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Particle Collisions in The Lower Dimensional Rotating Black Hole Space-time with Cosmological Constant

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Outline

Background and motivation I

- **Banãdos, Silk and West (BSW) found that the center-of-mass energy of two-particle collision on the horizon of the extremal Kerr black hole could be arbitrarily high when one particle has the critical angular momentum and hence the maximally rotating black hole might be regarded as a Planck-energy-scale collider. Phys. Rev. Lett. 103, 111102 (2009).**

Background and motivation II

- **Although it was pointed out [1,2] that the collision in fact takes an infinite proper time. Moreover, there are astrophysical limitations preventing a Kerr black hole to be an extremal one, and the gravitational radiation and backreaction effects should also be included in this process. But due to the potential interest in exploring ultra high energy physics, the BSW process has been studied extensively in other kinds of black holes or naked singularities**

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We investigate this on-horizon colliding process in the background of the 2+1 dimensional BTZ black hole,and our motivation is to examine whether the BSW effect remains valid in the lower dimensional space-time with cosmological constant case.

The horizons of 2+1 dimensional BTZ black hole I

The metric of the 2+1 Dimensional BTZ black holes with units $c = G = 1$ is given by

$$ds^2 = -N_r^2 dt^2 + N_r^{-2} dr^2 + r^2 (N_\phi dt + d\phi)^2 \tag{1}$$

where

$$N_r^2(r) = -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}, \tag{2}$$

$$N_\phi(r) = -\frac{J}{2r^2}. \tag{3}$$

M is related to the mass of the black hole, and J is related to the black hole's spin angular momentum. l^2 is related to the cosmological constant Λ by $l^{-2} = -\Lambda$.

The horizon r_{h} is given by $N_r|_{r=r_{\mathrm{h}}} = 0$

$$r_{\pm} = \sqrt{\frac{l}{2}(lM \pm \sqrt{l^2M^2 - J^2})} \tag{4}$$

Of these, $r_{\mathrm{h}} = r_{+}$is the black hole's outer horizon. In order for the horizon to exist

$$M > 0, |J| \leq Ml. \tag{5}$$

These two solutions are both coincided horizons of the extremal BTZ black hole when $|J| = Ml$, and the horizon of the extremal black hole

$$r_e = \sqrt{\frac{M}{2}}l \tag{6}$$

The Center-of-Mass Energy for the On-Horizon Collision in the Backgrounds of the BTZ Black Holes

To investigate the CM energy of the particle collision on the horizon, we first derive the 2+1 dimensional "4-velocity" of the colliding particles by solving the Hamilton-Jacobi equation.

$$\frac{dt}{d\tau} = \frac{2E - \frac{JL}{r^2}}{2N_r^2}, \tag{7}$$

$$\frac{d\phi}{d\tau} = \frac{J\left(2Er^2 - JL\right) + 4Lr^2N_r^2}{4r^4N_r^2}. \tag{8}$$

$$\frac{dr}{d\tau} = \frac{1}{2r^2}\sqrt{K^2 - 4r^2\left(L^2 + r^2\right)N_r^2} \tag{9}$$

$$\text{where } K = \left(JL - 2Er^2\right). \tag{10}$$

Next we compute the CM energy of the two-particle collision in the backgrounds of the 2+1 dimensional BTZ black holes by using

$$E_{\mathbf{CM}} = \sqrt{2}m_0\sqrt{1 - g_{\mu\nu}u_1^{\mu}u_2^{\nu}}, \tag{11}$$

where u_1^{μ}, u_2^{ν} **are the "4-velocity" vectors of the two particles** $(u = (\dot{t}, \dot{r}, \dot{\phi}))$**. We can obtain the rescaled CM energy**

$$\bar{E}_{\mathbf{CM}} = \frac{1}{2m_0^2}E_{\mathbf{CM}}^2 = \frac{1}{4r^4N_r^2}\left(K_1K_2 + 4r^2\left(-L_1L_2 + r^2\right)N_r^2 - H_1H_2\right) \tag{12}$$

where

$$H_i = \sqrt{(JL_i - 2E_ir^2)^2 - 4r^2\left(L_i^2 + r^2\right)N_r^2} \quad (i = 1, 2). \tag{13}$$

On the horizon, the denominator of the RHS of (12) is zero, and the numerator of it is

$$K_1 K_2 - \sqrt{K_1^2}\sqrt{K_2^2}, \tag{14}$$

$$K_i = K|_{E=E_i, L=L_i}, \quad i = 1, 2. \tag{15}$$

When $K_1 K_2 \geq 0$, the value of $\bar{E}_{\mathbf{CM}}$ undetermined. The limiting value of $\bar{E}_{\mathbf{CM}}$ as $r \to r_{\mathbf{h}}$ is given by the zero-order term in the expansion of $\bar{E}_{\mathbf{CM}}$ on the horizon

$$\bar{E}_{\mathbf{CM}}(r \to r_{\mathbf{h}}) = 2 + \frac{A}{2K_1 K_2}, \tag{16}$$

where

$$\begin{aligned} A &= J^2\left((L_1 - L_2)^2 - (E_1 - E_2)^2 l^2 - 2(L_1 - L_2)(L_{\mathbf{C}1} - L_{\mathbf{C}2})\right) \\ &\quad + 2l\left((E_2 L_1 - E_1 L_2)^2 + (E_1 - E_2)^2 l^2 M\right)\left(lM + \sqrt{l^2 M^2 - J^2}\right). \end{aligned} \tag{17}$$

- **So we can see that when $K_i = 0$, the CM energy on the horizon will blow up. We will call the angular momentum that make $K_i = 0$ the critical angular momentum $L_{\mathbf{C}i}$**

$$L_{\mathbf{C}i} = \frac{2r_{\mathbf{h}}^2 E_i}{J} = \frac{E_i l \left(lM + \sqrt{l^2 M^2 - J^2} \right)}{J}, \quad i = 1, 2. \qquad (18)$$

- **We can see that the critical angular momentum depends on the horizon $r_{\mathbf{h}}$, and when we consider different horizons of the black hole, the critical angular momentums of the particle will be different.**

The radial motion of the particle with the critical angular momentum

- **The colliding particle with the critical angular momentum must be able to reach the collision point on the horizon of the black hole.**
- **So $R(r) = \left(\frac{dr}{d\tau}\right)^2$ (equation (9)) of the colliding particle has to be positive in the neighborhood of the black hole's horizon.**

$$\left(\frac{dr}{d\tau}\right)^2 = \frac{K^2 - 4r^2\left(L^2 + r^2\right)N_r^2}{4r^4}. \qquad (19)$$

- **For particle with arbitrary energy E and angular momentum L**

$$R(r) = E^2 - \frac{L^2}{l^2} + M + \frac{1}{r^2}\left(L^2 M - EJL - \frac{J^2}{4}\right) - \frac{r^2}{l^2}. \qquad (20)$$

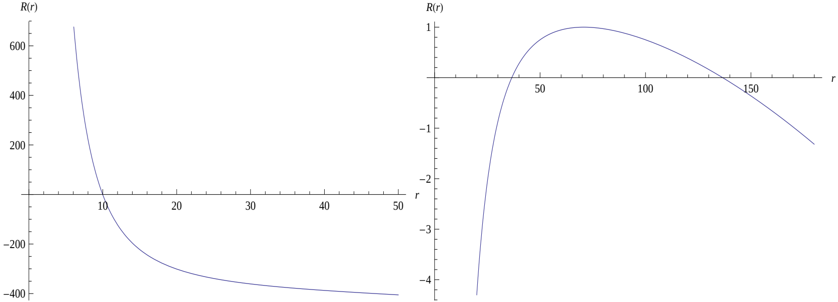


Figure: $L^2M - EJL - \frac{J^2}{4} > 0,$ $L^2M - EJL - \frac{J^2}{4} < 0$

Then we will study the motion of the particle has the critical angular momentum (Here we consider the critical angular momentum corresponding to the outer horizon)

$$R(r)|_{L=L_c} = 2E^2 + M - \frac{2E^2 l^2 M^2 + 2E^2 lM\sqrt{l^2M^2 - J^2}}{J^2} + \frac{W}{r^2} - \frac{r^2}{l^2} \tag{21}$$

where

$$W = \frac{E^2 l\left(2lM\left(l^2M^2 - J^2\right) + \left(2l^2M^2 - J^2\right)\sqrt{l^2M^2 - J^2}\right)}{J^2} - \frac{J^2}{4} \tag{22}$$

By solve $W = 0$ we can get the critical energy E_0.

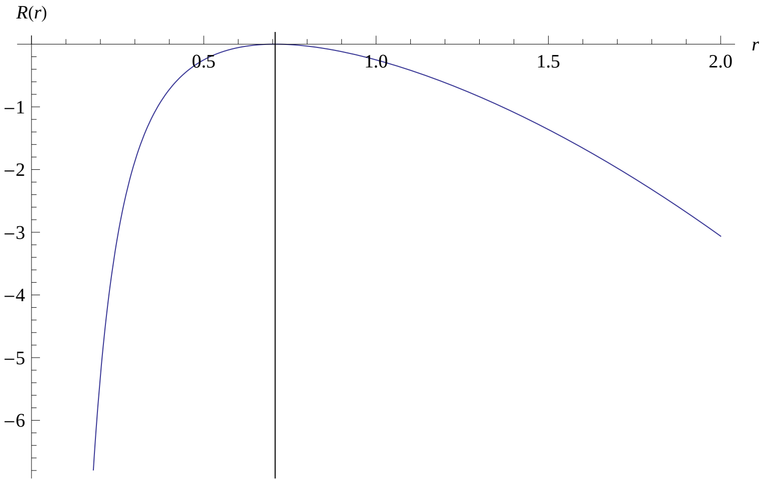
- **A. The extremal BTZ black hole case**
- **In the extremal BTZ black hole case, $R(r)$ for particle with critical angular momentum becomes very simple**

$$M - \frac{r^2}{l^2} - \frac{J^2}{4r^2}, \tag{23}$$

we solve $R(r) = 0$ and get

$$r_0 = \sqrt{\frac{M}{2}}\, l. \tag{24}$$

It is just the horizon of the extremal BTZ black hole.


R(r)
r
0.5
1.0
1.5
2.0
−1
−2
−3
−4
−5
−6

- **B. The non-extremal BTZ black hole case**
- **For the non-extremal BTZ black hole case, the bigger solution of $R(r) = 0$ is just the outer horizon of black hole**

$$r_0 = r_+ = \sqrt{\frac{l}{2}(lM + \sqrt{l^2M^2 - J^2})} \tag{25}$$

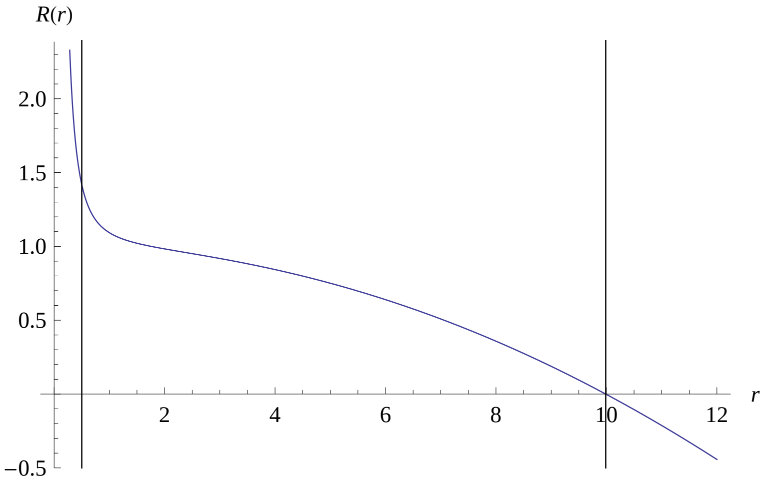


Figure: $E > E_0$.

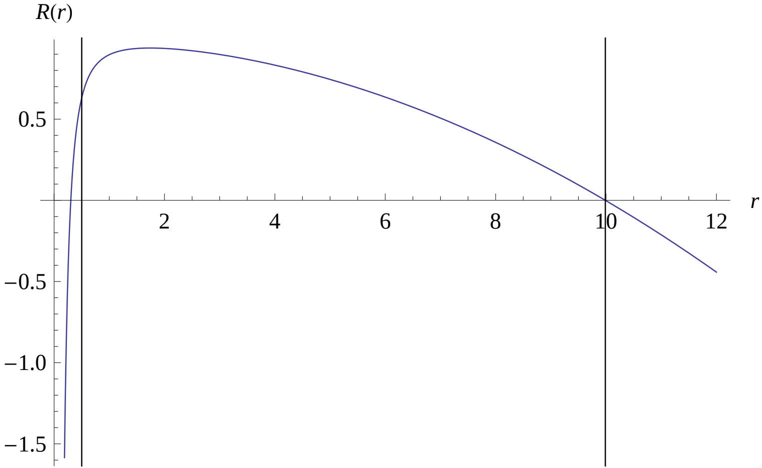


Figure: $E < E_0$.

For the particle with the critical angular momentum corresponding to the inner horizon

$$R(r)|_{L=L_{\mathbf{cl}}} = 2E^2 + M - \frac{2E^2 l^2 M^2 - 2E^2 lM\sqrt{l^2M^2 - J^2}}{J^2} + \frac{W}{r^2} - \frac{r^2}{l^2} \quad (26)$$

where

$$W = \frac{E^2 l\left(2lM\left(l^2M^2 - J^2\right) - \left(2l^2M^2 - J^2\right)\sqrt{l^2M^2 - J^2}\right)}{J^2} - \frac{J^2}{4} \quad (27)$$

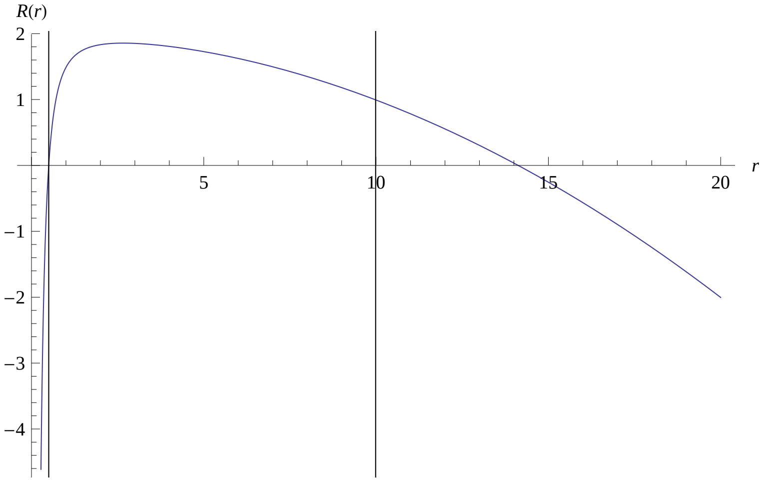


Figure: R(r) for the particle with the critical angular momentum corresponding to the inner horizon.

Conclusion

- **We have analyzed the possibility that the 2+1 dimensional BTZ black holes serve as particle accelerators.**
- **For the extremal BTZ black holes, particles with critical angular momentum can only exist on the outer horizon of the BTZ black hole.**
- **For the non-extremal BTZ black holes, particles can collide on the inner horizon with arbitrarily high CM energy. However, this requires certain condition of parameters which characterize the motion of particle, and we have obtained this additional condition.**

Thanks!