The NMSSM with CP violation and Singlino-driven electroweak baryogenesis

Hou, Tie-Jiun
Collaborators: Kingman Cheung, Jae Sik Lee, Eibun Senaha

IoP, Acdemia Sinica

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Outline

- Introduction to Baryogensis.
- EWBG in the MSM and MSSM.
- First order phase transition in NMSSM.
- EDMs of NMSSM as application on EWBG.
- Singlino-driven Baryon Asymmetry with CTP formalism.
- Summary

Introduction to Baryogensis

The baryon asymmetry of the universe is characterized by the Y_B

$$Y_B \equiv \frac{n_B}{s} = \begin{cases} (6.7 \quad - 9.2 \quad)x10^{-11}, & \text{BBN,} \quad \text{PDG} \\ (8.36 - 9.32)x10^{-11}, & \text{WMAP, 0803.0586} \end{cases}$$

This can not be a consequence of the special separation, there is no mechanism to separate matter from anti-matter in SM of cosmology.
At 1967, Sakharov proposed three conditions:

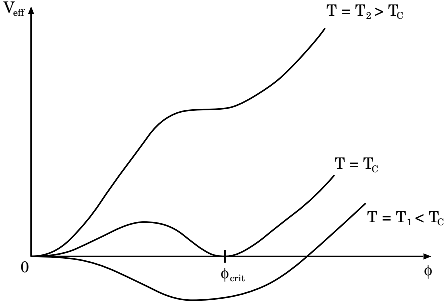
- Baryon number violation.
- C and CP violation.
  If C conserve, then $i \to f$ will exact equal to $\bar{i} \to \bar{f}$. CP conservation, with CPT theorem, expect the $i(r_i, p_i, s_i) \to f(r_j, p_j, s_j)$ equal to time reverse process $f(r_j, p_j, s_j) \to i(r_i, p_i, s_i)$. where r is coordinate, p is momentum, s is spin.
- Departure from thermal equilibrium.
  The CPT theorem expect exact the same mass, decay rate and opposite charge between particle and anti-particle. This symmetries lead us to conclude $n_b = n_{\bar{b}}$. Only when out-of-thermal equilibrium, we can avoid the control of CPT theorem and produce baryon number asymmetry.

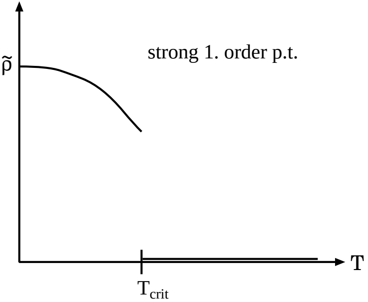
Electroweak Baryogenesis in the SM

In 1985, Kuzmin, Rubakov and Shaposhnikov suggest that anomalous baryon number violation in the SM to be the baryon number violation process.

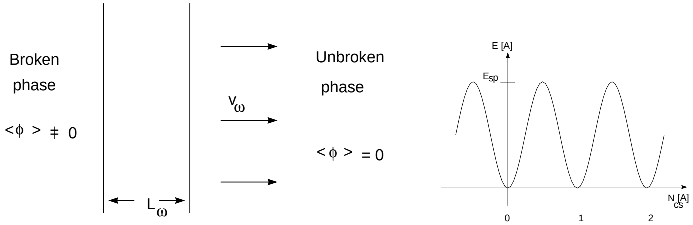
The SM satisfy the three conditions

- The baryon number violation is attained by sphaleron process.
- CP non-conservation happen in the CKM matrix. C is violated in weak interaction.
- The thermal non-equilibrium may be able to attained through the first order phase transition during the electroweak symmetry breaking.





Electroweak Baryogenesis



- At zero temperature, the reaction rate of sphaleron process is $e^{-\frac{2\pi}{\alpha_W}}$ suppressed.
- In the broken phase at finite temperature, $\Gamma_{sph}/V \propto (\frac{m_w(T)}{\alpha_W T})^3 M_W^4 e^{-\frac{E_{sph}}{T}}$, where $E_{sph} = \frac{2m_w(T)}{\alpha_w} B\left(\frac{\lambda}{g_w}\right)$ and $m_w(T) = g_w\langle\phi(T)\rangle/2$.
- In the symmetry phase, $\Gamma_{sph}/V \propto \alpha_w^5 T^4 > H = 1.66\sqrt{g_*}\frac{T^2}{M_P}$.
- In order to make sure the baryon number transform into the broken phase to be survived, it require strong first order phase transition with criteria $\frac{\langle\phi(T_C)\rangle}{T_C} \gtrsim 1$.

Electroweak Baryogenesis in the SM

The SM fail to explain the baryon asymmetry.

- The condition of strong first order phase transition require $m_H \lesssim 42$ GeV (with $m_H \lesssim 73$ GeV start to be first order). The baryon number generate in the symmetry phase would be washed-out during bubble expansion.
- $Y_B = \frac{n_B}{s} \sim \frac{\alpha_W^4 T^3}{s} \delta_{CP} \sim 10^{-8} \delta_{CP}$ which require the CP violating parameter $\delta_{CP} \gtrsim 10^{-3}$. However, $\delta_{CP} \sim \frac{A_{CP}}{T_C^{12}} \sim 10^{-20}$ is far too small to explain the observed baryon asymmetry.

Introduction of MSSM

The MSSM is motivated by the hierarchy problem.

$$W_{MSSM} = \hat{U}^c\,\mathbf{h_u}\hat{Q}\hat{H}_u - \hat{D}^c\,\mathbf{h_d}\hat{Q}\hat{H}_d - \hat{E}^c\,\mathbf{h_e}\hat{L}\hat{H}_d + \mu\hat{H}_u\hat{H}_d$$

In order to cause the SUSY breaking, there are lots of soft terms to be introduced into the MSSM, where the

$$\begin{aligned}
\mathcal{L}_{\rm soft}^{\rm MSSM} \quad = \quad & -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \text{c.c.}\right) \\
& -\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d + \text{c.c.}\right) \\
& -\widetilde{Q}^\dagger\,\mathbf{m_Q^2}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m_L^2}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m_{\overline{u}}^2}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m_{\overline{d}}^2}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m_{\overline{e}}^2}\,\widetilde{\overline{e}}^\dagger \\
& - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - \left(bH_uH_d + \text{c.c.}\right).
\end{aligned}$$

Higgs Sector of MSSM

There is no CP phase in the tree level Higgs sector of MSSM.

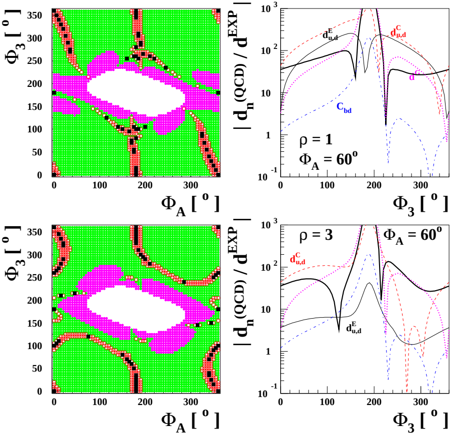
$$
\begin{aligned}
V_{eff} &= V_0 + \Delta V \\
V_0 &= \left(|\mu|^2 + M_{H_u}^2\right)|H_u|^2 + \left(|\mu|^2 + M_{H_d}^2\right)|H_d|^2 - \left(bH_uH_d + c.c.\right) \\
&\quad + \frac{1}{8}(g^2 + g'^2)\left(|H_u|^2 + |H_d|^2\right)^2 \\
\Delta_q V &= N_C \sum_q \left\{ 2 \sum_{j=1,2} \left[F_0(\bar{m}_{\tilde{q}_j}) + \frac{T^4}{2\pi^2} \mathcal{I}_B\left(\frac{\bar{m}_{\tilde{q}_j}}{T}\right) \right] \right. \\
&\quad \left. -4 \left[F_0(\bar{m}_{q_j}) + \frac{T^4}{2\pi^2} \mathcal{I}_F\left(\frac{\bar{m}_{q_j}}{T}\right) \right] \right\}
\end{aligned}
$$

$$
F_0(m^2) = \frac{1}{64\pi^2}(m^2)^2 \left(\log \frac{m^2}{M^2} - \frac{3}{2} \right), \quad \mathcal{I}_{B,F} = \int_0^\infty dx\, x^2 \log(1 \mp e^{-\sqrt{x^2+a^2}})
$$

The strong first order phase transition in MSSM satisfied by the one-loop thermal contribution, which require the top squark to be lighter than the top quark.

EDMs in the MSSM

According to Funakuba (Prog.Theor.Phys.109:415-432,2003), the strong first order phase transition in MSSM happens when $M_{\tilde{t}} < M_t$, $M_{H_1} \lesssim 110\mathrm{GeV}$, and $8 \lesssim \tan\beta \lesssim 30$.



Neutron EDM using QCD sum rule approach.

$|d_n/d^{EXP}| < 1$ (black)

$1 \leq |d_n/d^{EXP}| < 10$ (red)

$10 \leq |d_n/d^{EXP}| < 100$ (green)

$100 \leq |d_n/d^{EXP}|$ (magenta)

John Ellisa, Jae Sik Lee and Apostolos Pilaftsis JHEP 0810:049,2008

Higgs Sector of the NMSSM

The Next-to MSSM(NMSSM) is motivated by the the μ problem in the MSSM.

$$W_{MSSM} = \hat{U}^c\, \mathbf{h_u} \hat{Q} \hat{H}_u - \hat{D}^c\, \mathbf{h_d} \hat{Q} \hat{H}_d - \hat{E}^c\, \mathbf{h_e} \hat{L} \hat{H}_d + \mu \hat{H}_u \hat{H}_d$$

$$\frac{M_Z^2}{2} = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} \simeq -\mu^2 - m_{H_u}^2$$

By introducing the Higgs singlet, the effective μ is replaced by the VEV of the it.

$$W_{NMSSM} = \hat{U}^c\, \mathbf{h_u} \hat{Q} \hat{H}_u - \hat{D}^c\, \mathbf{h_d} \hat{Q} \hat{H}_d - \hat{E}^c\, \mathbf{h_e} \hat{L} \hat{H}_d + \lambda \hat{S}(\hat{H}_u \hat{H}_d) + \frac{1}{3}\kappa \hat{S}^3$$

$$\begin{aligned}
V &= V_F + V_D + V_{\rm soft} \\
V_F &= |\lambda|^2 |S|^2 (|H_u|^2 + |H_d|^2) + |\lambda|^2 |H_u H_d|^2 + |\kappa|^2 |S|^4 \\
&\quad + (|\lambda||\kappa| H_u H_d S^{2*} e^{i(\phi_\lambda - \phi_\kappa)} + \text{h.c.}) \\
V_D &= \frac{1}{8}\bar{g}^2 (|H_d|^2 - |H_u|^2)^2 + \frac{1}{2} g^2 |H_u^\dagger H_d|^2 \\
V_{\rm soft} &= m_1^2 |H_d|^2 + m_2^2 |H_u|^2 + m_S^2 |S|^2 \\
&\quad + (|\lambda||A_\lambda| S H_u H_d e^{i(\phi_\lambda + \phi_{A_\lambda})} - \frac{1}{3}|\kappa||A_\kappa| S^3 e^{i(\phi_\kappa + \phi_{A_\kappa})} + \text{h.c.}),
\end{aligned}$$

where

$$\lambda = |\lambda| e^{i\phi_\lambda}, \kappa = |\kappa| e^{i\phi_\kappa}, A_\lambda = |A_\lambda| e^{i\phi_{A_\lambda}}, A_\kappa = |A_\kappa| e^{i\phi_{A_\kappa}},$$

Electroweak Phase Transition of the NMSSM

It is possible to acquire strong first order phase transition without light top squark in the NMSSM. (Funakubo, Prog.Theor.Phys.114:369,2005)

phase	order parameters	symmetries
EW	$v \neq 0,\ v_s \neq 0$	fully broken
I, I′	$v = 0,\ v_s \neq 0$	local $SU(2)_L \times U(1)_Y$
II	$v \neq 0,\ v_s = 0$	global $U(1)$
SYM	$v = v_s = 0$	$SU(2)_L \times U(1)_Y$, global $U(1)$

Where $v = \sqrt{v_d^2 + (v_u \cos\theta)^2 + (v_u \sin\theta)^2}$. The effective potential is invariant under global $U(1)$, $(v_u \cos\theta + i v_u \sin\theta) \to e^{i\alpha}(v_u \cos\theta + i v_u \sin\theta)$, as $v_s = 0$. As a result, there are four types of phase transition.

A: SYM $\to$ I $\Rightarrow$ EW B: SYM $\to$ I′ $\Rightarrow$ EW

C: SYM $\Rightarrow$ II $\to$ EW D: SYM $\Rightarrow$ EW

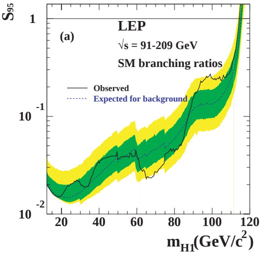
It shows that, strong first order phase transition can be achieved in type-B scenario without light top squark[1].

[1] Funakubo et al., Prog.Theor.Phys.114:369,2005

Conditions on the Higgs Sector

In our work[1], we study the allowed parameter space for the type B strong first order phase transition scenario with three conditions,

- Positivity of the Higgs mass square, $M^2_{H_i} \geq 0$;
- The prescribed vacuum to be global minimum, by numerical downhill simplex method.
- The LEP limit, from Eur. Phys. J. C**47**, 547 (2006).



Besides the one loop contribution, we also take into account the two-loop log enhenced contribution into the effective potential through renormalization group improvement.

The free parameters in Higgs boson sector of the NMSSM at one loop level are,

$$\begin{aligned}
\text{tree level} \quad &: \quad |\lambda|,\ |\kappa|,\ \tan\beta,\ v_s, |A_\lambda|,\ |A_\kappa|, (\phi'_\lambda - \phi'_\kappa), \\
&\quad\ \mathrm{sign}[\cos(\phi'_\lambda + \phi_{A_\lambda})], \mathrm{sign}[\cos(\phi'_\kappa + \phi_{A_\kappa})] \\
\text{one loop level} \quad &: \quad |A_t|,\ |A_b|,\ M_{Q_3},\ M_{U_3},\ M_{D_3},\ (\phi'_\lambda + \phi_{A_t}),\ (\phi'_\lambda + \phi_{A_b})
\end{aligned}$$

[1] Kingman Cheung, Tie-Jiun Hou, Jae Sik Lee, and Eibun Senaha Phys.Rev.D82:075007,2010.

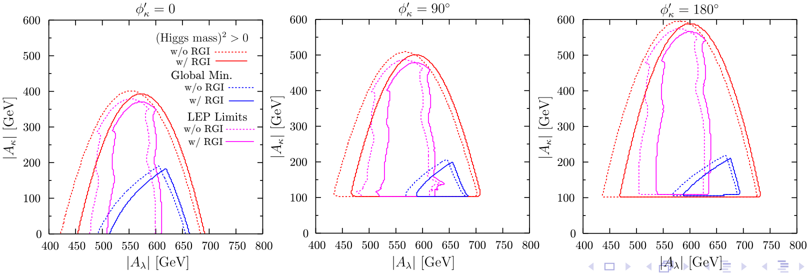
Numerical Result

In this work, we fix part of the parameters to be

$$|A_t| = |A_b| = M_{Q_3} = M_{U_3} = M_{D_3} = 1000\,\mathrm{GeV},$$
$$\phi'_\lambda = \phi_{A_t} = \phi_{A_b} = 0,\ \mathrm{sign}[\cos(\phi'_\lambda + \phi_{\mathrm{A}_\lambda})] = \mathrm{sign}[\cos(\phi'_\kappa + \phi_{\mathrm{A}_\kappa})] = +1.$$

With the type B scenario suggested by Funakubo,

$$\tan\beta = 5\,, \quad v_S = 200\ \mathrm{GeV}\,,$$
$$M_{\tilde{Q}} = M_{\tilde{U}} = M_{\tilde{D}} = |A_t| = |A_b| = 1000\ \mathrm{GeV}\,, \phi'_\lambda = 0\,,$$
$$\mathrm{sign}\,[\cos(\phi'_\kappa + \phi_{A_\kappa})] = \mathrm{sign}\,[\cos(\phi'_\lambda + \phi_{A_\lambda})] = +1\,,$$



EDMs in the NMSSM

Base on the experience of MSSM, we further consider the EDMs constraint on the CP phase ϕ'_κ [1].

$$\begin{aligned}\mathcal{L} &= -\frac{i}{2}\, d_f^E\, F^{\mu\nu}\, \bar{f}\, \sigma_{\mu\nu}\gamma_5\, f \;-\; \frac{i}{2}\, d_q^C\, G^{a\,\mu\nu}\, \bar{q}\, \sigma_{\mu\nu}\gamma_5\, T^a\, q \\ &\quad + \frac{1}{3}\, d^{\,G}\, f_{abc}\, G^a_{\rho\mu}\, \widetilde{G}^{b\,\mu\nu}\, G^c{}_\nu{}^\rho \;+\; \sum_{f,f'} C_{ff'}(\bar{f} f)(\bar{f}' i\gamma_5 f')\,,\end{aligned}$$

$$d_f^E = (d_f^E)^{\widetilde{\chi}^0} + (d_f^E)^{\rm BZ}\,; \quad d_q^C = (d_q^C)^{\widetilde{\chi}^0} + (d_q^C)^{\rm BZ}\,.$$

$$\left(d_f^E\right)^{\rm BZ} = \left(d_f^E\right)^{\gamma H^0} + \left(d_f^E\right)^{W^\mp H^\pm} + \left(d_f^E\right)^{W^\mp W^\pm} + \left(d_f^E\right)^{ZH^0}$$

[1] Kingman Cheung, Tie-Jiun Hou, Jae Sik Lee, and Eibun Senaha Phys.Rev.D84.015002,2011.

EDMs in the NMSSM

In this work, we consider three observable EDMs, Thallium(Tl), Neutron(n) and Mercury(Hg).

$$d_{\mathrm{Tl}}\,[e\,\mathrm{cm}] \;=\; -585 \cdot d_e^E\,[e\,\mathrm{cm}] \;-\; 8.5 \times 10^{-19}\,[e\,\mathrm{cm}] \cdot (C_S\,\mathrm{TeV}^2) \;+\; \cdots\,,$$

$$\mathcal{L}_{C_S} = C_S\,\bar{e} i \gamma_5\, e \bar{N} N$$

$$C_S = C_{de}\frac{29\,\mathrm{MeV}}{m_d} + C_{se}\frac{\kappa \times 220\,\mathrm{MeV}}{m_s} + (0.1\,\mathrm{GeV})\,\frac{m_e}{v^2}\sum_{i=1}^{3}\frac{g^S_{H_i gg}\, g^P_{H_i \bar{e} e}}{M^2_{H_i}}$$

EDMs in the NMSSM

For the neutron EDM, there are three hardronic approaches, the Chiral Quark Model (CQM), the Parton Quark Model (PQM) and QCD sum-rule technique.

$$\begin{aligned} d_n^{(CQM)} &= \frac{4}{3}\, d_d^{\rm NDA} - \frac{1}{3}\, d_u^{\rm NDA}\,, \\ d_{q=u,d}^{\rm NDA} &= \eta^E\, d_q^E + \eta^C\, \frac{e}{4\pi}\, d_q^C + \eta^G\, \frac{e\Lambda}{4\pi}\, d^{\,G}\,, \end{aligned}$$

$$d_n^{(PQM)} = \eta^E\, (\Delta_d^{\rm PQM}\, d_d^E + \Delta_u^{\rm PQM}\, d_u^E + \Delta_s^{\rm PQM}\, d_s^E)\,,$$

$$\Delta_d^{\rm PQM} = 0.746,\ \Delta_u^{\rm PQM} = -0.508,\ \Delta_s^{\rm PQM} = -0.226.$$

$$\begin{aligned} d_n^{(QCD)} &= d_n(d_q^E\,, d_q^C) + d_n(d^{\,G}) + d_n(C_{bd}) + \cdots\,, \\ d_n(d_q^E\,, d_q^C) &= (1.4 \pm 0.6)\,(d_d^E - 0.25\, d_u^E) + (1.1 \pm 0.5)\, e\,(d_d^C + 0.5\, d_u^C)/g_s\,, \\ d_n(d^{\,G}) &\sim \pm\, e\,(20 \pm 10)\ {\rm MeV}\, d^{\,G}\,, \\ d_n(C_{bd}) &\sim \pm\, e\, 2.6 \times 10^{-3}\ {\rm GeV}^2 \left[\frac{C_{bd}}{m_b} + 0.75\frac{C_{db}}{m_b}\right]\,, \end{aligned}$$

EDMs in the NMSSM

Using QCD sum-rule, the Mercury EDM is estimate with the uncertainty of Schiff-moment

$$d_{\mathrm{Hg}}^{\mathrm{I,II,III,IV}} = d_{\mathrm{Hg}}^{\mathrm{I,II,III,IV}}[S] + 10^{-2} d_e^E + (3.5\times 10^{-3}\mathrm{GeV})\, e\, C_S + (4\times 10^{-4}\ \mathrm{GeV})\, e \left[C_P + \left(\frac{Z-N}{A}\right)_{\mathrm{Hg}} C_P' \right],$$

where $\mathcal{L}_{C_P} = C_P\, \bar{e}e\, \bar{N} i\gamma_5 N + C_P'\, \bar{e}e\, \bar{N} i\gamma_5\tau_3 N$

$$C_P \simeq -375\ \mathrm{MeV} \sum_{q=c,s,t,b} \frac{C_{eq}}{m_q},$$

$$C_P' \simeq -806\ \mathrm{MeV}\, \frac{C_{ed}}{m_d} - 181\ \mathrm{MeV} \sum_{q=c,s,t,b} \frac{C_{eq}}{m_q}.$$

$$\begin{aligned}
d_{\mathrm{Hg}}^{\mathrm{I}}[S] &\simeq 1.8\times 10^{-3}\, e\, \bar{g}_{\pi NN}^{(1)}/\mathrm{GeV}, \\
d_{\mathrm{Hg}}^{\mathrm{II}}[S] &\simeq 7.6\times 10^{-6}\, e\, \bar{g}_{\pi NN}^{(0)}/\mathrm{GeV} + 1.0\times 10^{-3}\, e\, \bar{g}_{\pi NN}^{(1)}/\mathrm{GeV}, \\
d_{\mathrm{Hg}}^{\mathrm{III}}[S] &\simeq 1.3\times 10^{-4}\, e\, \bar{g}_{\pi NN}^{(0)}/\mathrm{GeV} + 1.4\times 10^{-3}\, e\, \bar{g}_{\pi NN}^{(1)}/\mathrm{GeV}, \\
d_{\mathrm{Hg}}^{\mathrm{IV}}[S] &\simeq 3.1\times 10^{-4}\, e\, \bar{g}_{\pi NN}^{(0)}/\mathrm{GeV} + 9.5\times 10^{-5}\, e\, \bar{g}_{\pi NN}^{(1)}/\mathrm{GeV}.
\end{aligned}$$

EDMs in the NMSSM

We also consider the deuteron EDM and EDM of ^{225}Ra for proposed future experiments.

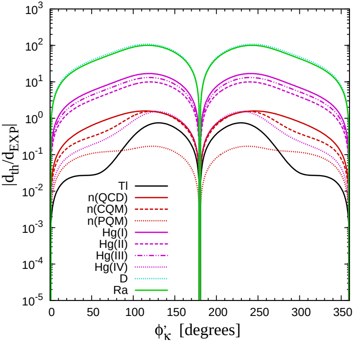
$$\begin{aligned} d_D \;\simeq\; & -\left[5^{+11}_{-3} + (0.6 \pm 0.3)\right]\, e\,(d_u^C - d_d^C)/g_s \\ & -(0.2 \pm 0.1)\, e\,(d_u^C + d_d^C)/g_s \;+\; (0.5 \pm 0.3)(d_u^E + d_d^E) \\ & +(1 \pm 0.2) \times 10^{-2}\, e\,\mathrm{GeV}^2 \left[\frac{0.5\,C_{dd}}{m_d} + 3.3\kappa\frac{C_{sd}}{m_s} + (1 - 0.25\kappa)\frac{C_{bd}}{m_b}\right] \\ & \pm\; e\,(20 \pm 10)\ \mathrm{MeV}\, d^{\,G}\ . \end{aligned}$$

$$d_{\mathrm{Ra}} \simeq d_{\mathrm{Ra}}[S] \simeq -8.7 \times 10^{-2}\, e\,\bar{g}^{(0)}_{\pi NN}\,/\mathrm{GeV} + 3.5 \times 10^{-1}\, e\,\bar{g}^{(1)}_{\pi NN}\,/\mathrm{GeV}\ .$$

Numerical Results

The parameter set is taken from the suggestion of Funakubo for type B parameter set.

$$
\begin{aligned}
&\tan\beta = 5\,, \quad v_S = 200 \text{ GeV}\,,\\
&M_{\widetilde{Q}_{1,2,3}} = M_{\widetilde{U}_{1,2,3}} = M_{\widetilde{D}_{1,2,3}} = M_{\widetilde{L}_{1,2,3}} = M_{\widetilde{E}_{1,2,3}} = 1 \text{ TeV}\,,\\
&|A_e| = |A_u| = |A_d| = |A_s| = |A_\tau| = |A_t| = |A_b| = 1 \text{ TeV}\,,\\
&\phi_{A_e} = \phi_{A_u} = \phi_{A_d} = \phi_{A_s} = \phi_{A_\tau} = \phi_{A_t} = \phi_{A_b} = 0\,,\ \phi_{1,2,3} = 0\,,\\
&\phi'_\lambda = 0\,; \quad \text{sign}\,[\cos(\phi'_\kappa + \phi_{A_\kappa})] = \text{sign}\,[\cos(\phi'_\lambda + \phi_{A_\lambda})] = +1\,,
\end{aligned}
$$



$$
\begin{aligned}
d_{\mathrm{Tl}}^{\mathrm{EXP}} &= 9 \quad \times 10^{-25}\, e\,\mathrm{cm}\,,\ \text{PRL,88,071805}\\
d_{\mathrm{n}}^{\mathrm{EXP}} &= 2.9 \times 10^{-26}\, e\,\mathrm{cm}\,,\ \text{PRL,97,131801}\\
d_{\mathrm{Hg}}^{\mathrm{EXP}} &= 3.1 \times 10^{-29}\, e\,\mathrm{cm}\,,\ \text{PRL,86,2505.}\\
d_{\mathrm{D}}^{\mathrm{EXP}} &= 3 \times 10^{-27}\, e\,\mathrm{cm}\ \text{AIP Conf.Proc.698(2004)200}\\
d_{\mathrm{Ra}}^{\mathrm{EXP}} &= 1 \times 10^{-27}\, e\,\mathrm{cm}\ \text{CERN-INTC-2010-049}
\end{aligned}
$$

Baryon Asymmetry

The NMSSM has collect the ingredients for EWBG:

- Baryon Asymmetry is done by the weak sphaleron effect.
- CP violation phase $\phi'_\lambda - \phi'_\kappa$ receive loose constraint from EDMs.
- Strong first order phase transition happens in the type-B scenario without light top squark.

In calculating the baryon asymmetry, we closely follow the method from Lee et. al.[1] and Huet et. al.[2]

The baryon number is determined by the diffusion equation.

$$\partial_t n_B(x) - D\nabla^2 n_B(x) = -\Gamma_{\rm ws} F_{\rm ws}(x)[n_L(x) + R n_B(x)]\,,$$

where the $\Gamma_{\rm ws} = 6\kappa\alpha_w^5 T$ is the weak sphaleron rate and the $F_{\rm ws}$ is the sphaleron transition profile function. The n_L is the number density of left-handed particle. The R is the relaxation coefficient for decay of baryon number through weak sphaleron transition.

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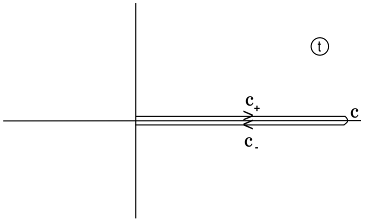
[1] C. Lee, V. Cirigliano, M. J. Ramsey-Musolf, Phys. Rev. D71 (2005) 075010.

[2] P. Huet, A. E. Nelson, Phys. Rev. D53 (1996) 4578-4597.

Close Time Path formalism

In order to consider the thermal non-equilibrium system, J. Schwinger suggest to consider the Close Time Path(CTP) formalism.



Consider scalar field as example,

$$
\begin{aligned}
G^{>}(x,y) &= \langle \phi_{-}(x)\phi_{+}^{\dagger}(y)\rangle \\
G^{<}(x,y) &= \langle \phi_{-}^{\dagger}(y)\phi_{+}(x)\rangle \\
G^{t}(x,y) &= \langle T\{\phi_{+}(x)\phi_{+}(y)\}\rangle = \theta(x_0-y_0)G^{>}(x,y)+\theta(y_0-x_0)G^{<}(x,y) \\
G^{\bar{t}}(x,y) &= \langle T\{\phi_{-}(x)\phi_{-}^{\dagger}(y)\}\rangle = \theta(x_0-y_0)G^{<}(x,y)+\theta(y_0-x_0)G^{>}(x,y)\,,
\end{aligned}
$$

Close Time Path formalism

These Green function can be written as 2×2 matrix,

$$\widetilde{G}(x,y) = \left(\begin{array}{cc} G_{++}(x,y) & G_{-+}(x,y) \\ G_{+-}(x,y) & G_{--}(x,y) \end{array} \right) = \left(\begin{array}{cc} G^{t}(x,y) & G^{<}(x,y) \\ G^{>}(x,y) & G^{\bar{t}}(x,y) \end{array} \right)$$

With the Schwinger-Dyson equation, they satisfy,

$$\widetilde{G}(x,y) = \widetilde{G}^{0}(x,y) + \int d^4w \int d^4z \; \widetilde{G}^{0}(x,w)\widetilde{\Sigma}(w,z)\widetilde{G}(z,y)$$

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With the current for boson to be determined as

$$\langle J^{\mu}_{\phi}(x)\rangle \equiv i\langle \phi^{\dagger}(x) \overset{\leftrightarrow\mu}{\partial_x} \phi(x)\rangle \equiv \left[n_{\phi}(x), \vec{J}_{\phi}(x)\right],$$

and

$$\left(\left(\Box_x + m^2\right) \widetilde{G}^{0}(x,y) = \left(\Box_y + m^2\right) \widetilde{G}^{0}(x,y) = -i\delta^{(4)}(x-y),$$

The quantum transport equation can be obtained

$$\frac{\partial n}{\partial X_0} + \nabla\cdot\mathbf{j}(X) = \int d^3z \int_{-\infty}^{X_0} dz_0 \left[\Sigma^{>}(X,z)G^{<}(z,X) - G^{>}(X,z)\Sigma^{<}(z,X) \right.$$
$$\left. + G^{<}(X,z)\Sigma^{>}(z,X) - \Sigma^{<}(X,z)G^{>}(z,X)\right].$$

Where $X = (x+y)/2$.

Source Term in Diffusion Equation

In the case of NMSSM without CP violation phase but $\phi'_\lambda - \phi'_\kappa$, the key contribution of CP violating source come from the singlino. In this work, we also consider no spontaneous CP phase θ and φ.

$$\mathcal{L} = \frac{|\lambda|}{\sqrt{2}} \overline{\tilde{H}^0}[v_u(x)e^{i(\phi_\lambda - \phi_\kappa/2)} P_L - v_d(x)e^{i\phi_\kappa/2} P_R]\tilde{S} + \text{H.C.}$$

and calculation, one obtain the CP violating source term from the singlet-Higgsino interaction would be

$$S^{\rm CPV}_{\widetilde{S}\widetilde{H}^0} = -2|\lambda|^2 |M_{\widetilde{S}}||\mu_{\rm eff}| v^2 \dot{\beta} \sin(\phi_\lambda - \phi_\kappa)\, \mathcal{I}^f_{\widetilde{S}\widetilde{H}^0} \,,$$

Where $|\mu_{\rm eff}| = |\lambda| v_S/\sqrt{2}$ and $|M_{\tilde{S}}(T)| = \left[2|\kappa|^2 v_S^2 + \frac{|\lambda|^2 + 2|\kappa|^2}{8}\, T^2\right]^{1/2}$.

The fermionic source function $\mathcal{I}^f$ takes the generic form of

$$\mathcal{I}^f_{ij} = \frac{1}{4\pi^2} \int_0^\infty dk \frac{k^2}{\omega_i \omega_j} \Big[(1 - 2{\rm Re}(n_j)) I(\omega_i, \Gamma_i, \omega_j, \Gamma_j) + (1 - 2{\rm Re}(n_i))\, I(\omega_j, \Gamma_j, \omega_i, \Gamma_i) - 2({\rm Im}(n_i) + {\rm Im}(n_j))\, G(\omega_i, \Gamma_i, \omega_j, \Gamma_j) \Big],$$

$$I(a,b,c,d) = \frac{1}{2} \frac{1}{[(a+c)^2 + (b+d)^2]} \sin\left[2 \arctan \frac{a+c}{b+d}\right] + \frac{1}{2} \frac{1}{[(a-c)^2 + (b+d)^2]} \sin\left[2 \arctan \frac{a-c}{b+d}\right],$$

$$G(a,b,c,d) = -\frac{1}{2} \frac{1}{[(a+c)^2 + (b+d)^2]} \cos\left[2 \arctan \frac{a+c}{b+d}\right] + \frac{1}{2} \frac{1}{[(a-c)^2 + (b+d)^2]} \cos\left[2 \arctan \frac{a-c}{b+d}\right].$$

Baryon Asymmetry

$$\partial^\mu Q_\mu = +\Gamma_M^-\left(\frac{T}{k_T}-\frac{Q}{k_Q}\right)+\Gamma_Y\left(\frac{T}{k_T}-\frac{H}{k_H}-\frac{Q}{k_Q}\right)-2\Gamma_{ss}\left(\frac{2Q}{k_Q}-\frac{T}{k_T}+\frac{9(Q+T)}{k_B}\right)$$
$$\partial^\mu T_\mu = -\Gamma_M^-\left(\frac{T}{k_T}-\frac{Q}{k_Q}\right)-\Gamma_Y\left(\frac{T}{k_T}-\frac{H}{k_H}-\frac{Q}{k_Q}\right)+\Gamma_{ss}\left(\frac{2Q}{k_Q}-\frac{T}{k_T}+\frac{9(Q+T)}{k_B}\right)$$
$$\partial^\mu H_\mu = -\Gamma_h\frac{H}{k_H}-\Gamma_Y\left(\frac{Q}{k_Q}+\frac{H}{k_H}-\frac{T}{k_T}\right)+S^{CPV}_{\widetilde{S}\widetilde{H}},$$

Where $k_{Q,T,B}=k_{q_L,t_R,b_R}+k_{\tilde{q}_L,\tilde{t}_R,\tilde{b}_R}$ and $k_H=k_{H_d}+k_{H_u}+k_{\tilde{H}}$,

$$\Gamma_M^-=\frac{6}{T^2}\left(\Gamma_t^-+\Gamma_{\tilde{t}}^-\right),\quad \Gamma_h=\frac{6}{T^2}\left(\Gamma_{\widetilde{H}^\pm\widetilde{W}^\pm}+\Gamma_{\widetilde{H}^0\widetilde{W}^0}+\Gamma_{\widetilde{H}^0\widetilde{B}^0}+\Gamma_{\widetilde{H}^0\widetilde{S}}\right),$$

with $\Gamma_{\widetilde{H}\widetilde{X}}=\Gamma^-_{\widetilde{H}\widetilde{X}}-\Gamma^+_{\widetilde{H}\widetilde{X}}$. The $\Gamma_{ss}=12\kappa'\alpha_s^4 T$ with $\kappa'\sim O(1)$, and $\Gamma_Y\sim\frac{27}{2}h_t^2\alpha_s[\frac{\zeta(3)}{\pi^2}]^2 T$ are much larger than the other, and thus

$$\begin{aligned}
Q &= \frac{(k_B-9k_T)k_Q}{(9k_T+9k_Q+k_B)k_H}H+\mathcal{O}(1/\Gamma_{ss},1/\Gamma_Y)\\
T &= \frac{(9k_T+2k_B)k_T}{(9k_T+9k_Q+k_B)k_H}H+\mathcal{O}(1/\Gamma_{ss},1/\Gamma_Y)
\end{aligned}$$

Baryon Asymmetry

The three diffusion equation than reduce into one,

$$\dot{H} - \bar{D}\nabla^2 H + \bar{\Gamma} H - \bar{S} = \mathcal{O}(1/\Gamma_{ss}, 1/\Gamma_Y) \quad ,$$

Where $\mathbf{T} = -D_q \nabla T$, $\mathbf{Q} = -D_q \nabla Q$, $\mathbf{H} = -D_h \nabla H$, and

$$\bar{D} = \frac{(9k_Q k_T + k_B k_Q + 4k_T k_B)D_q + k_H(9k_T + 9k_Q + k_B)D_h}{9k_Q k_T + k_B k_Q + 4k_T k_B + k_H(9k_T + 9k_Q + k_B)}$$

$$\bar{\Gamma} = \frac{(9k_Q + 9k_T + k_B)(\Gamma_M^- + \Gamma_h)}{9k_Q k_T + k_B k_Q + 4k_T k_B + k_H(9k_T + 9k_Q + k_B)}$$

$$\bar{S} = k_H(9k_Q + 9k_T + k_B)9k_Q k_T + k_B k_Q + 4k_T k_B + k_H(9k_T + 9k_Q + k_B)S_{\widetilde{H}}^{CPV}$$

With $\bar{z} > 0$ to be the broken phase and $\bar{z} < 0$ to be the symmetry phase, and assuming $\bar{\Gamma}$ to be non-zero and constant in $\bar{z} > 0$,

$$H(\bar{z}) = \mathcal{A}\, e^{v_w \bar{z}/\bar{D}}$$

Where $\mathcal{A} = \frac{1}{\bar{D}\kappa_+} \int_0^\infty \bar{S}(y)\, e^{-\kappa_+ y}\, dy$, $\kappa_+ = \frac{v_w + \sqrt{v_w^2 + 4\bar{\Gamma}\bar{D}}}{2\bar{D}} \simeq \sqrt{\frac{\bar{\Gamma}}{\bar{D}}}$. Further assuming the $\bar{S}$ to be constant and non-zero within the bubble wall,

$$\mathcal{A} \simeq k_H L_w \sqrt{\frac{r_\Gamma}{\bar{D}}} \; \frac{S_{\widetilde{S}\widetilde{H}^0}^{\mathrm{CPV}}}{\sqrt{\Gamma_M^- + \Gamma_h}} \, ,$$

where L_w denotes the bubble wall width and $r_\Gamma = \bar{\Gamma}/(\Gamma_M^- + \Gamma_h)$.

Baryon Asymmetry

With the assumption of $F_{\rm ws}$ to be step function, and $n_L = 5Q + 4T$, the diffusion equation of n_B becomes

$$D_q n_B''(\bar{z}) - v_w n_B'(\bar{z}) - \theta(-\bar{z})\,\mathcal{R}\,n_B = \theta(-\bar{z})\,\frac{n_F}{2}\Gamma_{\rm ws} n_L(\bar{z}) \ ,$$

Where $\mathcal{R} = \Gamma_{\rm ws}\left[\frac{9}{4}\,\left(1 + \frac{n_{\rm sq}}{6}\right)^{-1} + \frac{3}{2}\right]$, It turns out,

$$n_B(\bar{z} > 0) = -\frac{n_F\Gamma_{\rm ws}}{2\sqrt{v_w^2 + 4RD_q}}\int_{-\infty}^{0} n_L(x)\,e^{-\lambda_- x}\,dx \ .$$

Where $\lambda_- = \frac{v_w - \sqrt{v_w^2 + 4RD_q}}{2D_q} \approx -\frac{R}{v_w}$. Substitute the result above into the solution,

$$n_L = -H\left[r_1 + r_2\,\frac{v_w^2}{\Gamma_{\rm ss}\,\bar{D}}\left(1 - \frac{D_q}{\bar{D}}\right)\right] ,$$

Whree $r_1 = \frac{9k_Q k_T - 5k_Q k_B - 8k_T k_B}{k_H(9k_Q + 9k_T + k_B)}$, $r_2 = \frac{k_B^2(5k_Q + 4k_T)(k_Q + 2k_T)}{k_H(9k_Q + 9k_T + k_B)^2}$,

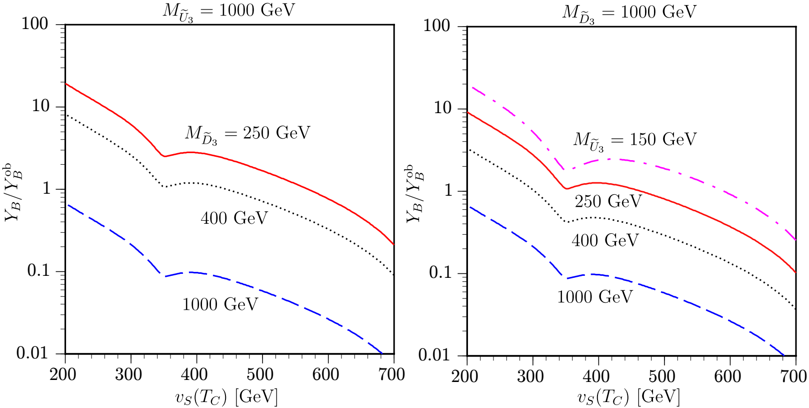
$$n_B \;=\; \frac{n_F\Gamma_{ws}}{2}\,\mathcal{A}\left[r_1 + r_2\frac{v_w^2}{\Gamma_{ss}\bar{D}}\left(1 - \frac{D_q}{\bar{D}}\right)\right]\frac{2\bar{D}D_q}{v_w\left[\bar{D}v_w + (2D_q - \bar{D})\sqrt{v_w^2 + 4\mathcal{R}D_q}\right] + 4\mathcal{R}\bar{D}D_q} ,$$

The $Y_B = n_B/s$ can then be able to work out by $s = \frac{2\pi^2}{45}g_{eff}(T)T^3 = 57.35\,T^3$.

Numerical Result

In this work (Phys.Lett.B710,188-191(2012)), we consider,

$$\tan\beta = 5\,,\quad v_S = 200\ \mathrm{GeV}\,,\quad |\lambda| = 0.81\,,\quad |\kappa| = 0.08\,;\quad \phi'_\lambda - \phi'_\kappa = \pm 90^\circ\,,$$
$$|A_\lambda| = 575\ \mathrm{GeV}\,,\quad |A_\kappa| = 110\ \mathrm{GeV}\,;\quad |A_t| = |A_b| = 1\ \mathrm{TeV}\,.$$
$$M_{\tilde{Q}_3} = 1\ \mathrm{TeV}\,, M_{\tilde{U}_3} = 150\ \mathrm{GeV} - 1\ \mathrm{TeV}\,, M_{\tilde{D}_3} = 250\ \mathrm{GeV} - 1\ \mathrm{TeV}\,;$$
$$T_C = 110\mathrm{GeV},\quad D_q = 6/T,\quad D_h = 110/T,$$



Summary

- Baryon asymmetry is possible to be explained by electroweak baryogenesis.
- The SM is fail to explain the baryon asymmetry because the parameter region producing strong first order phase transition has been excluded.
- It require $m_{\tilde{t}} < m_t$ in MSSM to produce strong first order phase transtion. The CP phases in MSSM are restricted by EDMs constraint.
- The corresponding soft term also provide cubic term at tree level. With the phase transition SYM $\rightarrow$ I$'$ $\rightarrow$ EW, strong first order phase transtion can be made without light top squark.
- With the virtue of the additional Higgs singlet in the NMSSM, there is one tree level physical CP phase which is compatible with the up-to-date EDMs constraint.
- Using CTP formalism, the singlino-driven baryon asymmetry has been estimated, which is compatible with the current observation.