# Thermal Mass Spectra of Vector and Axial-Vector Mesons in Predictive Soft-Wall AdS/QCD Model

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# Introduction

- Gauge/Gravity Correspondence
- Bottom-Up Holographic QCD

# The AdS/CFT Correspondence

*4 dimensional $\mathcal{N}=4$ Super Symmetric $SU(N_c)$ Yang-Mills theory ($N_c \to \infty$) is dual to Type II B Super String theory in an $AdS \times S_5$ background.*

*J. Maldacena 1998*

Gauge/Gravity Correspondence: Extend to less or non SUSY theory

- d+1 Gravity on the bulk ⟺ - d QFT on the boundary

- the classical gravity theory ⟺ - strongly coupled QFT

# AdS/CFT Dictionary

Witten 98; Gubser,Klebanov,Polyakov 98

| | | |
|---|---|---|
| • 5D bulk field $\phi$ | ⟺ | Operator $\mathcal{O}(x)$ |
| • 5D mass $m_5$ | ⟺ | Operator dimesion $\Delta$ |
| • 5D gauge symmetry | ⟺ | Current (global symmetry) |
| • small z | ⟺ | Large Q |
| • Confinement | ⟺ | (IR) cutoff $z_m$ (in Hard Wall) |
| • Kaluza-Klein states | ⟺ | Excited, Resonant spectrum |

$$\langle e^{\int d^d x\, \phi_0(\vec{x})\mathcal{O}(\vec{x})}\rangle_{4-\mathrm{D}\ \mathcal{N}=4\ \mathrm{SU(N)\ SYM}} = Z_{\mathrm{Sugra}}\Big|_{\phi(0,\vec{x})=\phi_0(\vec{x})}$$

GKP-W relation

# AdS/QCD

**<u>Goal</u> : Try to understand QCD using the 5 dimensional dual gravity theory (AdS/CFT correspondence)**

**<u>Approaches</u> :**

- **Top-down Approach : From String Theory**

  **Find brane config. for the gravity dual**

  D3-D7 system; D4-D8 system

- **Bottom-up Approach : From phenomenological**

  **Introduce fields, etc. as needed based on the AdS/CFT**

  *** Hard Wall Model - Introduce IR brane for confinement**

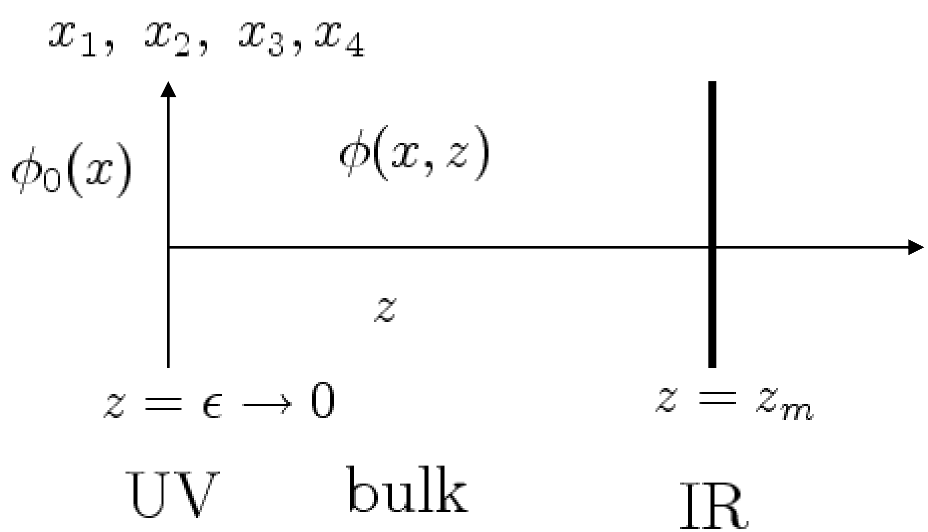
  *** Soft Wall Model – dilaton running**

# Hard wall model

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 95 (2005)

- Infrared Brane at $z = z_m \approx \frac{1}{\Lambda_{\rm QCD}}$ → Confinement
- Metric – Slice of AdS metric

$$ds^2 = \frac{1}{z^2}(-dz^2 + dx^\mu dx_\mu), \qquad 0 < z \le z_m.$$



# Hard wall model

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 95 (2005)

| Boundary | Bulk |
|---|---|
| $\bar{q}_L \gamma^\mu t^a q_L$ | $A^a_{L\mu}$ |
| $\bar{q}_R \gamma^\mu t^a q_R$ | $A^a_{R\mu}$ |
| $\bar{q}^\alpha_R q^\beta_L$ | $(2/z) X^{\alpha\beta}$ |

Global $SU(3)_L$ x $SU(3)_R$ symmetry

$SU(3)_L$ x $SU(3)_R$ gauge symmetry

5D action: $$S = \int d^5x \sqrt{g}\, \mathrm{Tr}\left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

**Mass term is determined by the scaling dimension**

$$(\Delta - p)(\Delta + p - 4) = m_5^2$$

**Xij has dimension $\Delta = 3$ and form p=0, AL & AR have dimension $\Delta = 3$ and form p=1**

# Gauge coupling

In AdS, correlation function of source fields on UV brane

$$\langle J_\mu J_\nu \rangle = \frac{\delta}{\delta V_\mu}\frac{\delta}{\delta V_\nu} e^{iS} = \frac{1}{g_5^2} V_\mu \frac{1}{z}\partial_z V(z)\Big|_{z=\epsilon}$$

Bulk to boundary propagator solution to the equation of motion with V(0)=1

$$V(Q,z) = 1 + \frac{Q^2 z^2}{4}\ln(Q^2 z^2) + \ldots \quad \Rightarrow \quad \Pi_V(Q^2) = -\frac{1}{2g_5^2}\ln Q^2$$

In QCD, correlation function of vector current is

$$\int d^4x \langle J_\mu^a(x) J_\nu^b(0)\rangle = \delta^{ab}\left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\Pi_V(Q^2) \qquad \Pi_V(Q^2) = -\frac{N_c}{24\pi^2}\ln(Q^2)$$

$$g_5^2 = \frac{12\pi^2}{N_c}$$

Large Nc ⟺ small coupling

# Chiral symmetry breaking by X

One can find the expectation value of *X*

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3$$

defined as the classical solutions of field equations, using UV boundary conditions

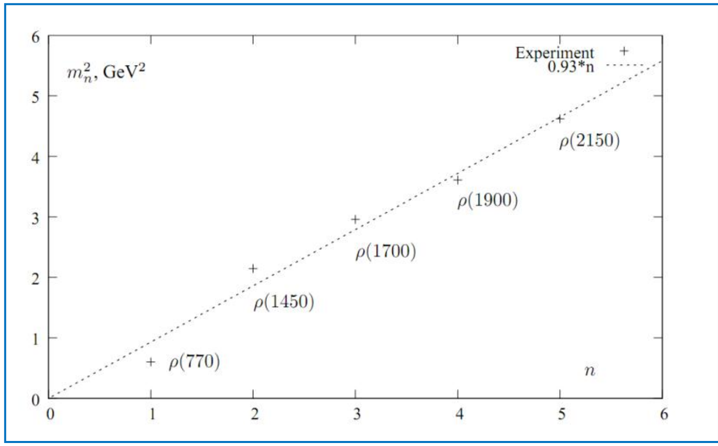
$$(2/\epsilon)X(\epsilon) = M$$

***M*** is quark mass matrix (Explicit chiral breaking)
**Σ** is chiral condensate (Spontaneous chiral breaking)

the model has three free parameters: ***M,* Σ, Zm,** which are fixed from the experiment data. (ρ meson, pion mass and pion decay constant)

# Soft wall model

Introducting a background dilaton $\Phi(z \to \infty) = \mu_d^2 z^2$ can lead to a linear trajectory for resonance vector meson mass




$$I = \int d^5x\, e^{-\Phi(z)} \sqrt{g} \left\{ -|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

Solving equations of motion for vector field

$$-\psi_n'' + V(z)\psi_n = m_n^2 \psi_n \qquad V(z) = \frac{1}{4}(B')^2 - \frac{1}{2}B''$$

$$B = \Phi - A = z^2 + \log z \qquad \psi_n(z) = e^{-\frac{z^2}{2}} z^{m+1/2} \sqrt{\frac{2n!}{(m+n)!}}\, L_n^m\left(z^2\right)$$

Linear trajectory for mass spectra of vector mesons

$$m_n^2 = 4(n+1)$$

# Modified Soft-Wall AdS/QCD

**Y.Q.Sui, YLW, Z.F.Xie, Y.B.Yang** PRD arXiv:0909.3887

- Hard-wall AdS/QCD models contain the chiral symmetry breaking, the resulting mass spectra for the excited mesons are contrary to the experimental data

- Soft-wall AdS/QCD models describe the linear confinement and desired mass spectra for the excited vector mesons, while the chiral symmetry breaking can't consistently be realized.

- A quartic interaction in the bulk scalar potential was introduced to incorporate linear confinement and chiral symmetry breaking. While it causes an instability of the scalar potential and a negative mass for the lowest lying scalar meson state.

- How to naturally incorporate two important features into a single AdS/QCD model and obtain the consistent mass spectra.

# Modified Soft-Wall AdS/QCD

**The 5D action with the background field of dilaton Φ(z) and a quartic term in the bulk scalar potential**

$$S_5 = \int d^5x\sqrt{g}e^{-\Phi(z)}\,\mathrm{Tr}\left[|DX|^2 - m_X^2|X|^2 - \lambda|X|^4 - \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right]$$

**Deformed 5D Metric in IR Region**

$$ds^2 = a^2(z)\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

$$a^2(z) = (1 + \mu_g^2\, z^2)/z^2$$

- IR region is modified
- UV region is not changed

**Equation for the bulk vacuum:**

$$\partial_z\left(a^3(z)e^{-\Phi}\partial_z v(z)\right) - a^5(z)e^{-\Phi}m_X^2 v(z) = 0$$

**IR&UV behaviour of dilaton and vacuum:**

$$v(z\to\infty) = \gamma\,(\mu_d z)^\alpha$$

$$v(z\to 0) = m_q\,\zeta\,z + \frac{\sigma\,z^3}{\zeta}$$

**Solutions for the dilaton field at the UV & IR boundary**

$$\Phi(z\to 0) = 3\mu_g^2\,z^2 + O(z^4)$$

$$\Phi(z\to\infty) = \mu_d^2\,z^2$$

# Three type of models for v(z)

| Models | $v(z)$ | 参数 | |
|---|---|---|---|
| Ia | $z(A+Bz^2)(1+Cz^2)^{-1}$ | $B=\frac{\sigma}{\zeta}+m_q\zeta C,$ | $C=B/\mu_d\gamma$ |
| Ib | $z(A+Bz^2)(1+Cz^2)^{-5/4}$ | $B=\frac{\sigma}{\zeta}+\frac{5}{4}m_q\zeta C,$ | $C=(B^2/\mu_d\gamma^2)^{2/5}$ |
| IIa | $z(A+Bz^2)(1+Cz^4)^{-1/2}$ | $B=\frac{\sigma}{\zeta},$ | $C=(B/\mu_d\gamma)^2$ |
| IIb | $z(A+Bz^2)(1+Cz^4)^{-5/8}$ | $B=\frac{\sigma}{\zeta},$ | $C=(B^2/\mu_d\gamma^2)^{4/5}$ |
| IIIa | $z[A+B\tanh(Cz^2)]$ | $B=\mu_d\gamma-m_q\zeta,$ | $C=\frac{\sigma}{\zeta B}$ |
| IIIb | $z[A+B\tanh(Cz^2)](1+Gz^4)^{-1/8}$ | $B=\mu_d^{1/2}\gamma G^{1/8}-m_q\zeta,$ | $C=\frac{\sigma}{\zeta B}$ |

$$A=m_q\zeta$$

**Two IR boundary conditions of the bulk VEV:**

$$\text{Case a}: \quad v(z\to\infty)=\gamma\left(\mu_d z\right)$$

$$\text{Case b}: \quad v(z\to\infty)=\gamma\left(\sqrt{\mu_d z}\right)$$

# 参数输入

- $\mu_g$ scales the mass spectra of meson resonances, it can be found out from a global fitting.

$$\mu_g = 363\,\mathrm{MeV} \quad (\mathrm{Case\ a}) \qquad \mu_g = 257\,\mathrm{MeV} \quad (\mathrm{Case\ b})$$

- The three parameters $m_q$, $\sigma$ and $\gamma$ are fixed by the known experimental values:

$$m_\pi = 139.6\ \mathrm{MeV} \qquad f_\pi = 92.4\ \mathrm{MeV} \qquad f_\pi^2 m_\pi^2 = 2 m_q \sigma$$

| Parameter | Ia | Ib | IIa | IIb | IIIa | IIIb |
|---|---|---|---|---|---|---|
| $m_q$ (MeV) | 4.16 | 4.64 | 4.44 | 4.07 | 4.98 | 4.25 |
| $\sigma^{\frac{1}{3}}$ (MeV) | 275 | 265 | 265 | 272 | 255 | 268 |
| $\gamma$ | 0.178 | 0.136 | 0.153 | 0.112 | 0.164 | 0.112 |

for $\lambda = 0$

# Solving Equations of Motion

Pseudoscalar Sector

$$\partial_z\left(a(z)e^{-\Phi}\partial_z\phi^a\right)+g_5^2\,a^3(z)\,v^2(z)e^{-\Phi}\left(\pi^a-\phi^a\right)=0$$

$$q^2\partial_z\phi^a-g_5^2\,a^2(z)\,v^2(z)\partial_z\pi^a=0$$

Scalar Sector

$$\partial_z\left(a^3(z)e^{-\Phi}\partial_z S_n(z)\right)-a^5(z)e^{-\Phi}m_X^2 S_n(z)=-a^3(z)e^{-\Phi}m_{S_n}^2 S_n(z)$$

Vector Sector

$$-\partial_z^2 V_n+\omega'\partial_z V_n=m_{V_n}^2 V_n$$

Axial-vector Sector

$$e^{\Phi}\partial_z(a(z)e^{-\Phi}\partial_z A_n)+a(z)q^2A_n-a^3(z)g_5^2v^2(z)A_n=0$$

IR & UV Boundary Condition：

$$s_n(z\to 0)=0,\quad \partial_z s_n(z\to\infty)=0,$$

# Mass Spectra of Vector Mesons

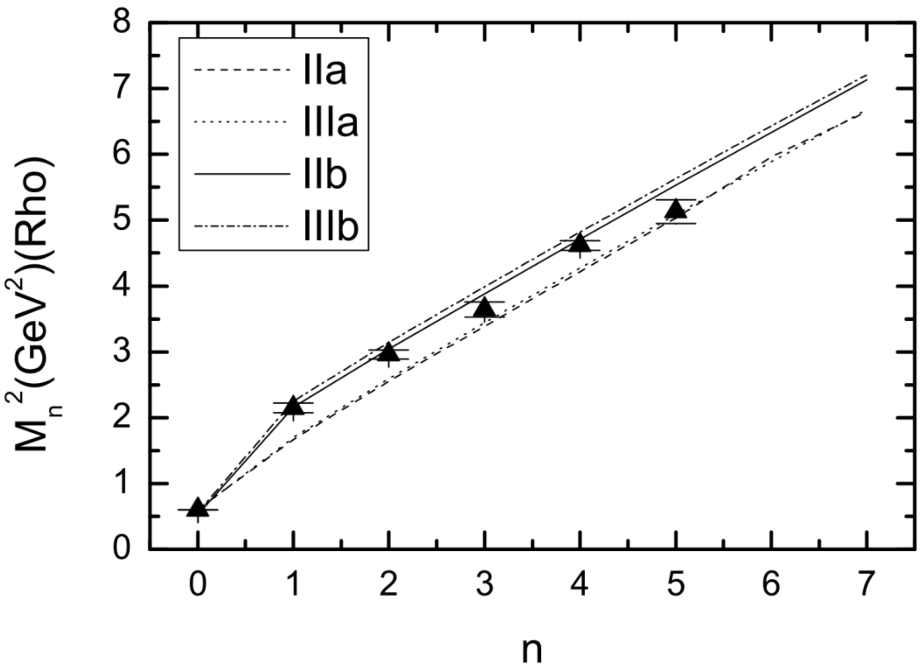
Without Quartic Interaction of bulk scalar $\boxed{\lambda = 0}$

| n | $\rho$实验值(MeV) | Ia | Ib | IIa | IIb | IIIa | IIIb |
|---|---|---|---|---|---|---|---|
| 0 | $775.5 \pm 1$ | 739 | 603 | 777 | 727 | 775 | 748 |
| 1 | $1465 \pm 25$ | 1223 | 1175 | 1292 | 1468 | 1303 | 1501 |
| 2 | $1720 \pm 20$ | 1534 | 1509 | 1596 | 1744 | 1610 | 1773 |
| 3 | $1909 \pm 30$ | 1784 | 1769 | 1842 | 1971 | 1856 | 1999 |
| 4 | $2149 \pm 17$ | 2000 | 1990 | 2054 | 2170 | 2068 | 2196 |
| 5 | $2265 \pm 40$ | 2193 | 2187 | 2249 | 2351 | 2255 | 2373 |
| 6 | —— | 2370 | 2367 | 2417 | 2516 | 2426 | 2535 |
| 7 | —— | 2534 | 2532 | 2578 | 2671 | 2584 | 2685 |

# Mass Spectra of Vector Mesons

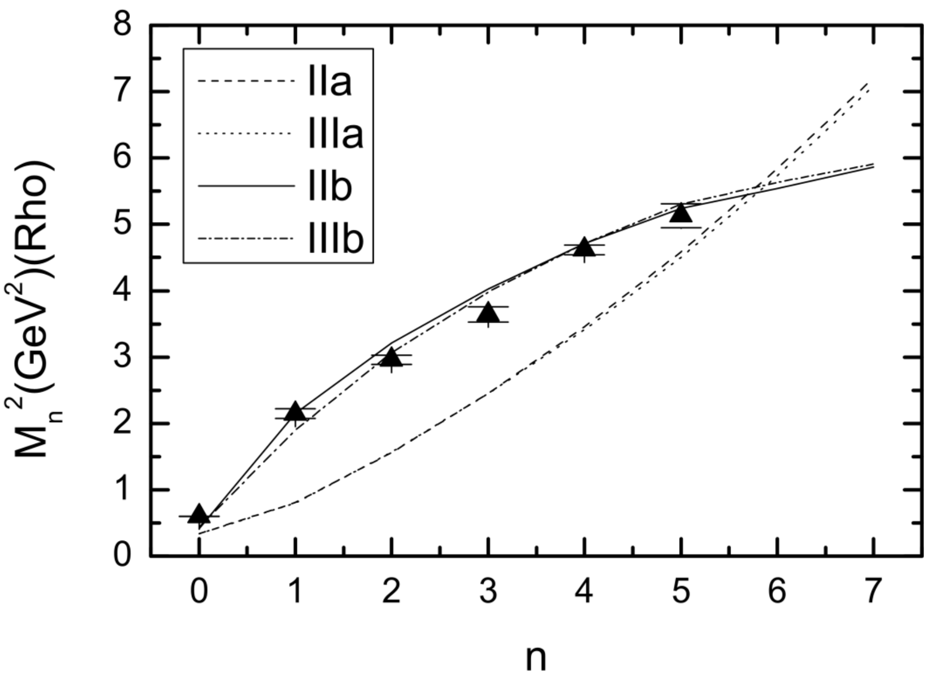
With Quartic Interaction of bulk scalar $\lambda = 9$

| n | $\rho$实验值(MeV) | IIa | IIb | IIIa | IIIb |
|---|---|---|---|---|---|
| 0 | $775.5 \pm 1$ | 583 | 646 | 584 | 661 |
| 1 | $1465 \pm 25$ | 900 | 1468 | 893 | 1378 |
| 2 | $1720 \pm 20$ | 1248 | 1793 | 1252 | 1753 |
| 3 | $1909 \pm 30$ | 1564 | 2008 | 1565 | 1994 |
| 4 | $2149 \pm 17$ | 1860 | 2170 | 1847 | 2174 |
| 5 | $2265 \pm 40$ | 2143 | 2289 | 2122 | 2332 |
| 6 | —— | 2417 | 2353 | 2395 | 2372 |
| 7 | —— | 2685 | 2420 | 2661 | 2432 |




$\lambda = 0$

$\lambda = 9$



# For finite temperature

| QCD | | gravity theory |
|---|---|---|
| deconfinement phase transition | ⟷ dual | Hawking-Page transition |

[ Herzog , Phys.Rev.Lett.98:091601,2007 ]

- AdS BH (Deconfinement) $ds^2 = \frac{L^2}{z^2}\left(f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right)$

$f(z) = 1 - (z/z_h)^4$   $z_h$ :Horizon

(HP)   Hawking temperature $T = 1/(\pi z_h)$ ➔ Temperature of the medium

- Thermal AdS (Confinement) $ds^2 = L^2\left(\frac{dt^2 + d\vec{x}^2 + dz^2}{z^2}\right)$

# Phase Transition

C.P.Herzog,PRL 98.091601 (2007)

On-shell action density ⟺ Free energy

$$I = -\frac{1}{2\kappa^2}\int d^5x\sqrt{g}\left(R + \frac{12}{L^2}\right)$$

On-shell →

$$I = \frac{4}{L^2\kappa^2}\int d^5x\sqrt{g}$$

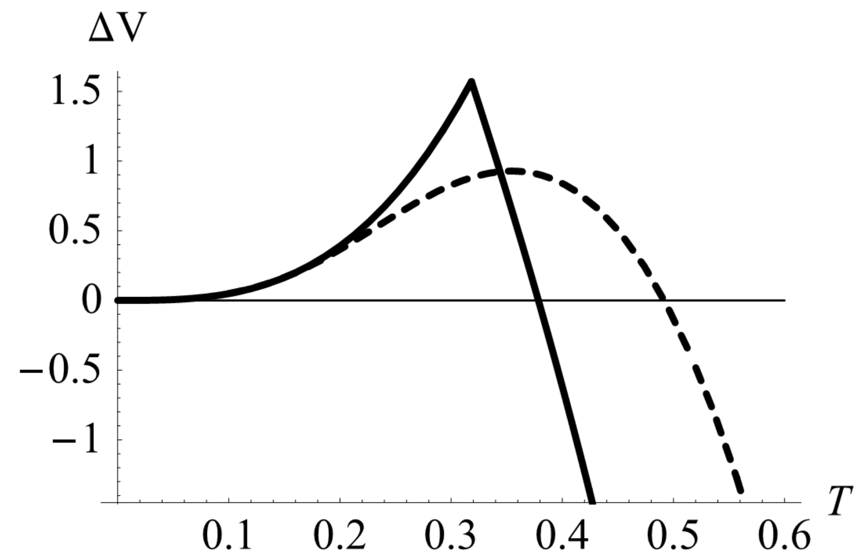
- tAdS: $V_1(\epsilon) = \frac{4L^3}{\kappa^2}\int_0^{\beta'} dt \int_\epsilon^\infty dz\, z^{-5} e^{-cz^2}$
- AdSBH: $V_2(\epsilon) = \frac{4L^3}{\kappa^2}\int_0^{\pi z_h} dt \int_\epsilon^{z_h} dz\, z^{-5} e^{-cz^2}$
- Difference between V1 and V2

$$\Delta V = \lim_{\epsilon\to 0}\left(V_2(\epsilon) - V_1(\epsilon)\right)$$

When △V is positive (negative), tAdS (AdSBH) is stable.

- Critical Temperature (1st order)

Tc,Hard = 122 and Tc,soft = 191 MeV.



# Extend to finite temperature

L.X.Cui, Shingo Takeuchi, Y.L.Wu arXiv:1112.5923

**5D action:**

$$S_5 = \int d^5x \sqrt{g} e^{-\Phi(z)} \,\mathrm{Tr}\left[ |DX|^2 - m_X^2 |X|^2 - \lambda |X|^4 - \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right]$$

**Deformed AdS Black Hole:** $ds^2 = a^2(z)\left(f(z)dt^2 - \sum_{i=1}^{3} dx_i^2 - \frac{dz^2}{f(z)}\right)$

with $a^2(z) = 1/z^2 + \mu_g^2$

- IR region is improved
- If $\mu_g = 0$ , it become an AdS BH

$$f(z) = 1 - (z/z_0)^4$$

$z_0$ : Horizon

**Hawking temperature is not changed:**

$$T = 1/(z_0 \pi)$$

# The EOM for Vector and Axial-Vector

Action at the quadratic order:

$$S = \int dx^5 e^{-\Phi(z)} \sqrt{g}\, \mathrm{Tr}\Big[-\frac{1}{2g_5^2}\left(F^V F^V + F^A F^A\right)$$
$$- \left(2X_0\partial\pi + [V, X_0] + \{A, X_0\}\right)\left(2X_0\partial\pi - [V, X_0] + \{A, X_0\}\right)\Big]$$

Vector: $0 = \widetilde{V}_x^{a\prime\prime}(p,u) + \left(\frac{a'(u)}{a(u)} + \frac{f'(u)}{f(u)} - \Phi'(u)\right)\widetilde{V}_x^{a\prime}(p,u) + \frac{1}{(\pi T)^2}\left(\frac{\omega^2}{f^2(u)} - \frac{q^2}{f(u)}\right)\widetilde{V}_x^a(p,u)$

Axial-Vector: $0 = \widetilde{W}_x^{a\prime\prime}(p,u) + \left(\frac{a'(u)}{a(u)} + \frac{f'(u)}{f(u)} - \Phi'(u)\right)\widetilde{W}_x^{a\prime}(p,u) + \frac{1}{(\pi T)^2}\left(\frac{\omega^2}{f^2(u)} - \frac{q^2}{f(u)}\right)\widetilde{W}_x^a(p,u)$

$$+ g_5^2 \frac{v^2(u)}{u^2 f(u)}\widetilde{W}_x^a(p,u).$$

# Taking in-falling boundary condition

From equation of V(z):

$$\Phi'(z) = \frac{3a'(z)}{a(z)} + \frac{\left(f(z)v'(z)\right)'}{f(z)v'(z)} - \frac{a^2(z)}{f(z)v'(z)} m_X^2 v(z)$$

We assume:

$$v(z) = v_1(z) \ln f(z) + v_0(z)$$

divergent term can be read:

$$\Phi'(z) = \frac{\ln f(z)}{v_1(z) f'(z)} \left\{ -a^2(z) v_1(z) m_X^2 + f'(z) v_1'(z) \right\} + \cdots$$

finite temperature part of vacuum:

$$v_1(z) = c_v \exp\left[ \frac{m_X^2}{8(\pi T)^4 z^4} \left( \frac{1}{2} + \mu_g^2 z^2 \right) \right]$$

# Retarded Green's function

◆ 2 independent solutions near the boundary

$$V(u \to 0) \quad \Rightarrow \quad \begin{aligned} \Phi_0^a(\omega, q, u=\epsilon) &= \epsilon\, Y_1\left(\frac{\sqrt{\omega^2 - q^2}}{\pi T}\epsilon\right) \\ \Phi_1^a(\omega, q, u=\epsilon) &= \epsilon\, J_1\left(\frac{\sqrt{\omega^2 - q^2}}{\pi T}\epsilon\right) \end{aligned}$$

◆ 2 independent solutions near the horizon

$$V(u \to 1) \quad \Rightarrow \quad V_\pm = (1-u)^{\pm i \frac{\omega}{4\pi T}}$$

$V_+$ : Out-coming from Black Hole → Advanced Green's function

$V_-$ : In-falling into Black Hole → Retarded Green's function

[D.T.Son and A.O.Starinets (2002)]

# Spectral Function

◆Linear combination of the 2 solution → In-falling solution

$$V(u) = A(\omega, q)\,\underline{\Phi_1(u)} + B(\omega, q)\underline{\Phi_2(u)} \xrightarrow{u \longrightarrow 1} V_{-}(u)$$

boundary condition: $\Phi_1 \approx 1$ $\Phi_2 \approx \epsilon^2$

◆Retarded Green's function

$$D^R(\omega, q) = \lim_{\epsilon \to 0} \frac{\delta^2}{\delta V_0(p)\delta V_0(p)} \exp(-S[V(p,\epsilon)])\Big|_{V_0=0}$$

:GKP-W relation

$$\propto \lim(\frac{1}{\epsilon} V(\epsilon)^* \partial_\epsilon V(\epsilon))$$

$$\propto -2\left[\frac{B^a(\omega, q)}{A^a(\omega, q)} + \frac{q^2 - \omega^2}{2}\left\{\gamma_E + \log\left(\frac{\epsilon}{2}\sqrt{\omega^2 - q^2}\right)\right\}\right]$$

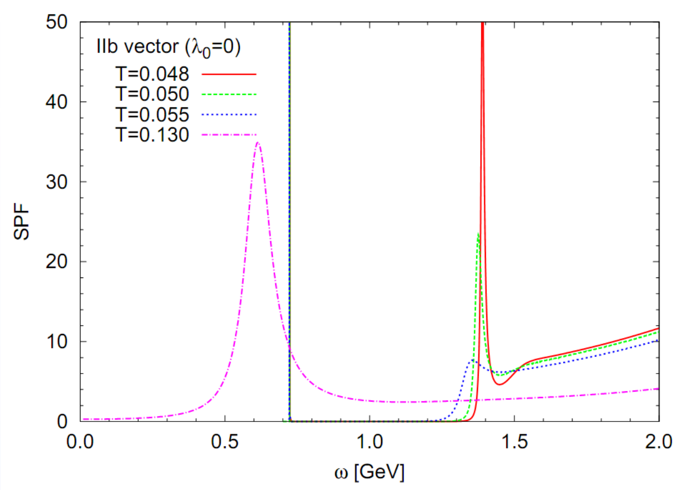
◆Spectral Function

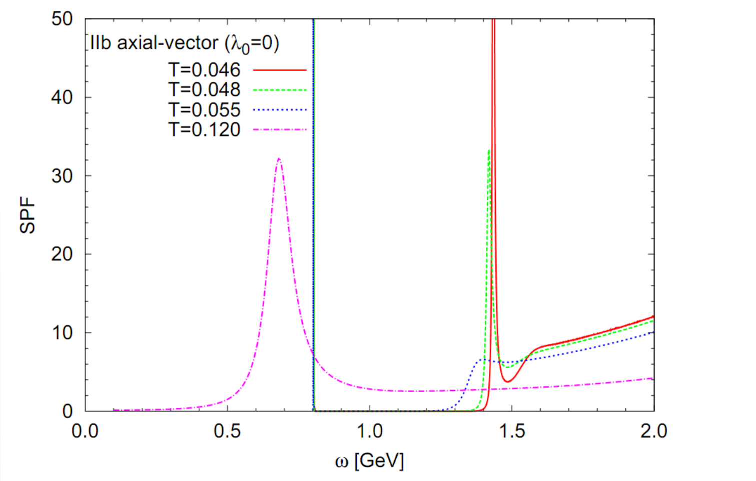
$$\rho(\omega, q) = -\frac{1}{\pi} \mathrm{Im}\, G^R(\omega, q)\,\theta(\omega^2 - q^2) \propto \frac{B^a(\omega, q)}{A^a(\omega, q)}$$

# Numerical Results

◆Low temperature results

$\lambda = 0$



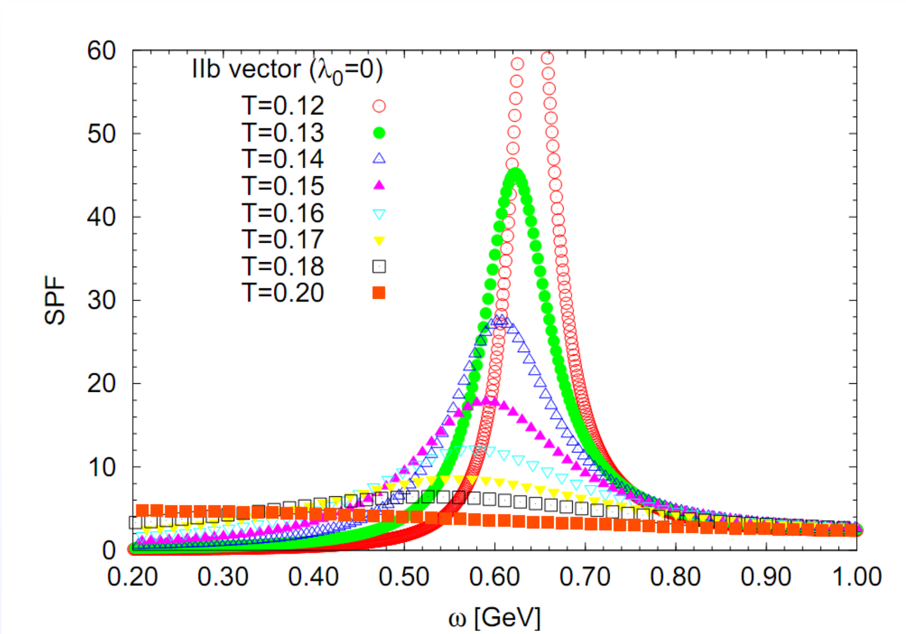

| n | $\rho_V$ (MeV) | Ia | Ib | IIa | IIb | $\rho_{AV}$ (MeV) | Ia | Ib | IIa | IIb |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $775.5 \pm 1$ | 739 | 603 | 777 | 727 | $1230 \pm 40$ | 934 | 714 | 940 | 807 |
| 1 | $1465 \pm 25$ | 1223 | 1175 | 1292 | 1468 | $1647 \pm 22$ | 1468 | 1247 | 1496 | 1507 |
| 2 | $1720 \pm 20$ | 1534 | 1509 | 1596 | 1744 | $1930^{+30}_{-70}$ | 1822 | 1573 | 1831 | 1778 |
| 3 | $1909 \pm 30$ | 1784 | 1769 | 1842 | 1971 | $2096 \pm 122$ | 2109 | 1829 | 2102 | 2003 |
| 4 | $2149 \pm 17$ | 2000 | 1990 | 2054 | 2170 | $2270^{+55}_{-40}$ | 2358 | 2049 | 2338 | 2202 |
| 5 | - | 2193 | 2187 | 2249 | 2351 | - | 2582 | 2049 | 2338 | 2202 |

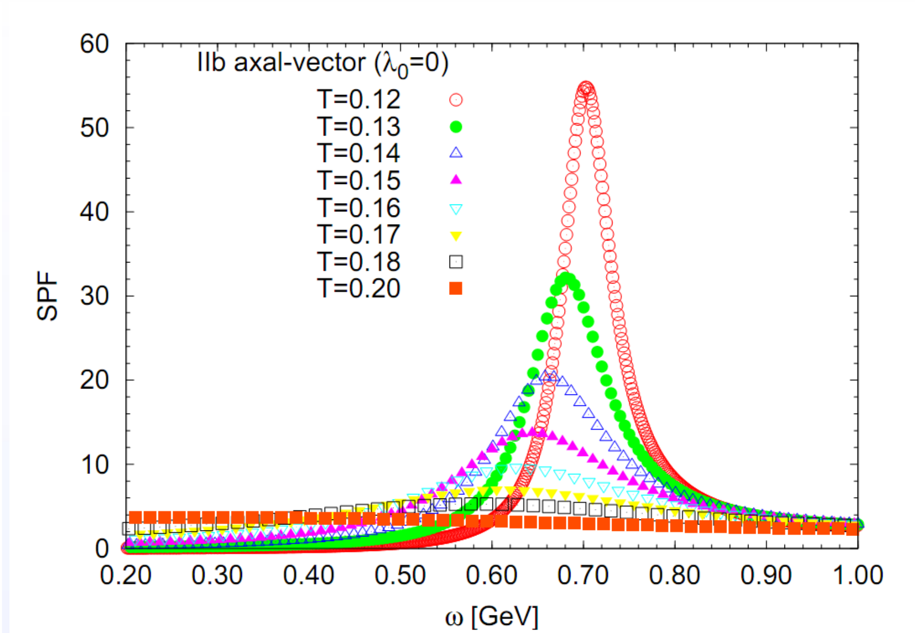
At low temperature, sharp peaks stand in accord with t=0 spectrum.

# Numerical Results

◆Higher temperature results

$\lambda = 0$



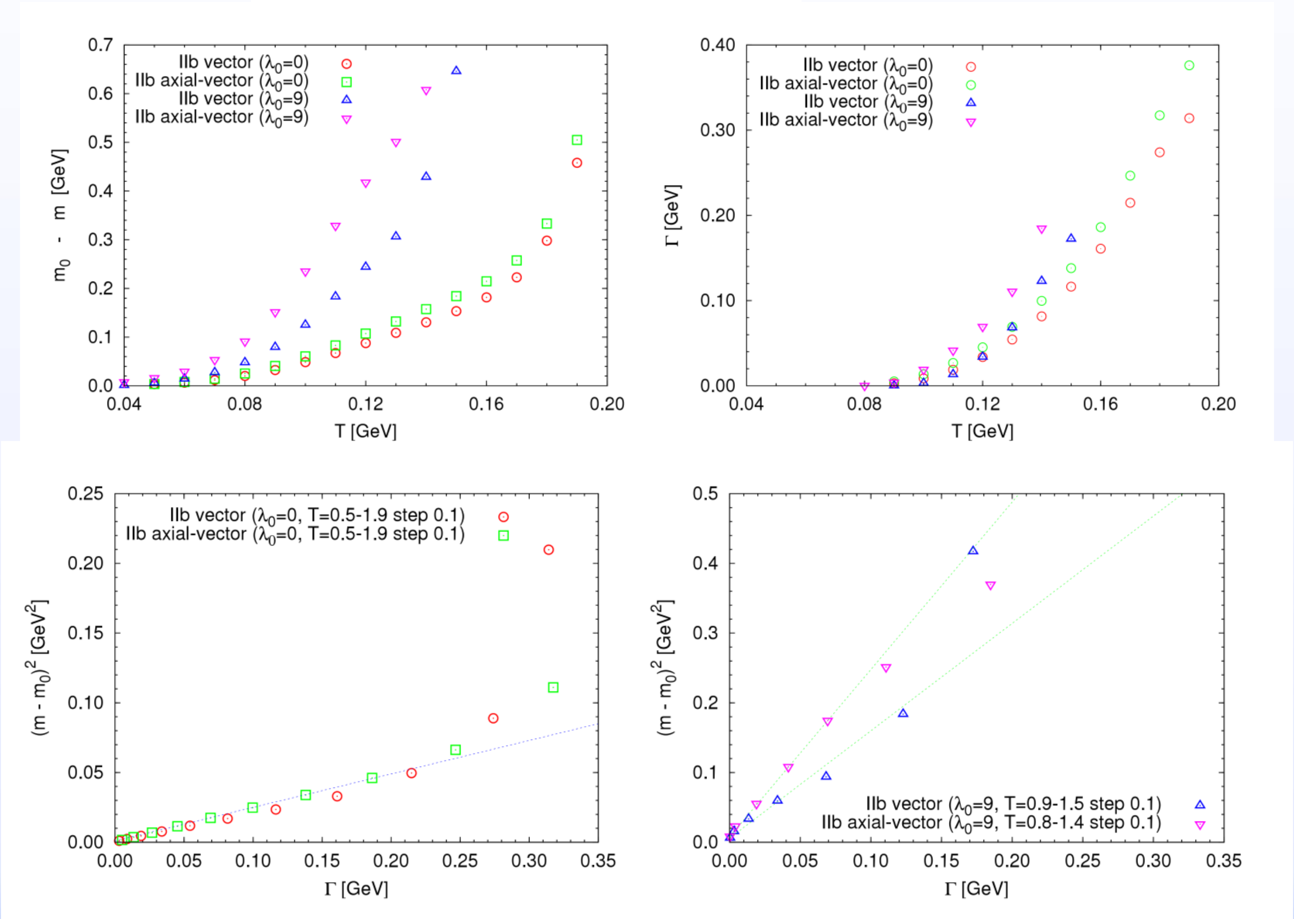

◆Properties

1. The lowest-lying state melts gradually as T increases.
2.  The peak moves to a smaller mass with increasing T.
3. The excited states melt much earlier and shift more.
4. The lowest-lying state survies until T~200MeV

# Mass shift and width of peak from spectral function

- fitting the spectral function by following Breit-Wigner form:

$$\frac{a\omega^b}{(\omega^2 - m^2)^2 + \Gamma^2} + P(\omega^2) \quad \text{with } P(\omega^2) = c_1 + c_2\omega^2 + c_3(\omega^2)^{c_4}$$



# comparison with other models

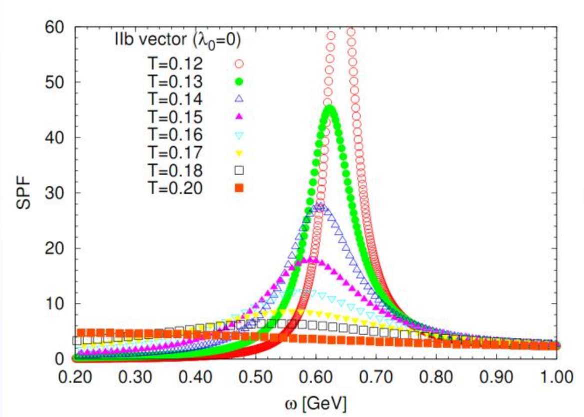
**Our results:**

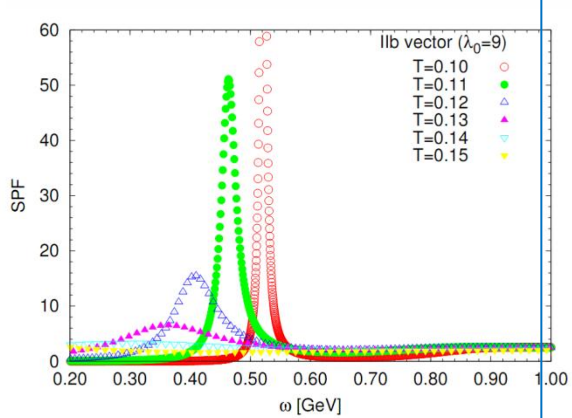
$\lambda = 0$:

$T_c = 200\,\mathrm{MeV}$

$\lambda = 9$:

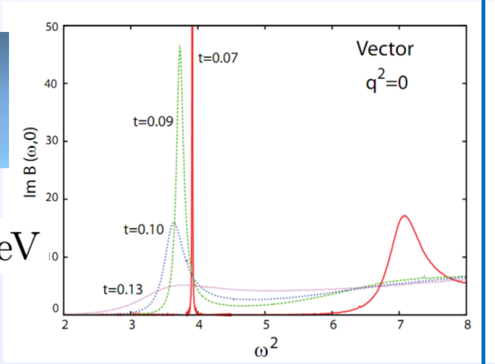
$T_c = 150\ \mathrm{MeV}$



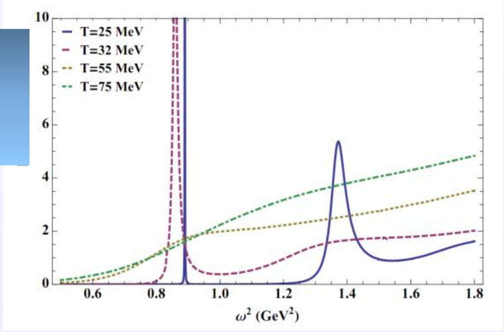

**Fukushima, et al (arXiv:0903.2316)**

$T_c = 0.15\sqrt{c} = 60\ \mathrm{MeV}$




**Colangelo,et al (arXiv:0909.1534)**

$T_c = 75\ \mathrm{MeV}$



# Why is this difference

- the introduction of the scale μg in the modified 5D metric will be crucial for causing the differences.

$$a^2(z) = 1/z^2 + \mu_g^2 \rightarrow \mu_g^2/(\pi^2 T^2)u^2$$

(μg couples temperature in the metric)

- dilaton and the scalar field
  - the dilaton in our model is determined as the solution of equation of motion with the given v(z) which consists of the finite temperature and the zero-temperature parts
  - to cooperate the chiral symmetry breaking and linear confinement we have to obtain the correct boundary conditions for both the VEV at the UV boundary and the dilaton at the IR boundary

Thanks you!