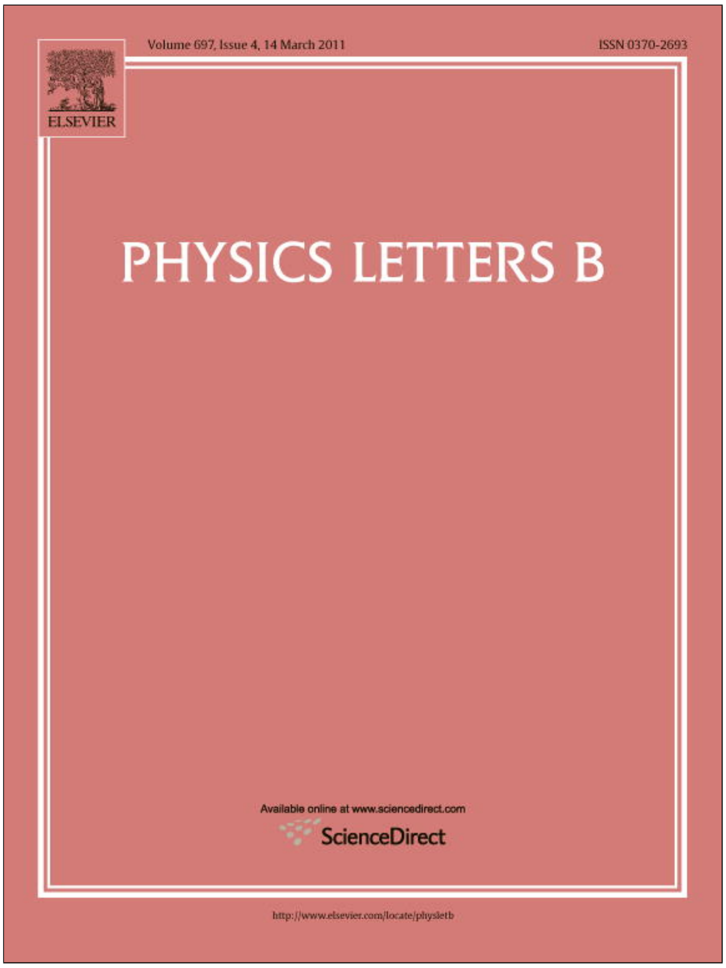
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First and second quantization theories of open p-brane and their spectra

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1. Introduction

String theory has been probably providing the most promising descriptions of nature, especially the realizations of spectra of particles in Standard Model. Besides, some kind of particles of spin two was assigned to gravitons [1–4]. On the analogy with particles, the dynamical properties of strings are described by Nambu–Goto action [5], given in terms of the area of string worldsheet. By involving an auxiliary worldsheet metric, it can also be replaced by a classically equivalent action, called Polyakov action with local conformal symmetries [6]. In contrast with the non-polynomial Nambu–Goto action the new action is quadratic in the derivatives of the coordinates. Some authors have investigated the connections between the two kinds of actions by introducing the interpolating actions [7–9].

Starting with the Letter of Kikkawa and Yamasaki [10], we have had an increasing interest in the theory of (two and more)-dimensional extended objects (membranes and p-brane) as unified theories containing non-abelian excitations [11]. The high-dimensional objects that motivated recent progress in string theory are extended structures embedded in a higher-dimensional spacetime from which it inherits an induced metric [12–15]. Over the last decade string theory has been gradually replaced by M-theory as the natural candidate for a fundamental description of nature. While a complete definition of M-theory is yet to be given, it is believed that the five perturbatively consistent string theories are different phases of this theory. Indeed it is known that membrane and five-brane occur naturally in eleven-dimensional supergravity, which is argued to be the low-energy limit of M-theory. Also, string theory is effectively described by the low-energy dynamics of a system of branes. For instance, the membrane of M-theory may be "wrapped" around the compact direction of radius R to become the fundamental string of type-IIA string theory, in the limit of vanishing radius [16–22]. A fundamental type-IIB string can be thought of as an M2-brane wrapped around x^{10}.

Ref. [22] gives the scheme of Dirac quantization of open p-brane in the D-brane background. In Ref. [23], we had discussed the quantization and spectrum of open 2-brane, from which we had seen more contents appeared than string case, especially the appearance of two types of tachyon states. In this Letter, we generalize the results to higher extended objects such as 3-brane and p-brane. As before, we also mainly work in 26-dimensional Minkowski spacetime, the arrangement is: Section 2 is the solution to Euler–Lagrange equation of p-brane and its quantization; Section 3 is the discussions on the second quantization of the

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p-brane; in Section 4, we give Hamiltonian of the open 3-brane and deduce its representation in terms of normal modes; in Section 5, we define the vacuum state and get a series of excited states by acting the raising operators on it and discuss some lower levels. Section 6 is summary and conclusion.

2. The solution to Euler–Lagrange equation of p-brane and its quantization

An open p-brane is a p-dimensional object which sweeps out a $(p+1)$-dimensional world volume parameterized by $\tau, \sigma^1, \ldots, \sigma^p$. And these parameters are collectively referred as ξ_i $(i = 0, 1, 2, \ldots, p)$. Then Polyakov action for the p-brane is given by [7]

$$S_P = -\frac{1}{4\pi\alpha'}\int d^{p+1}\xi\,\sqrt{-h}\big[h^{ab}\partial_a X^\mu \partial_b X_\mu - (p-1)\big], \tag{1}$$

where $h = \det|h_{ab}|$. It is well known that Eq. (1) is equivalent to Nambu–Goto action of the p-brane, and there are other lagrangian formalisms of p-branes, which do not require the fine tunned cosmological term [24]. Because $h = h_{ab}\Delta_{ab}$, $h^{ab} = \Delta_{ba}/h$, we have [1,2]

$$\delta h = -h h_{ab}\delta h^{ab}, \tag{2}$$

$$\delta\sqrt{-h} = -\frac{1}{2}\sqrt{-h}h_{ab}\delta h^{ab}, \tag{3}$$

$$\delta\big(\sqrt{-h}h^{cd}\big) = \sqrt{-h}\left(\delta h^{cd} - \frac{1}{2}h_{ab}h^{cd}\delta h^{ab}\right). \tag{4}$$

Let $\gamma_{ab} = \partial_a X^\mu \partial_b X_\mu$, Euler–Lagrange equation for h^{ab}

$$\frac{\delta S}{\delta h^{ab}} - \partial_c\frac{\delta S}{\delta(\partial_c h^{ab})} = 0 \tag{5}$$

gives

$$\frac{\delta S}{\delta h^{ab}} = -\frac{\sqrt{-h}}{2\pi\alpha'}\left\{\gamma_{ab} - \frac{1}{2}h_{ab}h^{cd}\gamma_{cd} + \frac{p-1}{2}h_{ab}\right\} = 0. \tag{6}$$

Define the energy–momentum tensor as

$$T_{ab} = \frac{-2\pi\alpha'}{\sqrt{-h}}\frac{\delta S}{\delta h^{ab}} = \gamma_{ab} - \frac{1}{2}h_{ab}h^{cd}\gamma_{cd} + \frac{p-1}{2}h_{ab}. \tag{7}$$

Because quantization of gravity field cannot very well be finished up to now, without losing the character of the open p-brane second quantization, we can choose the metric h_{ab} as $(-,+,+,\ldots,+)$, then we have

$$T_{00} = \frac{1}{2}(\gamma_{00} + \gamma_{11} + \gamma_{22} + \cdots + \gamma_{pp} - p + 1) = 0, \tag{8}$$

$$T_{11} = \frac{1}{2}(\gamma_{00} + \gamma_{11} - \gamma_{22} - \cdots - \gamma_{pp} + p - 1) = 0, \tag{9}$$

$$T_{22} = \frac{1}{2}(\gamma_{00} - \gamma_{11} + \gamma_{22} - \cdots - \gamma_{pp} + p - 1) = 0, \tag{10}$$

$$\vdots$$

$$T_{pp} = \frac{1}{2}(\gamma_{00} - \gamma_{11} - \gamma_{22} - \cdots + \gamma_{pp} + p - 1) = 0. \tag{11}$$

Eq. (8) indicates that the Hamiltonian of p-brane system vanishes, which we will discuss later. Eqs. (9)–(11) may be viewed constraint equations.

The Euler–Lagrange equation for X_μ can be derived from the variational principle as follows

$$\left(\partial_\tau^2 - \sum_{i=1}^{p}\partial_i^2\right)X^\mu\big(\tau, \sigma^1, \sigma^2, \ldots, \sigma^p\big) = 0 \tag{12}$$

with the Neumann boundary conditions

$$\partial_i X^\mu\big(\tau, \sigma^1, \ldots, \sigma^p\big)\big|_{\sigma^i=0} = \partial_i X^\mu\big(\tau, \sigma^1, \ldots, \sigma^p\big)\big|_{\sigma^i=\pi} = 0 \quad (i = 1, 2, \ldots, p). \tag{13}$$

On the other hand, because general physical processes should satisfy quantitative causal relation [25,26], some changes (cause) of some quantities in Eq. (12) must lead to the relative some changes (result) of the other quantities in Eq. (12) so that Eq. (12)'s right side keeps no-loss-no-gain, i.e., zero, namely, Eq. (12) also satisfies the quantitative causal relation, which just makes X^I relative to $(\tau, \sigma^1, \sigma^2, \ldots, \sigma^p)$ to form a coupling physical system of different variables.

The canonical momenta for canonical variables X_μ are defined as

$$P^\mu = \frac{\partial\mathcal{L}}{\partial\dot{X}_\mu} = \frac{1}{2\pi\alpha'}\dot{X}^\mu. \tag{14}$$

Therefore, we find out the solution satisfying the boundary conditions to the Euler–Lagrange equation as follows

$$X^0 = \frac{x^0}{\pi^{\frac{(p-1)}{2}}} + \frac{2\alpha' p^0}{\pi^{\frac{(p-1)}{2}}}\tau, \qquad X^1 = \frac{x^1}{\pi^{\frac{(p-1)}{2}}} + \frac{2\alpha' p^1}{\pi^{\frac{(p-1)}{2}}}\tau, \tag{15}$$

$$\begin{aligned} X^I\big(\tau, \sigma^1, \ldots, \sigma^p\big) &= \frac{x^I + 2\alpha' p^I\tau}{\pi^{\frac{(p-1)}{2}}} + i\sqrt{2\alpha'}\sum_{n_1,\ldots,n_p=0}^{+\infty}\left(\sum_{i=1}^{p}n_i^2\right)^{\frac{-1}{4}} \\ &\quad\times\big(X^I_{n_1n_2\cdots n_p}e^{i\tau\sqrt{\sum_{i=1}^p n_i^2}} - \big(X^I_{n_1n_2\cdots n_p}\big)^\dagger e^{-i\tau\sqrt{\sum_{i=1}^p n_i^2}}\big) \\ &\quad\times\prod_{i=1}^{p}\cos n_i\sigma^i, \end{aligned} \tag{16}$$

$$\begin{aligned} P^J\big(\tau, \sigma^1, \ldots, \sigma^p\big) &= \frac{1}{\pi}\Bigg[\frac{p^J}{\pi^{\frac{(p-1)}{2}}} + \sqrt{\frac{2}{\alpha'}}\sum_{n_1,n_2,\ldots,n_p=0}^{+\infty}\left(\sum_{i=1}^{p}n_i^2\right)^{\frac{1}{4}} \\ &\quad\times\big(\big(P^J_{n_1n_2\cdots n_p}\big)^\dagger e^{i\tau\sqrt{\sum_{i=1}^p n_i^2}} + P^J_{n_1n_2\cdots n_p}e^{-i\tau\sqrt{\sum_{i=1}^p n_i^2}}\big) \\ &\quad\times\prod_{i=1}^{p}\cos n_i\sigma^i\Bigg], \end{aligned} \tag{17}$$

where we have fixed the first two directions of the solution by two constraints of Eqs. (9)–(11), i.e. Eq. (15), which may make us able to gain reasonable spectrum of lower energy levels. And we have introduced $(X^I_{n_1n_2\cdots n_p})^\dagger$ and $(P^J_{n_1n_2\cdots n_p})^\dagger$ as Hermitian operators for $X^I_{n_1n_2\cdots n_p}$ and $P^J_{n_1n_2\cdots n_p}$, respectively, in order to guarantee the Hermiticity of $X^I(\tau, \sigma^1, \ldots, \sigma^p)$ and $P^I(\tau, \sigma^1, \ldots, \sigma^p)$, and $\{n_i\}$ cannot be zero simultaneously. Now there are still $p-2$ constraints left, which will restrict the p-brane on the $(D-p+2)$-dimensional hypersurface in D-dimensional Minkowski spacetime for $p \geqslant 2$. In this sense, we can conclude that $D - p + 2 \geqslant p + 1$, i.e., $p \leqslant \frac{D+1}{2}$.

Using (14), (16) and (17), we set up the following relations

$$X^I_{n_1n_2\cdots n_p} = -2\big(P^I_{n_1n_2\cdots n_p}\big)^\dagger; \qquad \big(X^I_{n_1n_2\cdots n_p}\big)^\dagger = -2P^I_{n_1n_2\cdots n_p}. \tag{18}$$

In order to determinate the commutative relations, we must calculate the commutative relations of $X^I(\tau, \sigma^1, \ldots, \sigma^p)$ and $P^J(\tau, \sigma^1, \ldots, \sigma^p)$ basing on the Delta function as [3]

$$\delta(\sigma - \sigma') = \frac{1}{\pi}\left(1 + 2\sum_{n=1}^{+\infty}\cos n\sigma\cos n\sigma'\right), \tag{19}$$

and in terms of the general rule of the standard quantization, we should take

$$\begin{aligned}&\left[X^I\left(\tau,\sigma^1,\ldots,\sigma^p\right),P^J\left(\tau,\sigma'^1,\ldots,\sigma'^p\right)\right]\\&\quad=i\eta^{IJ}\delta\left(\sigma^1-\sigma'^1\right)\cdots\delta\left(\sigma^p-\sigma'^p\right).\end{aligned}\tag{20}$$

Substituting Eqs. (16), (17), (19) into Eq. (20) and using Eq. (18), we deduce a series of new multiple commutative relations between the normal modes as follows

$$\begin{aligned}&\left[X^I_{n_1n_2n_3\cdots n_{p-1}n_p},P^J_{m_1m_2m_3\cdots m_{p-1}m_p}\right]\\&\quad=\frac{2^{p-2}}{\pi^{p-1}}\eta^{IJ}\delta_{n_1m_1}\delta_{n_2m_2}\delta_{n_3m_3}\cdots\delta_{n_{p-1}m_{p-1}}\delta_{n_pm_p},\end{aligned}\tag{21}$$

$$\begin{aligned}&\left[X^I_{0n_2n_3\cdots n_{p-1}n_p},P^J_{0m_2m_3\cdots m_{p-1}m_p}\right]\\&\quad=\frac{2^{p-3}}{\pi^{p-1}}\eta^{IJ}\delta_{n_2m_2}\delta_{n_3m_3}\cdots\delta_{n_{p-1}m_{p-1}}\delta_{n_pm_p},\end{aligned}\tag{22}$$

$$\vdots$$

$$\begin{aligned}&\left[X^I_{00n_3\cdots n_{p-1}n_p},P^J_{00m_3\cdots m_{p-1}m_p}\right]\\&\quad=\frac{2^{p-4}}{\pi^{p-1}}\eta^{IJ}\delta_{n_3m_3}\cdots\delta_{n_{p-1}m_{p-1}}\delta_{n_pm_p},\end{aligned}\tag{23}$$

$$\vdots$$

$$\left[X^I_{00\cdots n_{p-1}n_p},P^J_{00\cdots m_{p-1}m_p}\right]=\frac{1}{\pi^{p-1}}\eta^{IJ}\delta_{n_{p-1}m_{p-1}}\delta_{n_pm_p},\tag{24}$$

$$\vdots$$

$$\left[X^I_{00\cdots 0n_p},P^J_{00\cdots 0m_p}\right]=\frac{1}{2\pi^{p-1}}\eta^{IJ}\delta_{n_pm_p},\tag{25}$$

for $n_i,m_i>0$, where the first "$\cdots$" means that only one of $\{n_i\}$ is zero, and they have the similar commutative relations with Eq. (22). The second "$\cdots$" means that only two of $\{n_i\}$ are zero, which have the similar commutative relations with Eq. (23). Further, the same procedure can be used for the other "$\cdots$", and the commutative relations of the other forms are zero.

3. The second quantization of the open *p*-brane

Now we need to generalize the discussions of the usual second quantization to p-brane and to continue finishing the second quantization in Section 2, i.e., we further give the second quantization of the p-brane, namely, we use Eqs. (21)–(25) and the discussions after Eq. (25) in Section 2 to construct the lowering and raising multiple operators in the state space as $\phi^I_{n_1}\otimes\phi^I_{n_2}\otimes\cdots\otimes\phi^I_{n_p}$, $\phi^{J\dagger}_{m_1}\otimes\phi^{J\dagger}_{m_2}\otimes\cdots\otimes\phi^{J\dagger}_{m_p}$, and then we obtain a series of the new fundamental multiple commutative relations of corresponding Eqs. (21)–(25) as follows

$$\begin{aligned}&\left[\phi^I_{n_1}\otimes\phi^I_{n_2}\otimes\cdots\otimes\phi^I_{n_p},\ \phi^{J\dagger}_{m_1}\otimes\phi^{J\dagger}_{m_2}\otimes\cdots\otimes\phi^{J\dagger}_{m_p}\right]\\&\quad=\eta^{IJ}\delta_{n_1m_1}\otimes\delta_{m_2m_2}\otimes\cdots\otimes\delta_{n_pm_p},\\&\left[\phi^I_{n_1}\otimes\phi^I_{n_2}\otimes\cdots\otimes\phi^I_{n_p},\phi^J_{m_1}\otimes\phi^J_{m_2}\otimes\cdots\otimes\phi^J_{m_p}\right]=0,\\&\left[\phi^{I\dagger}_{n_1}\otimes\phi^{I\dagger}_{n_2}\otimes\cdots\otimes\phi^{I\dagger}_{n_p},\phi^{J\dagger}_{m_1}\otimes\phi^{J\dagger}_{m_2}\otimes\cdots\otimes\phi^{J\dagger}_{m_p}\right]=0;\\&\left[\phi^I_0\otimes\phi^I_{n_2}\otimes\cdots\otimes\phi^I_{n_p},\ \phi^{J\dagger}_0\otimes\phi^{J\dagger}_{m_2}\otimes\cdots\otimes\phi^{J\dagger}_{m_p}\right]\\&\quad=\eta^{IJ}\delta_{m_2m_2}\otimes\cdots\otimes\delta_{n_pm_p},\\&\left[\phi^I_0\otimes\phi^I_{n_2}\otimes\cdots\otimes\phi^I_{n_p},\phi^J_0\otimes\phi^J_{m_2}\otimes\cdots\otimes\phi^J_{m_p}\right]=0,\\&\left[\phi^{I\dagger}_0\otimes\phi^{I\dagger}_{n_2}\otimes\cdots\otimes\phi^{I\dagger}_{n_p},\phi^{J\dagger}_0\otimes\phi^{J\dagger}_{m_2}\otimes\cdots\otimes\phi^{J\dagger}_{m_p}\right]=0;\\&\vdots\\&\left[\phi^I_0\otimes\phi^I_0\otimes\cdots\otimes\phi^I_0\otimes\phi^I_{n_p},\ \phi^{J\dagger}_0\otimes\phi^{J\dagger}_0\otimes\cdots\otimes\phi^{J\dagger}_0\otimes\phi^{J\dagger}_{m_p}\right]\\&\quad=\eta^{IJ}\delta_{n_pm_p},\\&\left[\phi^I_0\otimes\cdots\otimes\phi^I_0\otimes\phi^I_{n_p},\phi^J_0\otimes\cdots\otimes\phi^J_0\otimes\phi^J_{m_p}\right]=0,\\&\left[\phi^{I\dagger}_0\otimes\cdots\otimes\phi^{I\dagger}_0\otimes\phi^{I\dagger}_{n_p},\phi^{J\dagger}_0\otimes\cdots\otimes\phi^{J\dagger}_0\otimes\phi^{J\dagger}_{m_p}\right]=0;\end{aligned}\tag{26}$$

where we have used the generalized expressions of Eq. (29) below for the p-brane (the concretely deducing process of the generalized expressions see Appendix A). Therefore, we have continued to do the second quantization and finished the second quantization of the open p-brane. In Section 2, we have given the first quantization of the open p-brane by finding out the solution of satisfying the boundary conditions to the Euler–Lagrange equation (12). The set (26) of the fundamental multiple commutative relations of corresponding Eqs. (21)–(25) are a series of the new fundamental multiple commutative relations of the lowering and raising multiple operators in the state space, which cannot be got before.

In order to show both wave properties and extended object (for $p=0$, particle) properties of the open p-brane, we, in Section 2, have given the operatorization of the wave function of the open p-brane, i.e., making the Fourier expanding coefficient became the multiple lowering and raising operators, and the second quantization of the canonical variables requires a kind of multiple commutative relations (26) between the normal modes, which is important to construct the spectrum of the open p-brane.

Not losing the general property of the research of this Letter, and to avoid to be too lengthy for the equations, we will concretely take $p=3$ case for example, finish and see what kind of contents of the spectrum appear in the following sections. For the spectra of the p-brane ($p>3$), one can do the whole analogous research on $p=3$ except more complex.

4. Hamiltonian of the open 3-brane and its representation in the second quantization theory

In order to obtain the mass-squared operators, we must calculate the Hamiltonian in terms of the raising and lowering operators. The Hamiltonian of the 3-brane can be derived from the Polyakov action (1) for $p=3$ as follows

$$H=\int_0^\pi d\sigma^1\int_0^\pi d\sigma^2\int_0^\pi d\sigma^3\left(P_\mu\dot{X}^\mu-\mathcal{L}\right).\tag{27}$$

Here, different from the string case, there is an extra cosmological constant term, which is proportional to $\frac{1}{\alpha'}$. In the zero slope limit, this term will be very large because of $\alpha'\sim\ell^2\sim10^{-66}$ cm, and then we can omit this term as the contributions of background fields.

Substituting Eqs. (15) and (16) into the Hamiltonian, we obtain the total Hamiltonian for the 3-brane in terms of normal modes as follows

$$\begin{aligned}4\pi\alpha'H&=2\alpha'\pi^3\eta_{IJ}\sum_{n=1}^{+\infty}n\left[\left(X^I_{n00}\right)^\dagger X^J_{n00}+X^I_{n00}\left(X^J_{n00}\right)^\dagger\right]\\&\quad+2\alpha'\pi^3\eta_{IJ}\sum_{m=1}^{+\infty}m\left[\left(X^I_{0m0}\right)^\dagger X^J_{0m0}+X^I_{0m0}\left(X^J_{0m0}\right)^\dagger\right]\\&\quad+2\alpha'\pi^3\eta_{IJ}\sum_{l=1}^{+\infty}l\left[\left(X^I_{00l}\right)^\dagger X^J_{00l}+X^I_{00l}\left(X^J_{00l}\right)^\dagger\right]\end{aligned}$$

$$
\begin{aligned}
&+\alpha'\pi^3\eta_{IJ}\sum_{n,m=1}^{+\infty}(n^2+m^2)^{\frac{1}{2}}\\
&\times\big[(X^I_{nm0})^\dagger X^J_{nm0}+X^I_{nm0}(X^J_{nm0})^\dagger\big]\\
&+\alpha'\pi^3\eta_{IJ}\sum_{n,l=1}^{+\infty}(n^2+l^2)^{\frac{1}{2}}\\
&\times\big[(X^I_{n0l})^\dagger X^J_{n0l}+X^I_{n0l}(X^J_{n0l})^\dagger\big]\\
&+\alpha'\pi^3\eta_{IJ}\sum_{m,l=1}^{+\infty}(m^2+l^2)^{\frac{1}{2}}\\
&\times\big[(X^I_{0ml})^\dagger X^J_{0ml}+X^I_{0ml}(X^J_{0ml})^\dagger\big]\\
&+\frac{\alpha'\pi^3}{2}\eta_{IJ}\sum_{n,m,l=1}^{+\infty}(n^2+m^2+l^2)^{\frac{1}{2}}\\
&\times\big[(X^I_{nml})^\dagger X^J_{nml}+X^I_{nml}(X^J_{nml})^\dagger\big]\\
&+4\pi\alpha'^2p^2.
\end{aligned}
\tag{28}
$$

Now we use Eqs. (21)–(25) to construct the raising and lowering multiple operators in the case of $p=3$

$$
\begin{aligned}
&\begin{cases}
X^I_{nml}=\dfrac{2}{\pi}\phi^{I\dagger}_n\otimes\phi^{I\dagger}_m\otimes\phi^{I\dagger}_l,\\
(X^I_{nml})^\dagger=\dfrac{2}{\pi}\phi^I_n\otimes\phi^I_m\otimes\phi^I_l,
\end{cases}\\
&\begin{cases}
X^I_{0ml}=\dfrac{\sqrt{2}}{\pi}\phi^{I\dagger}_0\otimes\phi^{I\dagger}_m\otimes\phi^{I\dagger}_l,\\
(X^I_{0ml})^\dagger=\dfrac{\sqrt{2}}{\pi}\phi^I_0\otimes\phi^I_m\otimes\phi^I_l,
\end{cases}\\
&\begin{cases}
X^I_{n0l}=\dfrac{\sqrt{2}}{\pi}\phi^{I\dagger}_n\otimes\phi^{I\dagger}_0\otimes\phi^{I\dagger}_l,\\
(X^I_{n0l})^\dagger=\dfrac{\sqrt{2}}{\pi}\phi^I_n\otimes\phi^I_0\otimes\phi^I_l,
\end{cases}\\
&\begin{cases}
X^I_{nm0}=\dfrac{\sqrt{2}}{\pi}\phi^{I\dagger}_n\otimes\phi^{I\dagger}_m\otimes\phi^{I\dagger}_0,\\
(X^I_{nm0})^\dagger=\dfrac{\sqrt{2}}{\pi}\phi^I_n\otimes\phi^I_m\otimes\phi^I_0,
\end{cases}\\
&\begin{cases}
X^I_{n00}=\dfrac{1}{\pi}\phi^{I\dagger}_n\otimes\phi^{I\dagger}_0\otimes\phi^{I\dagger}_0,\\
(X^I_{n00})^\dagger=\dfrac{1}{\pi}\phi^I_n\otimes\phi^I_0\otimes\phi^I_0,
\end{cases}\\
&\begin{cases}
X^I_{0m0}=\dfrac{1}{\pi}\phi^{I\dagger}_0\otimes\phi^{I\dagger}_m\otimes\phi^{I\dagger}_0,\\
(X^I_{0m0})^\dagger=\dfrac{1}{\pi}\phi^I_0\otimes\phi^I_m\otimes\phi^I_0,
\end{cases}\\
&\begin{cases}
X^I_{00l}=\dfrac{1}{\pi}\phi^{I\dagger}_0\otimes\phi^{I\dagger}_0\otimes\phi^{I\dagger}_l,\\
(X^I_{00l})^\dagger=\dfrac{1}{\pi}\phi^I_0\otimes\phi^I_0\otimes\phi^I_l,
\end{cases}
\end{aligned}
\tag{29}
$$

where we have used Eq. (18). Using the on-shell conditions: $P^2=-M^2$, we have

$$
\begin{aligned}
H={}&\eta_{IJ}\sum_{n=1}^{+\infty}n\Big[\big[\phi^{I\dagger}_n\otimes\phi^{I\dagger}_0\otimes\phi^{I\dagger}_0\big]\big[\phi^J_n\otimes\phi^J_0\otimes\phi^J_0\big]+\frac{1}{2}\eta^{IJ}\Big]\\
&+\eta_{IJ}\sum_{m=1}^{+\infty}m\Big[\big[\phi^{I\dagger}_0\otimes\phi^{I\dagger}_m\otimes\phi^{I\dagger}_0\big]\big[\phi^J_0\otimes\phi^J_m\otimes\phi^J_0\big]+\frac{1}{2}\eta^{IJ}\Big]\\
&+\eta_{IJ}\sum_{l=1}^{+\infty}l\Big[\big[\phi^{I\dagger}_0\otimes\phi^{I\dagger}_0\otimes\phi^{I\dagger}_l\big]\big[\phi^J_0\otimes\phi^J_0\otimes\phi^J_l\big]+\frac{1}{2}\eta^{IJ}\Big]\\
&+\eta_{IJ}\sum_{n,m=1}^{+\infty}(n^2+m^2)^{\frac{1}{2}}\Big[\big[\phi^{I\dagger}_n\otimes\phi^{I\dagger}_m\otimes\phi^{I\dagger}_0\big]\\
&\times\big[\phi^J_n\otimes\phi^J_m\otimes\phi^J_0\big]+\frac{1}{2}\eta^{IJ}\Big]\\
&+\eta_{IJ}\sum_{n,l=1}^{+\infty}(n^2+l^2)^{\frac{1}{2}}\Big[\big[\phi^{I\dagger}_n\otimes\phi^{I\dagger}_0\otimes\phi^{I\dagger}_l\big]\\
&\times\big[\phi^J_n\otimes\phi^J_0\otimes\phi^J_l\big]+\frac{1}{2}\eta^{IJ}\Big]\\
&+\eta_{IJ}\sum_{m,l=1}^{+\infty}(m^2+l^2)^{\frac{1}{2}}\Big[\big[\phi^{I\dagger}_0\otimes\phi^{I\dagger}_m\otimes\phi^{I\dagger}_l\big]\\
&\times\big[\phi^J_0\otimes\phi^J_m\otimes\phi^J_l\big]+\frac{1}{2}\eta^{IJ}\Big]\\
&+\eta_{IJ}\sum_{n,m,l=1}^{+\infty}(n^2+m^2+l^2)^{\frac{1}{2}}\Big[\big[\phi^{I\dagger}_n\otimes\phi^{I\dagger}_m\otimes\phi^{I\dagger}_l\big]\\
&\times\big[\phi^J_n\otimes\phi^J_m\otimes\phi^J_l\big]+\frac{1}{2}\eta^{IJ}\Big]\\
&+\alpha'p^2\\
={}&N_1+N_2+N_3+N_{12}+N_{23}+N_{13}\\
&+N_{123}+\frac{3a}{2}+\frac{3b}{2}+\frac{c}{2}-\alpha'M^2,
\end{aligned}
\tag{30}
$$

where

$$
N_1=\eta_{IJ}\sum_{n=1}^{+\infty}n\big[\phi^{I\dagger}_n\otimes\phi^{I\dagger}_0\otimes\phi^{I\dagger}_0\big]\big[\phi^J_n\otimes\phi^J_0\otimes\phi^J_0\big],\tag{31}
$$

$$
N_2=\eta_{IJ}\sum_{m=1}^{+\infty}m\big[\phi^{I\dagger}_0\otimes\phi^{I\dagger}_m\otimes\phi^{I\dagger}_0\big]\big[\phi^J_0\otimes\phi^J_m\otimes\phi^J_0\big],\tag{32}
$$

$$
N_3=\eta_{IJ}\sum_{l=1}^{+\infty}l\big[\phi^{I\dagger}_0\otimes\phi^{I\dagger}_0\otimes\phi^{I\dagger}_l\big]\big[\phi^J_0\otimes\phi^J_0\otimes\phi^J_l\big],\tag{33}
$$

$$
\begin{aligned}
N_{12}={}&\eta_{IJ}\sum_{n,m=1}^{+\infty}(n^2+m^2)^{\frac{1}{2}}\big[\phi^{I\dagger}_n\otimes\phi^{I\dagger}_m\otimes\phi^{I\dagger}_0\big]\\
&\times\big[\phi^J_n\otimes\phi^J_m\otimes\phi^J_0\big],
\end{aligned}\tag{34}
$$

$$
\begin{aligned}
N_{23}={}&\eta_{IJ}\sum_{m,l=1}^{+\infty}(m^2+l^2)^{\frac{1}{2}}\big[\phi^{I\dagger}_0\otimes\phi^{I\dagger}_m\otimes\phi^{I\dagger}_l\big]\\
&\times\big[\phi^J_0\otimes\phi^J_m\otimes\phi^J_l\big],
\end{aligned}\tag{35}
$$

$$
\begin{aligned}
N_{13}={}&\eta_{IJ}\sum_{n,l=1}^{+\infty}(n^2+l^2)^{\frac{1}{2}}\big[\phi^{I\dagger}_n\otimes\phi^{I\dagger}_0\otimes\phi^{I\dagger}_l\big]\\
&\times\big[\phi^J_n\otimes\phi^J_0\otimes\phi^J_l\big],
\end{aligned}\tag{36}
$$

$$
N_{123}=\eta_{IJ}\sum_{n,m,l=1}^{+\infty}(n^2+m^2+l^2)^{\frac{1}{2}}\big[\phi^{I\dagger}_n\otimes\phi^{I\dagger}_m\otimes\phi^{I\dagger}_l\big]
$$

$$\times \left[\phi_n^J \otimes \phi_m^J \otimes \phi_l^J\right], \tag{37}$$

$$a = \eta_I^I \sum_{n=1}^{+\infty} n = (D-2) \sum_{n=1}^{+\infty} n, \tag{38}$$

$$b = \eta_I^I \sum_{n,m=1}^{+\infty} \sqrt{n^2+m^2} = (D-2) \sum_{n,m=1}^{+\infty} \sqrt{n^2+m^2}, \tag{39}$$

$$c = (D-2) \sum_{n,m,l=1}^{+\infty} \sqrt{n^2+m^2+l^2}. \tag{40}$$

Without involving Fermi fields, and in the limit of contracting to zero of two spacial dimensions of the 3-brane, the 3-brane system should be reduced to string case, then we can choose the space-time dimensional number 26, and use the Riemann Zeta function

$$\zeta(-1) = \sum_n n = -\frac{1}{12}, \tag{41}$$

then we have

$$a = (D-2) \sum_{n=1}^{+\infty} n = -\frac{(D-2)}{12} = -2. \tag{42}$$

However, the number b and c are still infinity, and we have to remove them by viewing them as the effects of background fields or the vacuum zero point energies.

Thus, we give both Hamiltonian of the open 3-brane and the its representation in terms of normal modes.

5. Spectrum of the open 3-brane in the second quantization theory

Just as what we have pointed out from Eq. (8), the Hamiltonian of the 3-brane system vanishes. Besides, Refs. [7–9] investigated the connections between Nambu–Goto action and Polyakov action by introducing the interpolating actions. Because of the internal constraints in phase space, one can find that the canonical Hamiltonian vanishes, and the total Hamiltonian would be made up of internal constraints. So we can set the Hamiltonian to be zero, and then the mass-squared operator is

$$\begin{aligned}\alpha' M^2 &= N_1 + N_2 + N_3 + N_{12} + N_{23} + N_{13} + N_{123} + \frac{3a}{2} \\ &= N_1 + N_2 + N_3 + N_{12} + N_{23} + N_{13} + N_{123} - 3. \end{aligned} \tag{43}$$

The vacuum state $|0,0,0\rangle \equiv |0\rangle \otimes |0\rangle \otimes |0\rangle$ is defined to be annihilated by the lowering operators for $n, m, l \geqslant 0$

$$\phi_n \otimes \phi_m \otimes \phi_l |0,0,0\rangle = 0, \tag{44}$$

where n, m and l cannot be zero simultaneously.

In general, a basis for the Fock space states can be taken of the form with the raising operators

$$|\lambda, \kappa, \chi\rangle = \prod_{n_i, m_i, l_i, I} \left\{\phi_{n_i}^{I\dagger} \otimes \phi_{m_i}^{I\dagger} \otimes \phi_{l_i}^{I\dagger}\right\} |0,0,0\rangle, \tag{45}$$

where $\sum n = \lambda$, $\sum m = \kappa$ and $\sum l = \chi$. Because the mass-squared operator is symmetrical with respect to n, m and l in Eq. (43), the states $|\lambda, \kappa, \chi\rangle$, $|\kappa, \lambda, \chi\rangle$ and $|\kappa, \chi, \lambda\rangle$ must have the same mass-square.

Now, let us see how this works for the first some levels of the open 3-brane in flat spacetime by removing the contributions of background fields.

At the lowest mass level, the only state is $|0,0,0\rangle$, which means $N_1 = N_2 = N_3 = N_{12} = N_{23} = N_{13} = N_{123} = 0$. Then the mass-squared operator can be calculated

$$\begin{aligned}\alpha' M^2 &= N_1 + N_2 + N_3 + N_{12} + N_{23} + N_{13} + N_{123} - 3 \\ &= -3. \end{aligned} \tag{46}$$

Obviously, it is a tachyon state.

At the next level there are three kinds of excited states corresponding to

$$\begin{cases} n = 1, \\ m = 0, \\ l = 0, \end{cases} \quad \begin{cases} n = 0, \\ m = 1, \\ l = 0, \end{cases} \quad \begin{cases} n = 0, \\ m = 0, \\ l = 1, \end{cases} \tag{47}$$

respectively

$$|1,0,0\rangle = r_I^{(1)} \left(\phi_1^{I\dagger} \otimes \phi_0^{I\dagger} \otimes \phi_0^{I\dagger}\right) |0,0,0\rangle, \tag{48}$$

$$|0,1,0\rangle = s_I^{(1)} \left(\phi_0^{I\dagger} \otimes \phi_1^{I\dagger} \otimes \phi_0^{I\dagger}\right) |0,0,0\rangle, \tag{49}$$

$$|0,0,1\rangle = t_I^{(1)} \left(\phi_0^{I\dagger} \otimes \phi_0^{I\dagger} \otimes \phi_1^{I\dagger}\right) |0,0,0\rangle. \tag{50}$$

The mass-squared operators can be calculated as $\alpha' M_r^2 = N_1 + N_2 + N_3 + N_{12} + N_{23} + N_{13} + N_{123} - 3 = -2$. Similarly

$$M_r^2 = M_s^2 = M_t^2 = -\frac{2}{\alpha'}. \tag{51}$$

Obviously, they are still the tachyon states with the same mass-square.

The next level is the second excited states, and there are six kinds of states

$$|2,0,0\rangle_1 = r_{IJ}^{(2)} \left(\phi_1^{I\dagger} \otimes \phi_0^{I\dagger} \otimes \phi_0^{I\dagger}\right)\left(\phi_1^{J\dagger} \otimes \phi_0^{J\dagger} \otimes \phi_0^{J\dagger}\right) |0,0,0\rangle, \tag{52}$$

$$|2,0,0\rangle_2 = r_I^{(2)} \left(\phi_2^{I\dagger} \otimes \phi_0^{I\dagger} \otimes \phi_0^{I\dagger}\right) |0,0,0\rangle, \tag{53}$$

$$|0,2,0\rangle_1 = s_{IJ}^{(2)} \left(\phi_0^{I\dagger} \otimes \phi_1^{I\dagger} \otimes \phi_0^{I\dagger}\right)\left(\phi_0^{J\dagger} \otimes \phi_1^{J\dagger} \otimes \phi_0^{J\dagger}\right) |0,0,0\rangle, \tag{54}$$

$$|0,2,0\rangle_2 = s_I^{(2)} \left(\phi_0^{I\dagger} \otimes \phi_2^{I\dagger} \otimes \phi_0^{I\dagger}\right) |0,0,0\rangle, \tag{55}$$

$$|0,0,2\rangle_1 = t_{IJ}^{(2)} \left(\phi_0^{I\dagger} \otimes \phi_0^{I\dagger} \otimes \phi_1^{I\dagger}\right)\left(\phi_0^{J\dagger} \otimes \phi_0^{J\dagger} \otimes \phi_1^{J\dagger}\right) |0,0,0\rangle, \tag{56}$$

$$|0,0,2\rangle_2 = t_I^{(2)} \left(\phi_0^{I\dagger} \otimes \phi_0^{I\dagger} \otimes \phi_2^{I\dagger}\right) |0,0,0\rangle. \tag{57}$$

The mass-squared operators can be calculated as $\alpha' M_r^2 = N_1 + N_2 + N_3 + N_{12} + N_{23} + N_{13} + N_{123} - 3 = -1$. Similarly

$$\alpha' M_r^2 = \alpha' M_s^2 = \alpha' M_t^2 = -1. \tag{58}$$

Now let us discuss these 2-rank tachyon states. We will take the states $|2,0,0\rangle_i$ $(i = 1,2)$ for example. Here $r_{IJ}^{(2)}$ can be viewed as the elements of an arbitrary square matrix of size $D-2$. According to the irreducible representation theory in the group theory, any square matrix can be decomposed into its traceless symmetric part, its antisymmetric part and a multiple of the unit matrix,

$$r_{IJ}^{(2)} = \hat{S}_{IJ} + A_{IJ} + \delta_{IJ} S', \tag{59}$$

where $\hat{S}_{IJ}$ denotes the traceless symmetric part of $r_{IJ}^{(2)}$, and A_{IJ} denotes the antisymmetric part of $r_{IJ}^{(2)}$, and S' the trace of $r_{IJ}^{(2)}$. Then the states can be decomposed into the following forms

$$\hat{S}_{IJ} \left(\phi_1^{I\dagger} \otimes \phi_0^{I\dagger} \otimes \phi_0^{I\dagger}\right)\left(\phi_1^{J\dagger} \otimes \phi_0^{J\dagger} \otimes \phi_0^{J\dagger}\right) |0,0,0\rangle, \tag{60}$$

$$A_{IJ} \left(\phi_1^{I\dagger} \otimes \phi_0^{I\dagger} \otimes \phi_0^{I\dagger}\right)\left(\phi_1^{J\dagger} \otimes \phi_0^{J\dagger} \otimes \phi_0^{J\dagger}\right) |0,0,0\rangle, \tag{61}$$

$$S' \left(\phi_1^{I\dagger} \otimes \phi_0^{I\dagger} \otimes \phi_0^{I\dagger}\right)\left(\phi_{1I}^{\dagger} \otimes \phi_{0I}^{\dagger} \otimes \phi_{0I}^{\dagger}\right) |0,0,0\rangle. \tag{62}$$

So far, we have found all the tachyon states of the open 3-brane, where there are two kinds of scalar tachyon states coming from the usual ground states and the second excited states, respectively. Besides, we obtain other types of tachyon states, including the first excited states which associate with three kinds of vector states, and the second exited states which associate with symmetric and antisymmetric 2-rank tensor states. In string theory, a series of developments starting in 1999 have essentially elucidated the role of the open string tachyon. And the presence of tachyon indicates instability of open string theory. More precisely, there is some instability in the theory of open string on the background of a space-filling $D25$-brane. For a quite few years, superstring theories, the kind of string theories that also include fermions, seemed blessedly devoid of tachyons. Later studies, however, showed that tachyons can appear when we construct realistic models based on superstrings. In 3-brane model, we find that there are tachyon states, including not only scalar states, but vector states and tensor states, appearing.

The next level is the third excited states, and there are *nine* kinds of states

$$|3,0,0\rangle_1 = r_I^{(3)}\big(\phi_3^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_0^{I\dagger}\big)|0,0,0\rangle, \tag{63}$$

$$|3,0,0\rangle_2 = r_{IJ}^{(3)}\big(\phi_2^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_0^{I\dagger}\big)\big(\phi_1^{J\dagger}\otimes\phi_0^{J\dagger}\otimes\phi_0^{J\dagger}\big)|0,0,0\rangle, \tag{64}$$

$$|3,0,0\rangle_3 = r_{IJK}^{(3)}\big(\phi_1^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_0^{I\dagger}\big)\big(\phi_1^{J\dagger}\otimes\phi_0^{J\dagger}\otimes\phi_0^{J\dagger}\big)\big(\phi_1^{K\dagger}\otimes\phi_0^{K\dagger}\otimes\phi_0^{K\dagger}\big)|0,0,0\rangle, \tag{65}$$

$$|0,3,0\rangle_1 = s_I^{(3)}\big(\phi_0^{I\dagger}\otimes\phi_3^{I\dagger}\otimes\phi_0^{I\dagger}\big)|0,0,0\rangle, \tag{66}$$

$$|0,3,0\rangle_2 = s_{IJ}^{(3)}\big(\phi_0^{I\dagger}\otimes\phi_2^{I\dagger}\otimes\phi_0^{I\dagger}\big)\big(\phi_0^{J\dagger}\otimes\phi_1^{J\dagger}\otimes\phi_0^{J\dagger}\big)|0,0,0\rangle, \tag{67}$$

$$|0,3,0\rangle_3 = s_{IJK}^{(3)}\big(\phi_0^{I\dagger}\otimes\phi_1^{I\dagger}\otimes\phi_0^{I\dagger}\big)\big(\phi_0^{J\dagger}\otimes\phi_1^{J\dagger}\otimes\phi_0^{J\dagger}\big)\big(\phi_0^{K\dagger}\otimes\phi_1^{K\dagger}\otimes\phi_0^{K\dagger}\big)|0,0,0\rangle, \tag{68}$$

$$|0,0,3\rangle_1 = t_I^{(3)}\big(\phi_0^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_3^{I\dagger}\big)|0,0,0\rangle, \tag{69}$$

$$|0,0,3\rangle_2 = t_{IJ}^{(3)}\big(\phi_0^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_2^{I\dagger}\big)\big(\phi_0^{J\dagger}\otimes\phi_0^{J\dagger}\otimes\phi_1^{J\dagger}\big)|0,0,0\rangle, \tag{70}$$

$$|0,0,3\rangle_3 = t_{IJK}^{(3)}\big(\phi_0^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_1^{I\dagger}\big)\big(\phi_0^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_1^{I\dagger}\big)\big(\phi_0^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_1^{I\dagger}\big)|0,0,0\rangle. \tag{71}$$

Now we have $\alpha' M_r^2 = N_1+N_2+N_3+N_{12}+N_{23}+N_{13}+N_{123}-3=0$. Similarly

$$\alpha' M_r^2 = \alpha' M_s^2 = \alpha' M_t^2 = 0. \tag{72}$$

Now, we can deal with the 2-rank massless states Eq. (64) in the same way as Eq. (59). Then we have the following forms

$$\hat{S}_{IJ}\big(\phi_2^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_0^{I\dagger}\big)\big(\phi_1^{J\dagger}\otimes\phi_0^{J\dagger}\otimes\phi_0^{J\dagger}\big)|0,0,0\rangle, \tag{73}$$

$$A_{IJ}\big(\phi_2^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_0^{I\dagger}\big)\big(\phi_1^{J\dagger}\otimes\phi_0^{J\dagger}\otimes\phi_0^{J\dagger}\big)|0,0,0\rangle, \tag{74}$$

$$S'\big(\phi_2^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_0^{I\dagger}\big)\big(\phi_{1I}^{\dagger}\otimes\phi_{0I}^{\dagger}\otimes\phi_{0I}^{\dagger}\big)|0,0,0\rangle. \tag{75}$$

By analogy with the closed bosonic string theory, Eq. (73) represents one-particle graviton states; Eq. (74) corresponds to the one-particle states of Kalb–Ramond fields; Eq. (75) has no free indices, so it represents one scalar state called dilaton state. While the states $|3,0,0\rangle_1$ simply represent the massless vector states, which can be related to the photon states. Different from the 2-brane model [23], $|3,0,0\rangle_3$ denote 3-rank tensor states, and we can construct totally symmetric and totally antisymmetric 3-rank massless tensor states S_{IJK} and A_{IJK}. The same procedure can be practiced to the states $|0,3,0\rangle_i$ and $|0,0,3\rangle_i$ $(i=1,2,3)$. So we have three sets of spectra of massless states resulted from the symmetric directions of σ^1, σ^2 and σ^3. Here we also find more massless states than string case, especially 3-rank, 2-rank tensor states, vector states and scalar states all have been produced at the same level.

Then we consider the first massive level, which has six kinds of excited states

$$|1,1,0\rangle_1 = r_I^{(4)}\big(\phi_1^{I\dagger}\otimes\phi_1^{I\dagger}\otimes\phi_0^{I\dagger}\big)|0,0,0\rangle, \tag{76}$$

$$|1,1,0\rangle_2 = r_{IJ}^{(4)}\big(\phi_1^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_0^{I\dagger}\big)\big(\phi_0^{J\dagger}\otimes\phi_1^{J\dagger}\otimes\phi_0^{J\dagger}\big)|0,0,0\rangle, \tag{77}$$

$$|1,0,1\rangle_1 = s_I^{(4)}\big(\phi_1^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_1^{I\dagger}\big)|0,0,0\rangle, \tag{78}$$

$$|1,0,1\rangle_2 = s_{IJ}^{(4)}\big(\phi_1^{I\dagger}\otimes\phi_0^{I\dagger}\otimes\phi_0^{I\dagger}\big)\big(\phi_0^{J\dagger}\otimes\phi_0^{J\dagger}\otimes\phi_1^{J\dagger}\big)|0,0,0\rangle, \tag{79}$$

$$|0,1,1\rangle_1 = t_I^{(4)}\big(\phi_0^{I\dagger}\otimes\phi_1^{I\dagger}\otimes\phi_1^{I\dagger}\big)|0,0,0\rangle, \tag{80}$$

$$|0,1,1\rangle_2 = t_{IJ}^{(4)}\big(\phi_0^{I\dagger}\otimes\phi_1^{I\dagger}\otimes\phi_0^{I\dagger}\big)\big(\phi_0^{J\dagger}\otimes\phi_0^{J\dagger}\otimes\phi_1^{J\dagger}\big)|0,0,0\rangle, \tag{81}$$

then we have the mass-squared operator as

$$\begin{aligned}\alpha' M_r^2 &= N_1+N_2+N_3+N_{12}+N_{23}+N_{13}+N_{123}-3\\ &= 1+1+0+\sqrt{2}+0+0+0+(-3)=\sqrt{2}-1,\end{aligned} \tag{82}$$

and similarly

$$\alpha' M_s^2 = \alpha' M_t^2 = \sqrt{2}-1. \tag{83}$$

They belong to the first massive level, including vector states, 2-rank traceless tensor states and scalar states. Now we have obtained some lower levels of Fock states. In these levels, tachyon states, massless states and massive states all appeared, then we find that there are more fruitful contents than the string case.

By comparing the results obtained above with the 2-brane case [23], we can conclude that higher rank tensor states will appear at lower level when the higher-dimensional objects considered.

6. Summary and conclusion

In this Letter, we have given the first and second quantization theories and the spectra of the open p-brane from the free Polyakov action. According to the Euler–Lagrange equation and the Neumann boundary conditions, we have gained the solution to the equation by analogy with the string case. Further, we not only deduce a series of new multiple commutative relations between the different normal modes of the p-brane, which cannot be obtained in the past, but also obtain the new lowering and raising multiple operators in the state space as $\phi_{n_1}^I\otimes\phi_{n_2}^I\otimes\cdots\otimes\phi_{n_p}^I$, $\phi_{m_1}^{J\dagger}\otimes\phi_{m_2}^{J\dagger}\otimes\cdots\otimes\phi_{m_p}^{J\dagger}$, and then we give a series of the new fundamental multiple commutative relations of the lowering and raising multiple operators in the state space, which cannot be obtained before. To investigate the spectra of the p-brane, we naturally and consistently relate the commutative relationships between the different normal modes to the commutative relationships between the different multiple raising and lowering operators, and define the vacuum states which must be annihilated by the lowering operators. And the basis for the Fock space states can be taken of the form with the raising operators acting on the vacuum state. Benefit from the work of Ref. [7], we can choose the vanishing of the Hamiltonian. On the construction of the spectrum, we have removed the infinite contribution of the background.

In the fourth section, we have discussed the spectrum of the open 3-brane explicitly. At the first three levels, there are three classes of tachyon states appearing as scalar states, vector states and 2-rank states, respectively, which are different from the string theory and 2-brane model. In string theory, a tachyon state has the mass-square $-1/\alpha'$ and it represents an excitation of the D-brane with open strings attached, which can lower the energy of the D-brane. So the existence of the tachyon is telling us that the D-brane is unstable and will decay. In 3-brane model, we find from Eqs. (46) and (62) that there are two kinds of scalar tachyon states, one of which has lower mass-square than that of string case and that of 2-brane case [23]. The tachyon state as vectors and 2-rank tensors will also contribute to the lowering of the energy of the D-brane with open 3-branes attached. As a result, all the contributions of the three types of tachyon states must be incorporated into the tachyon potential, then the tachyon potential will depend on all the three types of field configurations which make the D-brane more unstable and decay more easily.

More interestingly, some massless states, such as the graviton states, Kalb–Ramond fields, dilaton states and photon states are all produced at the same level in the open 3-brane model. Besides, 3-rank tensor states also appear at the same level. From the angle of differential geometry, this 3-rank tensor can always be used to construct totally symmetric and totally antisymmetric forms. In string theory, however, graviton fields, Kalb–Ramond fields and dilaton field only appear in closed bosonic string theory and photon states only appear in the open string theory. Whereafter, we also give two massive levels.

This procedure to achieve the spectrum of open 3-brane can be generalized directly to higher-dimensional objects, i.e., p-brane. Due to more and more spacial directions considered, the commutative relations or the constructions of raising and lowering operators will be more and more complicated. The direct result is that all the energy levels will be suppressed greatly, and higher rank tensor states will be produced at lower levels. What attract us is still the tachyon states which will contribute to the instability of the D-brane with p-brane attached. While the appearance of higher rank tachyon states will make the tachyon potential more and more complicated.

The motivation to study the spectra of the p-brane is that the p-brane is the natural generalization of the string as the strings had done to the particles. In Ref. [23], we had investigated the quantization and the spectra of open 2-brane and found some particle states which are not found in string theory, such as vector tachyon states. That is to say, the increase of dimensions will increase the types of the particle states. A technical difference between the p-brane and string theory is that the creating and annihilating operators are difficult to be defined because of the increasing numbers and types of the oscillating modes. In this Letter, we have defined the operators and states by introducing the tensor product forms of single-operator and single-states, respectively. The novel feature in our treatment is that we have defined the creating and annihilating operators and the corresponding states, which possess multidimensional oscillating modes, and we found very fruitful types and numbers of particle states as predicted. Therefore, a general useful theory of the first and second quantizations and spectra of the open p-brane are given in this Letter.

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Appendix A

Now we give the generalized expressions of Eq. (29) for the p-brane.

Substituting the second expression of Eq. (18) into Eq. (21), we have

$$\left[X^I_{n_1n_2n_3\cdots n_{p-1}n_p}, \left(X^J_{m_1m_2m_3\cdots m_{p-1}m_p}\right)^\dagger\right] = -\frac{2^{p-1}}{\pi^{p-1}}\eta^{IJ}\delta_{n_1m_1}\delta_{n_2m_2}\delta_{n_3m_3}\cdots\delta_{n_{p-1}m_{p-1}}\delta_{n_pm_p}. \tag{84}$$

Using Eq. (84) and the first multiple commutative relation of Eq. (26), we obtain

$$\begin{cases} X^I_{n_1n_2n_3\cdots n_p} = \sqrt{\dfrac{2^{p-1}}{\pi^{p-1}}}\phi^{I\dagger}_{n_1}\otimes\phi^{I\dagger}_{n_2}\otimes\cdots\otimes\phi^{I\dagger}_{n_p}, \\ \left(X^J_{m_1m_2m_3\cdots m_p}\right)^\dagger = \sqrt{\dfrac{2^{p-1}}{\pi^{p-1}}}\phi^J_{m_1}\otimes\phi^J_{m_2}\otimes\cdots\phi^J_{m_p}. \end{cases} \tag{85}$$

The other discussions are similar, we don't repeat again.

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