Cook's Matter Field in the Stephenson-Kilmister-Yang Theory of Gravity

Nien-En Tung

May 10, 2012

Outline

- Introduction
- Results of Cook's theory
- Summary

- Alternative gravitational theory.
- General relativity (GR)

$$S_{EH} = \frac{1}{\kappa} \int_{\Sigma} \sqrt{-g}\; R d^4 x \ .$$

- Other Lagrangian densities: $\sqrt{R}$, $R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu}$, $f(R)$, etc.

Introduction

- Weyl's Unification: To combine the gravitation with the electromagnetism.
- A conformal structure. All physics should not be altered after the measurement unit is changed.
- In his work, he proposed the contracted Riemann tensor $R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu}$ as a new Lagrangian for the unification.

- The equation of motion (EOM) of $R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu}$: Stephenson (1958), Kilmister and Newman (1961).
- Stephenson showed the EOM:

$$\nabla_\nu R^\nu{}_{\mu\alpha\beta} = 0. \tag{1}$$

- With the use of the second Bianchi identity,

$$\nabla_\mu R_{\alpha\beta} - \nabla_\beta R_{\alpha\mu} = 0 \tag{2}$$

- In fact, $\nabla_\mu R_{\alpha\beta} - \nabla_\beta R_{\alpha\mu} = 0$ is the same as Yang's gravitational equation by applying the $GL(n)$-gauge theory.
- The SKY equation: $\nabla_\nu R^{\nu}{}_{\mu\alpha\beta} = 0$.
- Stephenson, Kilmister and Newman showed the solution of the vacuum Einstein equation satisfies the SKY equation.
- Although Pavelle found unphysical solution to Eq. (2), Stephenson's equation could be improved by some regulation or restriction. (Thompson 1975)

- The matter field for the SKY equation.
- Kilmister obtained the gravitational equation with the matters

$$\nabla_\mu R_{\alpha\beta} - \nabla_\beta R_{\alpha\mu} = I_{\alpha\mu\beta}, \tag{3}$$

- He characterised that the I has 20 independent components with restricting by conservation identities:

$$\nabla_\beta I^{\alpha\mu\beta} = 0. \tag{4}$$

- He did not describe the explicit form for the current density I.

- Camenzind (1975) wrote down a Yang-Mills field equation for $SO(3,1)$:

$$\nabla_\nu R^\nu{}_{\mu\alpha\beta} = 8\pi G_N \widehat{J}_{\alpha\beta\mu}, \tag{5}$$

- he chose the current density $\widehat{J}$ as:

$$\widehat{J}^{\alpha\beta\mu} = \nabla^\beta \left(T^{\mu\alpha} - 1/2 g^{\mu\alpha} T^\lambda_\lambda \right) - \nabla^\mu \left(T^{\beta\alpha} - 1/2 g^{\beta\alpha} T^\lambda_\lambda \right), \tag{6}$$

  where the tensor T is the energy-momentum tensor

- Camenzind's theory is one's inability to draw Eq. (5) from the principle of variation.

- Cook (2008) can derive Camenzind-like equation by the variation method of Palatini
-

$$S_G = \frac{-1}{16\pi} \int_{\Sigma} \sqrt{-g}\, d^4x \, (R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu} + 16\pi J^{\alpha\beta}{}_{\mu} \Gamma^{\mu}{}_{\alpha\beta}), \quad (7)$$

where Σ is a spacetime region. The J-term is

$$J_{\alpha\beta}{}^{\mu} = \frac{2G_N}{c^4} \left(\nabla_\alpha \bar{T}^{\mu}_{\beta} - \nabla_\beta \bar{T}^{\mu}_{\alpha} \right), \quad (8)$$

where $\bar{T}^{\mu}_{\beta} = T_{\beta}{}^{\mu} - \frac{1}{2}\delta_{\beta}{}^{\mu} T$ and $T = T_{\mu}{}^{\mu}$ with the Newton constant G_N.

- The EOM:

$$\nabla_\nu R^{\nu}{}_{\mu\alpha\beta} = 4\pi J_{\alpha\beta\mu}. \quad (9)$$

- The SKY-Cook theory is analogous in mathematical structure with the Maxwell theory.
- Cook views the Christoffel symbol as an gauge potential.
- Therefore, the Riemann tensor can be viewed as the field strength.
- In the Lagrangian density, he coupled the current density J with the connection.

- To investigate the Friedmann-Robertson-Walker (FRW) solution in Cook's theory.
- The FRW metric in GR.

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \qquad (10)$$

- The Friedmann equation with scalar curvature $k = 0$

$$H^2 = \frac{8\pi G_N}{3}\rho, \qquad (11a)$$

$$\dot{H} + H^2 = -\frac{4\pi G_N}{3}(\rho + 3p), \qquad (11b)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter.

- Dominant species: Radiation $a(t) \propto t^{1/2}$.; matter $a(t) \propto t^{2/3}$.; vacuum $a(t) \propto e^{\Lambda t}$.

- The FRW metric in the SKY-Cook theory.

$$\ddot{H} + 4H\dot{H} = 8\pi G_N \left(\frac{1}{2} (\dot{\rho} - \dot{p}) + H(\rho + p) \right), \qquad (12)$$

one can see that Eq. (12) involves a higher-order derivative.

- Matter Dominant Case
- $$\frac{\dot{a}^2}{2} - \frac{4\pi G_N M}{3a} + \frac{B}{4a^2} = \frac{Ca^2}{4}, \tag{13}$$
  where B and C are two integral constants.
- When B and C are null, we can show that $a(t) \propto t^{2/3}$.

- Radiation Dominant case
- The dynamical equation in the radiation case is

$$\nabla_\nu R^\nu{}_{i0i} = 0, \tag{14}$$

-
$$\frac{\dot{a}^2}{2} + \frac{A_0}{4a^2} = -\frac{k}{2} + B_0 a^2, \tag{15}$$

  where A_0 and B_0 are two integral constants.
- If the universe is at the late time, then the curvature term and the potential term will not be important. That means:

$$\frac{\dot{a}^2}{2} \simeq B_0 a^2. \tag{16}$$

  In this case, $a \simeq C_0 \exp(\sqrt{2B_0}t)$ with a constant C_0.

- Th initial condition of the universe. $a(0) = 0, \dot{a} > 0$.
- The solution of the $a(t)$ is $a = C_0 \exp(\sqrt{2B_0}t)$.

- In the above calculation, one may note that the current density tensor J is null while the universe is filled with the radiation.
- The radiation solution now belongs to the set of the vacuum solutions of Eq. (9) and therefore the light is excluded from the species of the ordinary matters.

- In this definition of the current density J, we can not distinguish the gravitational effect of the radiation from that of the vacuum energy.
- Therefore, such a definition of the current density of the matter seems to be improper.
- Without the matter field, Eq. (1) or Eq. (2) contains the all solution space of the Einstein equation and shows no serious contradiction to Einstein's formula.
- The matter field for the SKY equation is an open question.