The thermal entropy density of spacetime

Rongjia Yang (杨荣佳)
Hebei University(河北大学)

Outline

- Introduction
- Thermal entropy density of spacetime
- Discussion and conclusion

Introduction

1、The first equivalence principle

In the gravitational field, the gravity is also the inertial force.

Galileo

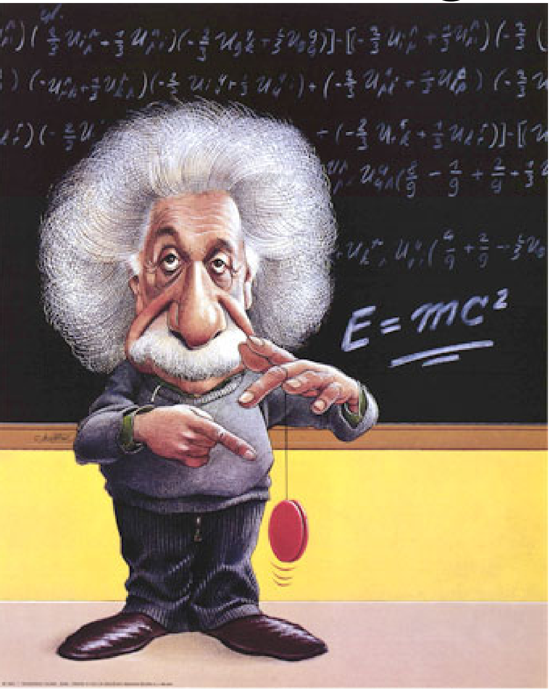
gravitational mass = inertial mass

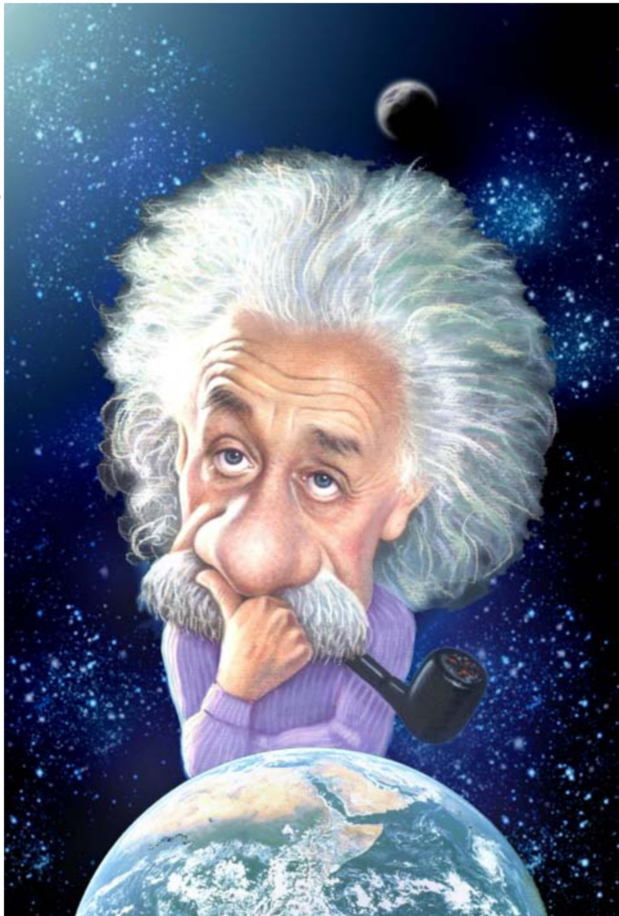
Introduction

2、The second equivalence principle

gravitational force=inertial force

On which general relativity bases





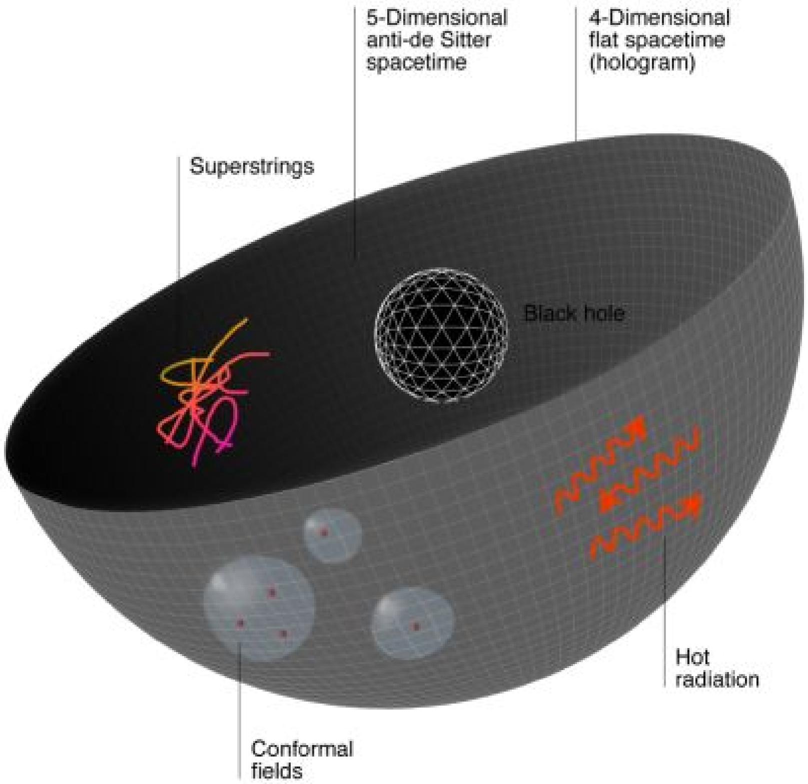
Introduction

3、Black-hole thermodynamics ： There is an profound connection between gravitation and thermodynamics:

- Cocke, Bekenstein, Hawking, Davies and Unruh, Jacobson, Padmanabhan,…
- Cai Ronggen,Wang Bin,Jing Jiliang,Yu Hongweil,Gong yungui,Gao Sijie,…
- Verlinde: gravity can be explained as entropic force
- Li Miao,Cai Ronggen,Gao Changjun,…

Introduction

4、The Holographic Principle



Introduction

- All these investigations were in specific contexts, or based on some assumptions, such as Unruh temperature, the existence of horizon, null surfaces, the apparent horizon, and so on.
- Does the connection between gravity and therm holds in any arbitrary spacetime? Can the analysis be carried out without assumptions about temperature or horizon?

Thermal entropy density of spacetime

- In general relativity or thermodynamics, both the energy density and the pressure play important roles.

ρ the energy density

p the pressure

$\rho + p$ What is it

Thermal entropy density of spacetime

- Let's start with the first law of thermodynamics in curved spacetime

$$dE = TdS - p\sqrt{h}dV.$$

For a very small volume, the energy density can be considered as unchanged

$$(\rho + p)\sqrt{h}dV = TdS \equiv Ts\sqrt{h}dV,$$

Thermal entropy density of spacetime

- To avoid the difficulty of defining temperature, we introduce the thermal entropy density

$$\sigma \equiv Ts$$

Then we have

$$\sigma = \rho + p.$$

For radiation, $\rho = \alpha T^4$ with α a constant, $p = \rho/3$, and $\sigma = 4aT^4/3$, it is obvious that $\sigma = \rho + p$.

Thermal entropy density of spacetime

- Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}$$

The stress energy tensor of the perfect fluid

$$T_{\mu\nu} = g_{\mu\nu} p + (\rho + p) u_\mu u_\mu$$

Thermal entropy density of spacetime

- We obtain

$$R - 4\Lambda = -8\pi(3p - \rho).$$

3+1 Einstein equations

$$n^\mu n^\nu R_{\mu\nu} + \frac{1}{2}R - \Lambda = 8\pi\mathcal{E}$$

$$\mathcal{E} = \Gamma^2(\rho + p) - p$$

Thermal entropy density of spacetime

- According to the scalar Gauss relation

$$\mathcal{R} + K^2 - K_{ij}K^{ij} - 2\Lambda = 16\pi\mathcal{E}$$

We obtain

$$\rho + p = \frac{1}{4\pi(4\Gamma^2 - 1)}\left[\mathcal{R} + K^2 - K_{ij}K^{ij} - \frac{1}{2}R\right]$$

Thermal entropy density of spacetime

$$\boxed{\Gamma = -\boldsymbol{n} \cdot \boldsymbol{u}}$$

$$\boxed{\Gamma = (1 - \boldsymbol{U} \cdot \boldsymbol{U})^{-1/2}}$$

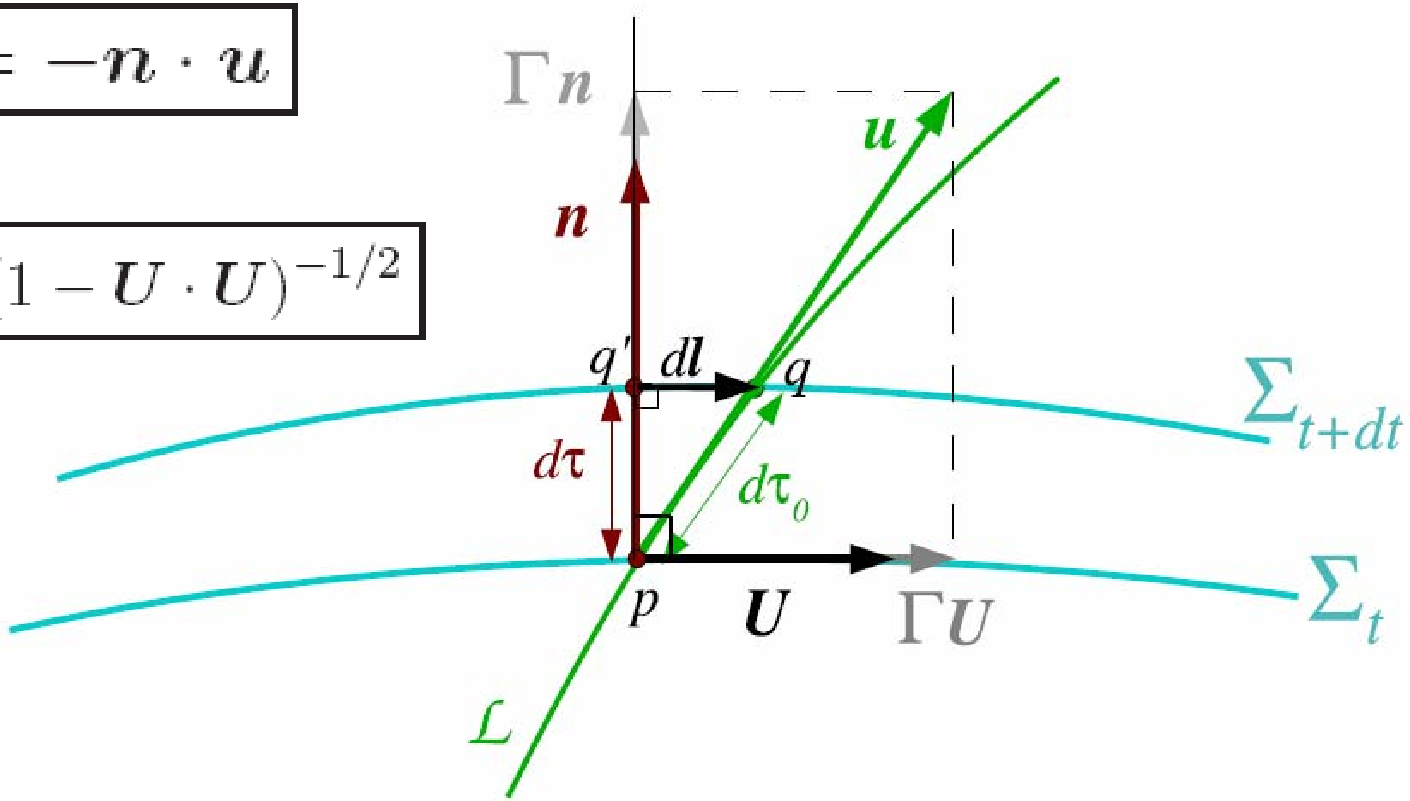


Figure 5.1: Worldline $\mathcal{L}$ of a fluid element crossing the spacetime foliation $(\Sigma_t)_{t\in\mathbb{R}}$. $\boldsymbol{u}$ is the fluid 4-velocity and $\boldsymbol{U} = d\boldsymbol{\ell}/d\tau$ the relative velocity of the fluid with respect to the Eulerian observer, whose 4-velocity is $\boldsymbol{n}$. $\boldsymbol{U}$ is tangent to Σ_t and enters in the orthogonal decomposition of $\boldsymbol{u}$ with respect to Σ_t, via $\boldsymbol{u} = \Gamma(\boldsymbol{n} + \boldsymbol{U})$. *NB:* contrary to what the figure might suggest, $d\tau > d\tau_0$ (conflict between the figure's underlying Euclidean geometry and the actual Lorentzian geometry of spacetime).

Thermal entropy density of spacetime

- We can express the thermal entropy density with 3 dimension spacial geometrical quantities as

$$\sigma = \frac{1}{8\pi(4\Gamma^2 - 1)} \times \tag{12}$$

$$\times \left[\mathcal{R} + K^2 - 3K_{ij}K^{ij} + \frac{2}{N}\mathcal{L}_m K + \frac{2}{N}D_i D^i N\right],$$

Thermal entropy density of spacetime

- In co-moving coordinate

$$\sigma = \frac{1}{24\pi}(4R_{\mathrm{s}} - 3R).$$

$$R_{\mathrm{s}} = g_{11}R^{11} + g_{22}R^{22} + g_{33}R^{33}$$

In FRW universe，the thermal entropy density

$$-(\dot{H} - k/a^2)/4\pi$$

Discussion and conclusion

- We first obtained the therm entropy density of any arbitrary spacetime without depending on the definition of temperature or horizon, that is to say, gravity can possess thermal effects, or, therm entropy density can possess effects of gravity. This may shed light on the nature of gravity.

Thank you !