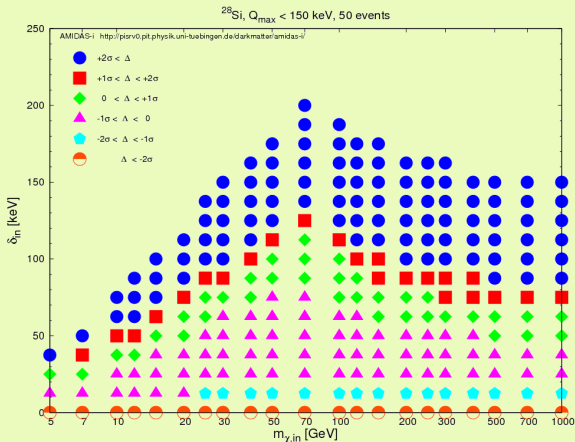
Reconstruction of the recoil spectrum - inelastic scattering

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Reconstruction of the recoil spectrum - inelastic scattering

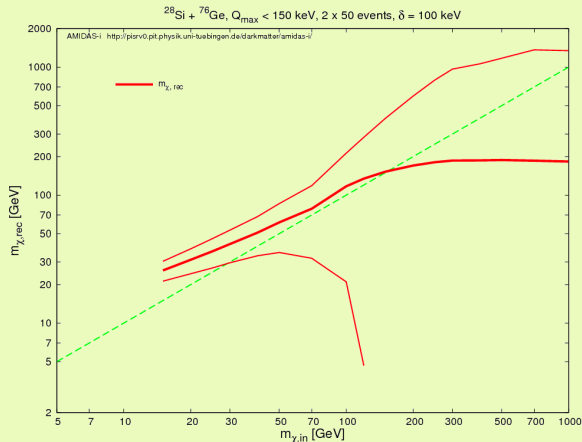
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Determinations of the WIMP mass and the mass splitting

- Reconstructed $m_{\chi, \text{rec}}$

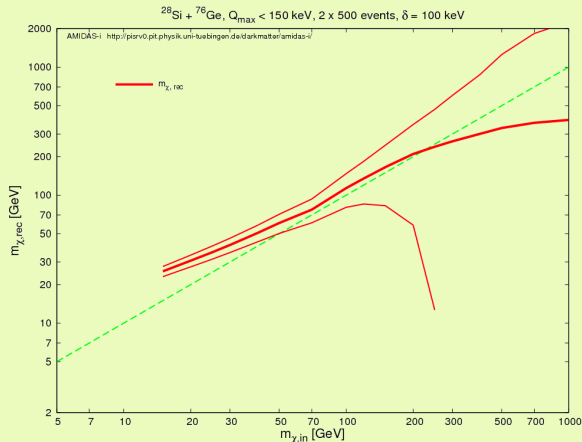
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Determinations of the WIMP mass and the mass splitting

- Reconstructed $m_{\chi, \text{rec}}$

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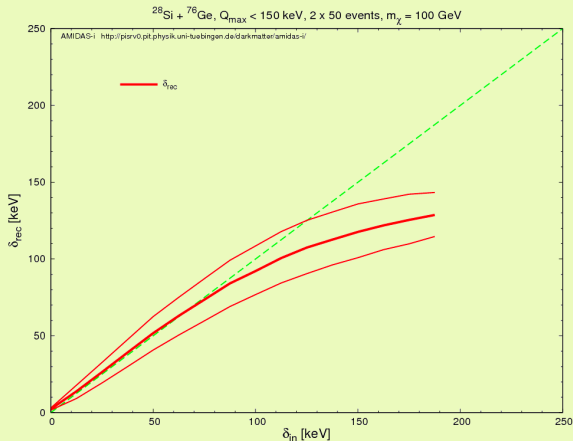




Determinations of the WIMP mass and the mass splitting

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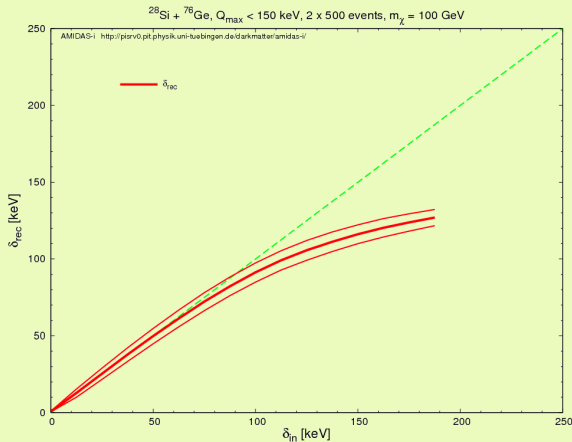




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- └ Model-independent data analysis
 - └ Reconstruction of the recoil spectrum



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- Can we identify **inelastic** WIMP from the elastic one?



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- For **inelastic** ($\delta > 0$) case, we could
 - observe **positive** $Q_{v_{\text{thre}}}$ with a 2σ - 5σ confidence level
 - observe **positive** δ , although would be **underestimated** for $\delta > 75$ keV
 - determine (the upper bound of) m_χ



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 - observe **positive** δ , although would be **underestimated** for $\delta > 75$ keV
 - determine (the upper bound of) m_χ
- For **elastic** ($\delta = 0$) case, we could observe
 - very small, but non-zero positive $Q_{v_{\text{thre}}}$...
 - very small, but non-zero positive δ ...
 - (unphysically) **negative** m_χ : the larger the input m_χ , the larger the absolute value of the reconstructed m_χ



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$$\begin{aligned}
 f_1(v) &= \mathcal{N} \left\{ \left(\alpha\sqrt{Q} + \frac{\alpha_\delta}{\sqrt{Q}} \right) \left(\alpha\sqrt{Q} - \frac{\alpha_\delta}{\sqrt{Q}} \right)^{-1} \left\{ -2Q \cdot \frac{d}{dQ} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] \right\} \right\} \\
 &= \mathcal{N} \left\{ \left(\alpha\sqrt{Q} + \frac{\alpha_\delta}{\sqrt{Q}} \right) \left(\alpha\sqrt{Q} - \frac{\alpha_\delta}{\sqrt{Q}} \right)^{-1} \right. \\
 &\quad \left. \times \left\{ \frac{2Q}{F^2(Q)} \left[\frac{d}{dQ} \ln F^2(Q) + \left(k - \frac{k'}{Q^2} \right) \right] \left(\frac{dR}{dQ} \right) \right\} \right\}
 \end{aligned}$$

with

$$Q(v) = \frac{v^2 - 2\alpha\alpha_\delta \pm v\sqrt{v^2 - 4\alpha\alpha_\delta}}{2\alpha^2}$$

and

$$\mathcal{N} = 2 \left\{ \int_0^\infty \left[\frac{1}{Q} \left(\alpha\sqrt{Q} - \frac{\alpha_\delta}{\sqrt{Q}} \right) \right] \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ \right\}^{-1}$$



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Thank you very much for your attention!