Suppression of the CMB quadrupole mode due to the primordial black hole remnants

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緊急
First LeCosPA Symposium:
Towards Ultimate Understanding
of the Universe


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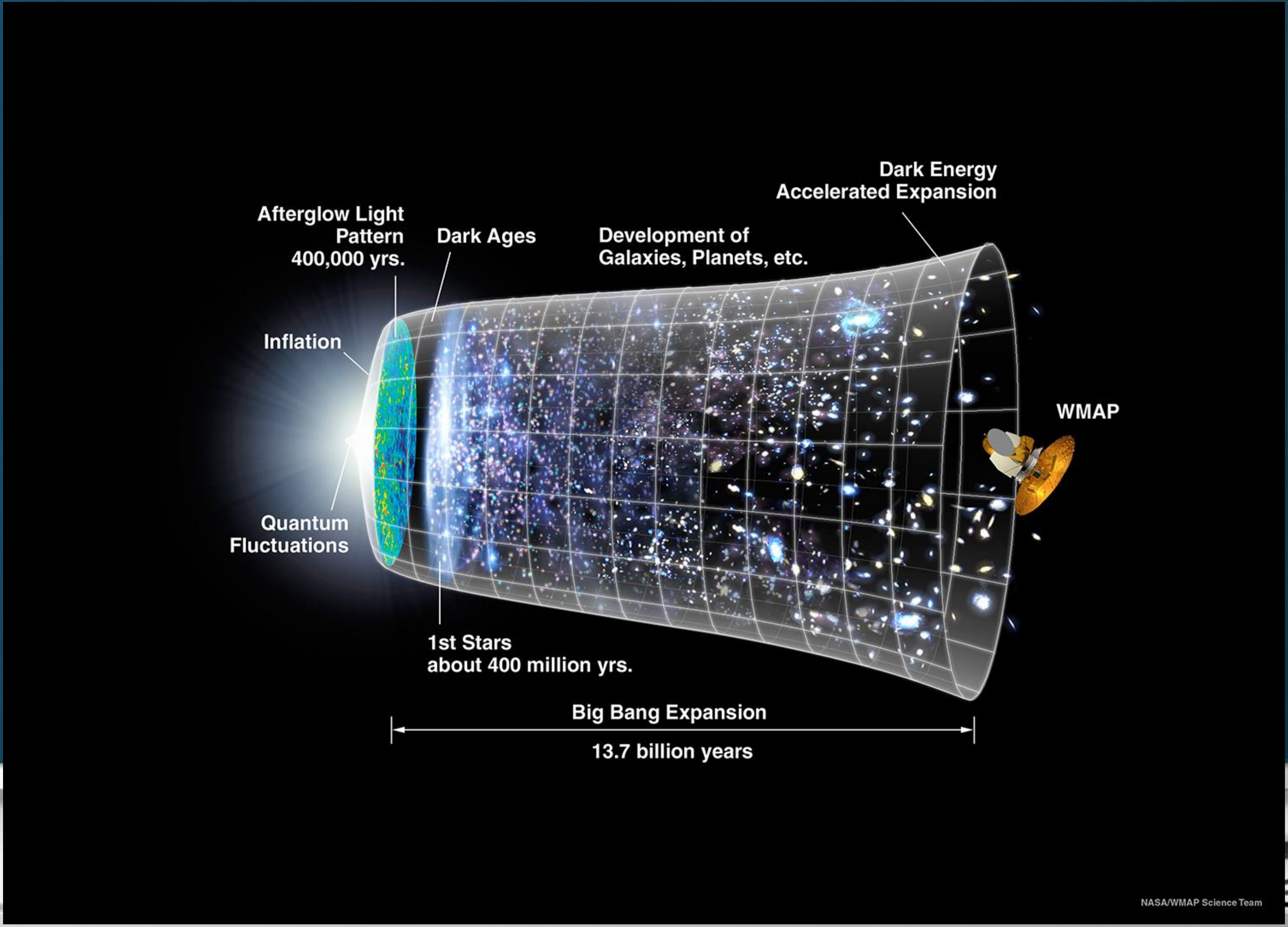
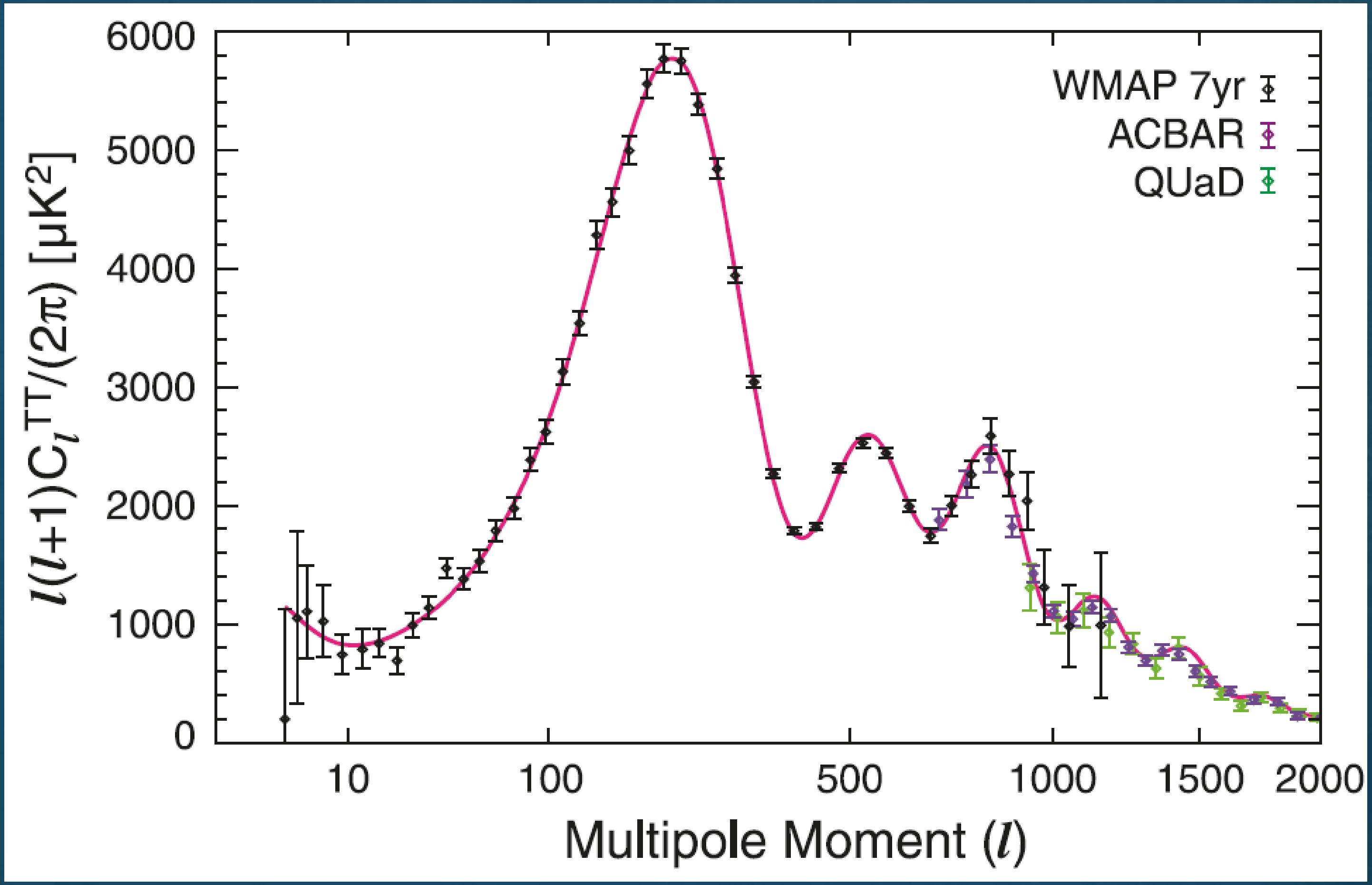
Outline

- Our understanding to the history of the universe
- The primordial black hole remnants and the pre-inflation matter era
- Issues of the present treatment and the *ab initio* treatment proposed
- Quantization of the scalar field in the pre-inflation era
- The resulting mode evolution and primordial spectrum

The Evolution of the Universe

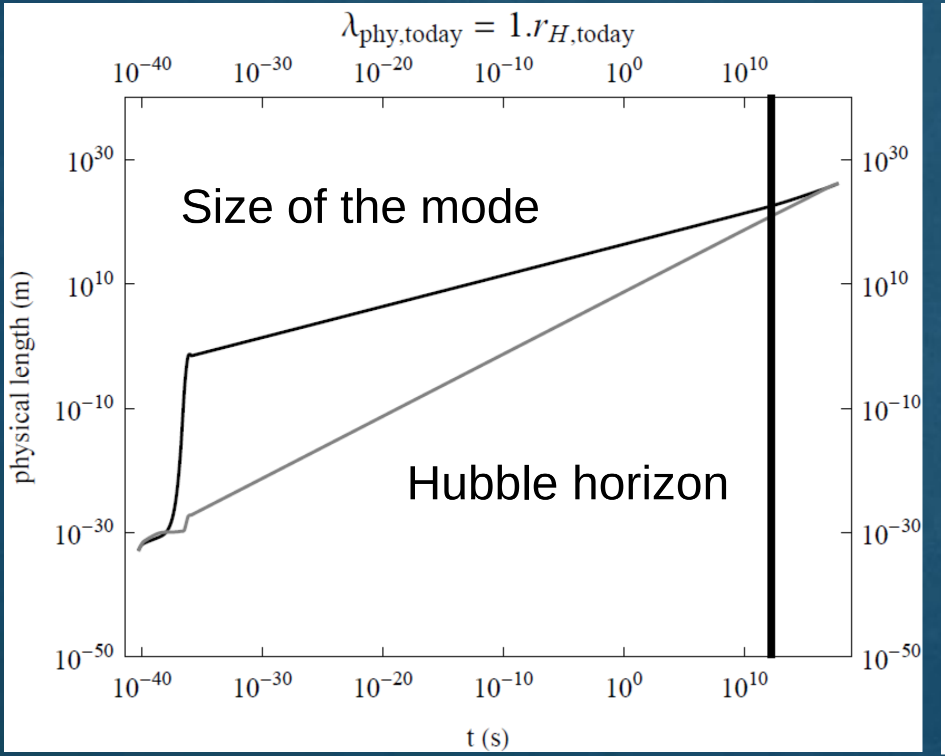


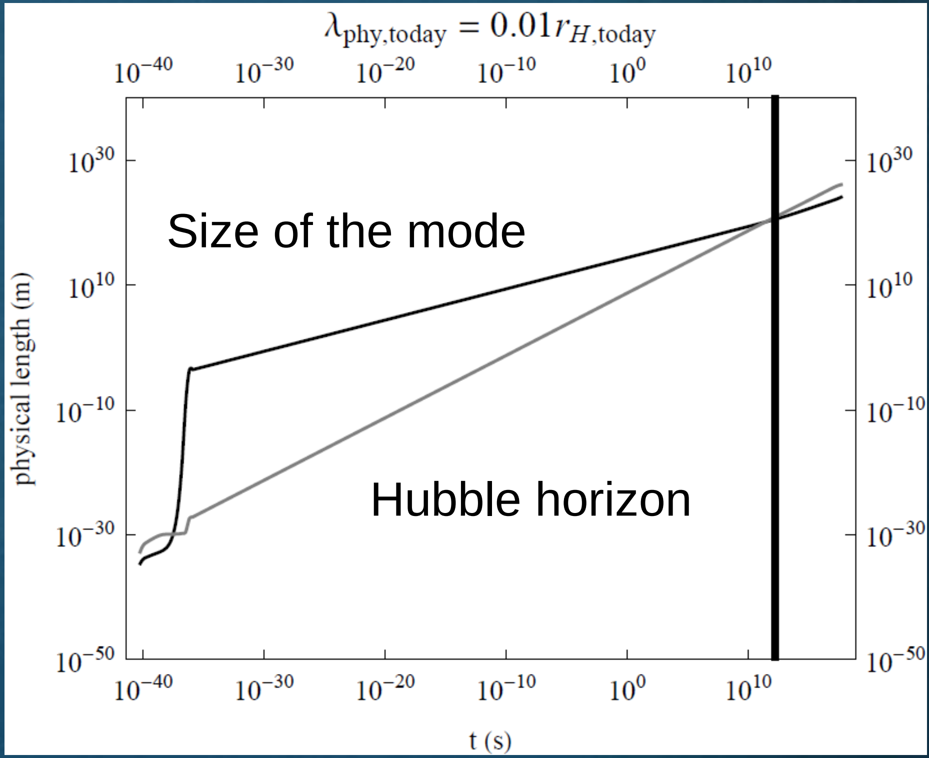
WMAP 7-Year Temperature Power Spectrum



- E. Komatsu et al., *Astrophys. J. Suppl. Ser.* **192** 18 (2011).

Mode Evolution





Various Scenarios

- Initially kinetic energy dominated
  - C. R. Contaldi, M. Peloso, L. Kofman, and A. Linde, *J. Cosmol. Astropart. Phys.* **07**, 002 (2003).
  - J. F. Donoghue, K. Dutta, and A. Ross, *Phys. Rev. D* **80**, 023526 (2009).
- Pre-inflation radiation-dominated era
  - B. A. Powell and W. H. Kinney, *Phys. Rev. D* **76**, 063512 (2007).
  - I-C. Wang and K.-W. Ng, *Phys. Rev. D* **77**, 083501 (2008).
- Pre-Inflation matter-dominated era
  - F. Scardigli, C. Gruber, and P. Chen, *Phys. Rev. D* **83**, 063507 (2011).
  - C. Gruber, *Cosmic Microwave Background Anomaly and Its Indication of a Pre-Inflation Black Hole Universe* (Master Thesis, Johannes Kepler Universität, Linz, Austria, 2010).
  - Y.-H. Lin, *Ab initio Investigation on the infrared cutoff of the primordial power spectrum* (Master Thesis, National Taiwan University, Taipei, Taiwan, 2011)

Various Scenarios

- General forms of initial conditions
  - L. Sriramkumar and T. Padmanabhan, *Phys. Rev. D* **71**, 103512 (2005).
  - D. Boyanovsky, H. J. de Vega, and N. G. Sanchez, *Phys. Rev. D* **74**, 123006 (2006).
- Double inflation
  - B. Feng and X. Zhang, *Phys. Lett. B* **570**, 145 (2003).
- Punctuated inflation
  - R. K. Jain, P. Chingangbam, L. Sriramkumar, and T. Souradeep, *Phys. Rev. D* **82**, 023509 (2010).
- Model-independent reconstruction
  - P. Mukherjee and Y. Wang, *Astrophys. J.* **599**, 1 (2003).

Pre-Inflation Matter Era:
The Black Hole Remnants

- *Black hole remnants in the early universe*
  - F. Scardigli, C. Gruber, and P. Chen, *Phys. Rev. D* **83**, 063507 (2011)

- Primordial micro black holes
- Produced as the spontaneous formation of black holes out of the gravitational instabilities of space-time

- D. J. Gross, M. J. Perry, and L. G. Yaffe, *Phys. Rev. D* **25**, 330 (1982)

Pre-Inflation Matter Era: The Black Hole Remnants

- Evaporate after created
- Left as remnants with minimum mass restricted by the general uncertainty principle
  - R. J. Adler, P. Chen, and D. I. Santiago, *Gen. Relativ. Gravit.* **33**, 2101 (2001)

- Total amount of the black hole remnants are then limited by the holographic principle
  - G. ’t Hooft, arXiv:gr-qc/9310026
  - L. Susskind, *J. Math. Phys. (N.Y.)* **36**, 6377 (1995)
  - R. Bousso, *Rev. Mod. Phys.* **74**, 825 (2002)

Evolution of the Energy Density in the Pre-inflation Matter Era

- In (pre-inflation) matter dominated era, Friedmann equation reads

$$H^2 = \frac{8\pi}{3}\frac{\rho_i a_i^3}{a^3}$$

- where we set the initial condition at

$$t_i = 10^3 t_p, \eta_i = 0, a_i = 1$$

- and

$$\rho_i = 10^{-5}\rho_p (\rho_p = \frac{m_p}{l_p^3})$$

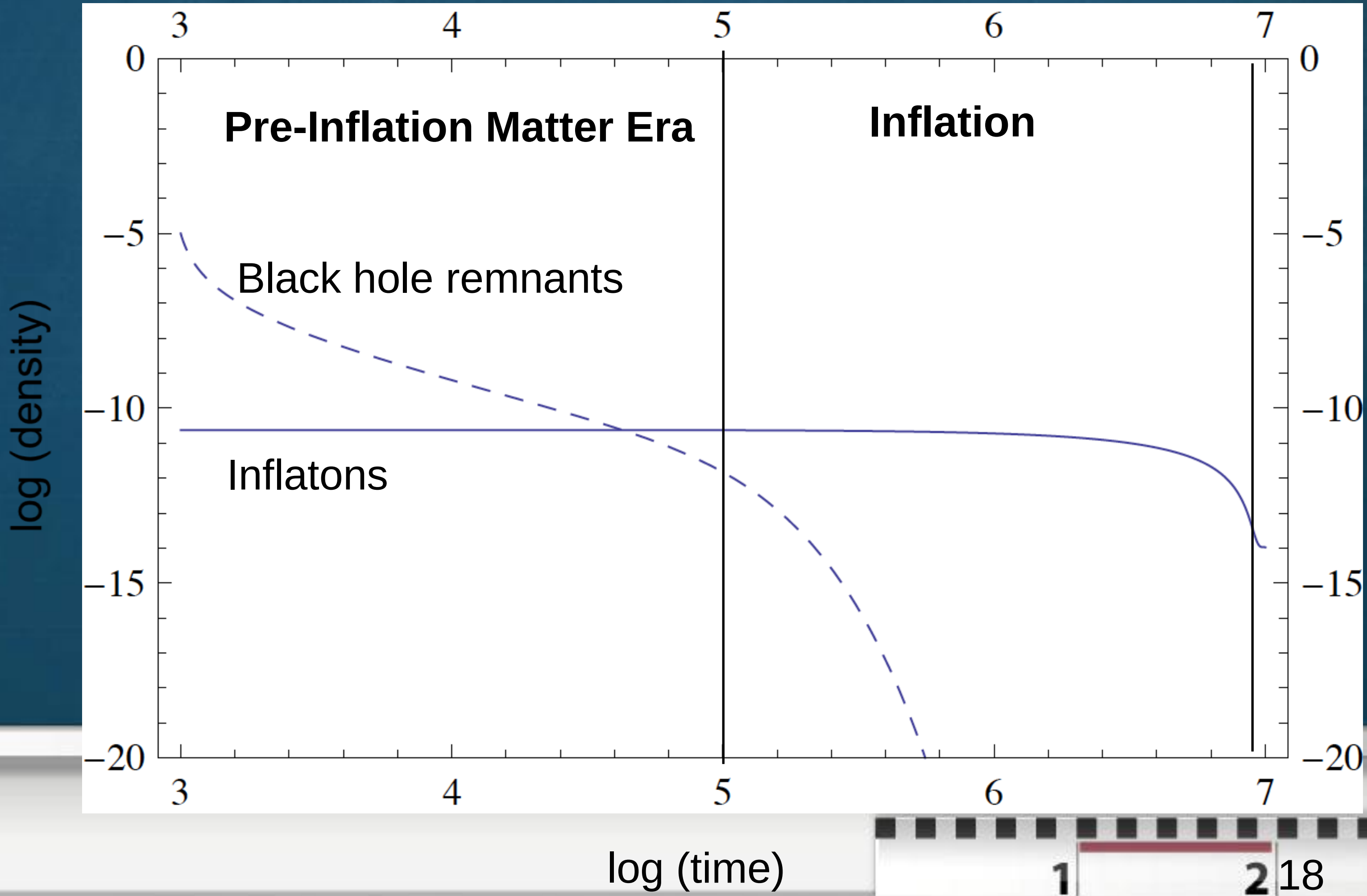
- while

$$\bar{\rho}_s = 2.38\times 10^{-11}\rho_p$$

- The time is chosen to be that when the production of black holes is cut off due to the holographic principle

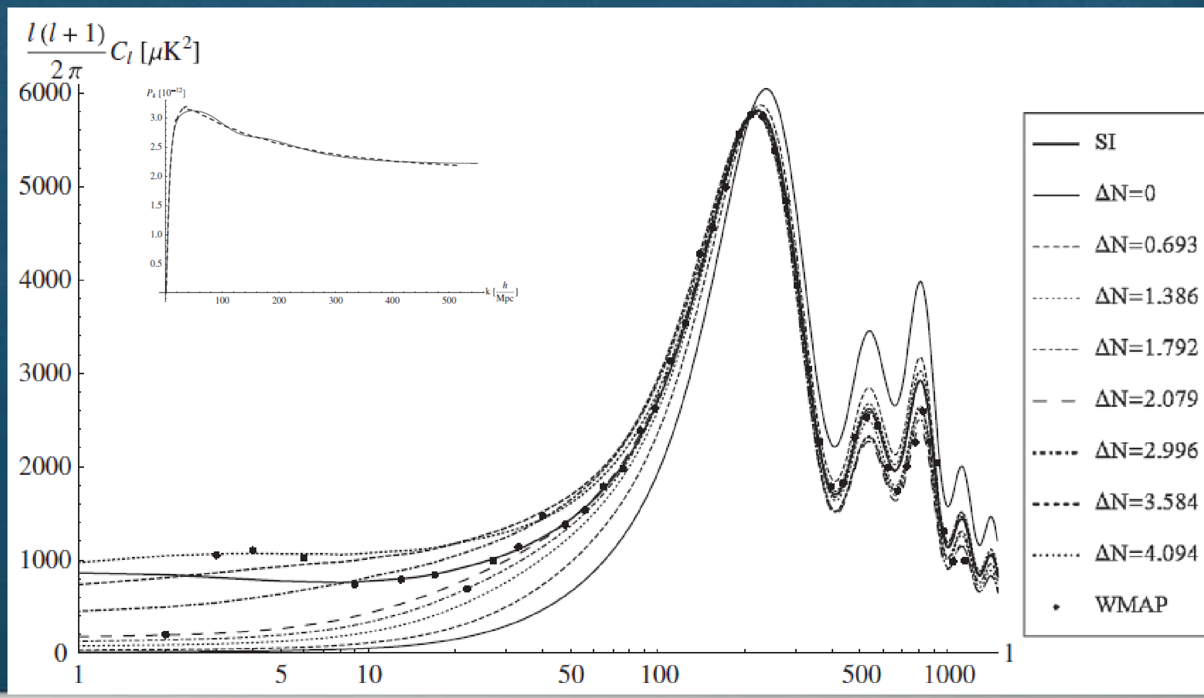
Evolution of the Energy Density in the Pre-inflation Matter Era

- Energy density against time (log-log)
- Solid: scalar field; Dashed: pre-inflation matter density



Issues about the treatment

- *Black hole remnants in the early universe*
  - F. Scardigli, C. Gruber, and P. Chen, *Phys. Rev. D* **83**, 063507 (2011)
- Disturbed higher moments
- Improper estimation of the evolution of modes



Issues about the treatment

- Massless scalar field can not provide the exponential expansion
- Metric perturbations are not taken into account
- The conservative curvature perturbation is not considered
- Boundary conditions are obtained by the matching to the CMB data

The Conservative Quantity: The Adiabatic Modes

- On the super-horizon scales, the following quantity is conserved (S. Weinberg, *Phys. Rev. D* **67**, 123504 (2003); *ibid* **69**, 023503 (2004).)

$$R = -\Psi - \frac{H}{\dot{\phi}_0}\delta\phi$$

- Serves as the bridge connecting the inflation and the radiation eras

Ab initio treatment proposed

- Y.-H. Lin, *Ab initio Investigation on the infrared cutoff of the primordial power spectrum* (Master Thesis, National Taiwan University, Taipei, Taiwan, 2011)
- Adopt a massive scalar field responsible for the inflation
- Perturbations to the scalar field and the metric are both included
- Using the conservative variable for the primordial power spectrum
- Obtain the initial conditions to the differential equations by quantizing the scalar field

Inflation Model

$$L = -\frac{1}{2}\partial_\alpha \phi \partial^\alpha \phi - V(\phi)$$

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

- See, for example, V. Mukhanov, *Physical Foundations of Cosmology* (Cambridge University Press, Cambridge, 2005)

Inflation Model

$$\phi(\vec{x},t) = \phi_0(t) + \delta\phi(\vec{x},t)$$

$$g_{\mu\nu} = \begin{pmatrix} -1-2\Psi & 0 & 0 & 0 \\ 0 & a^2(1+2\Phi) & 0 & 0 \\ 0 & 0 & a^2(1+2\Phi) & 0 \\ 0 & 0 & 0 & a^2(1+2\Phi) \end{pmatrix}$$

Change the form of the equation

- The original form of the perturbed part of the Einstein equation
  - Newtonian gauge
  - Scalar field

$$c_s = -\frac{\sqrt{\bar{\rho}_s + \bar{P}_s}}{\dot{\bar{\varphi}}\sqrt{\left(\frac{\partial \bar{\rho}_s}{\partial \bar{X}}\right)_{\bar{\varphi}}}} = 1$$

$$\nabla^2\Phi - 3Ha^2\left(\dot{\Phi} - H\Psi\right) = 4\pi a^2\left(\bar{\rho}_s + \bar{P}_s\right)\left\{\frac{1}{c_s^2}\left[\Psi - \frac{\partial}{\partial t}\left(\frac{\delta\varphi}{\dot{\bar{\varphi}}}\right)\right] + 3H\frac{\delta\varphi}{\dot{\bar{\varphi}}}\right\}$$

$$\dot{\bar{\varphi}} < 0$$

$$\dot{\Phi} - H\Psi = -4\pi\left(\bar{\rho}_s + \bar{P}_s\right)\frac{\delta\varphi}{\dot{\bar{\varphi}}}$$

$$X = -\frac{1}{2}\partial_\alpha\varphi\partial^\alpha\varphi$$

$$\bar{X} = \frac{1}{2}\dot{\bar{\varphi}}^2$$

- Mukhanov, *Physical Foundations of Cosmology*

Change the form of the equation

- By new variables u and v, and new parameters z

$$z = \frac{a\sqrt{\overline{\rho}_s + \overline{P}_s}}{c_s H}$$

$$R = \Phi - H\frac{\delta\varphi}{\dot{\overline{\varphi}}} = \frac{v}{z}$$

$$u = \frac{\Phi}{4\pi\sqrt{\overline{\rho}_s + \overline{P}_s}}$$

Change the form of the equation

- The equation reduces to
- (16) $$\frac{\partial^2 v}{\partial \eta^2} + \left(k^2 - \frac{1}{z} \frac{d^2 z}{d\eta^2} \right) v = 0$$
- with
- (15) $$z = -\frac{a}{H} \dot{\varphi}$$
- Rearrange the equation of motion (16) in terms of R
- (17) $$\frac{d^2 R}{d\eta^2} + 2 \frac{1}{z} \frac{dz}{d\eta} \frac{dR}{d\eta} + k^2 R = 0$$
- This is also known as the “Mukhanov-Sasaki equation”

Quantization of R

- The Lagrangian of R is

$$S = \int L d\eta dx dy dz$$

$$L = \frac{1}{2}\left\{\left[\frac{\partial}{\partial \eta}(Rz)\right]^{2^{2}} + c_s{}^2 Rz^2 \nabla^2 R + R^2 z \frac{d^2 z}{d\eta^2}\right\}$$

- The canonical conjugate of R is

$$\pi = \frac{\partial L}{\partial \frac{\partial R}{\partial \eta}} = \frac{\partial R}{\partial \eta} z^2 + Rz \frac{dz}{d\eta}$$

- To invoke the commutation relation, we need the *analytical* solution of R

$$\left[\hat{R}(\vec{x},\eta), \hat{\pi}(\vec{x}',\eta)\right] = i\delta^3(\vec{x} - \vec{x}')$$

Quantization of R

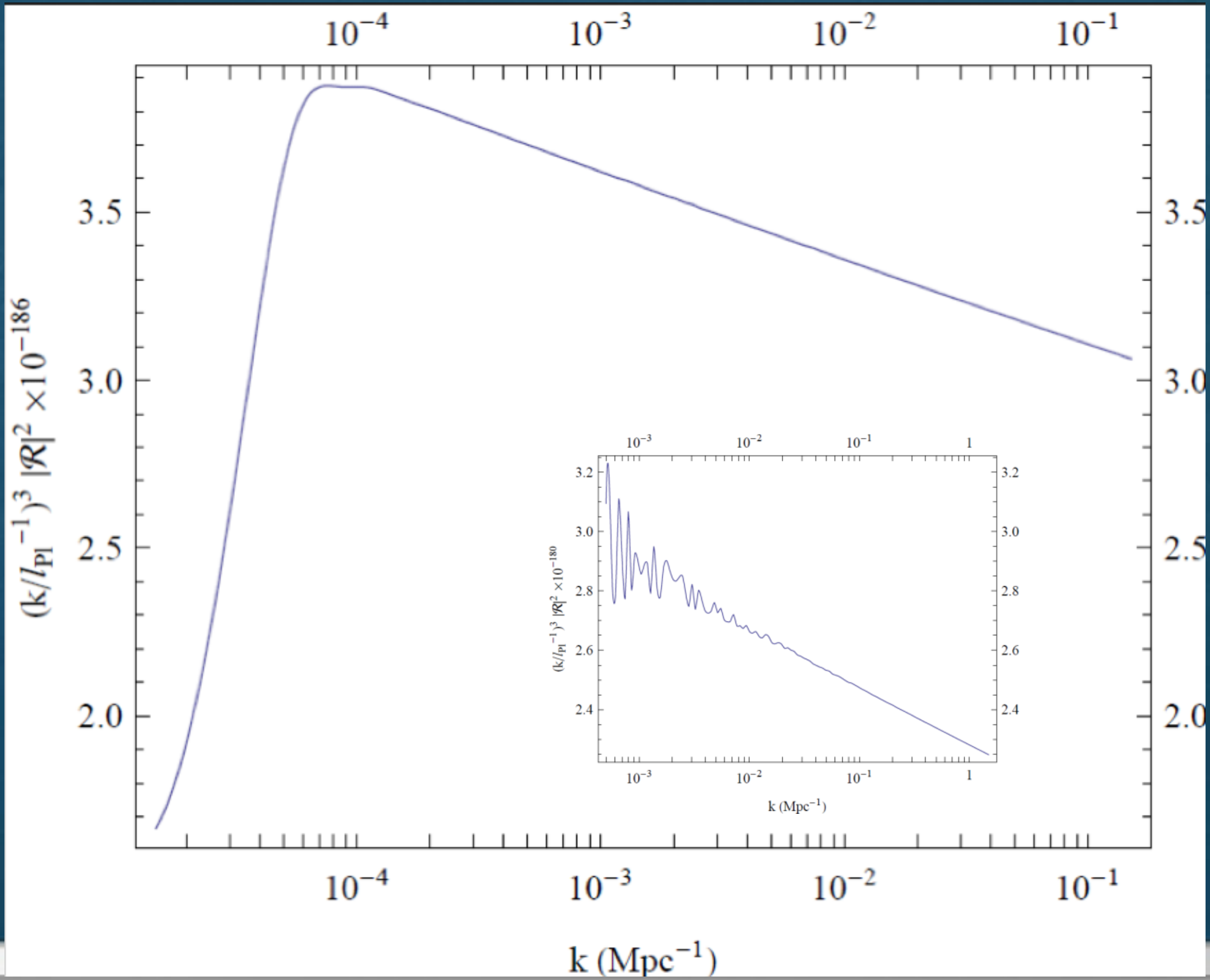
- Obtain the normalized solution at the early times

$$\delta\phi_k = \frac{3}{8\pi^{\frac{5}{2}}\sqrt{k}\,\rho_0\bar{\eta}^2}\left(1-\frac{i}{k\bar{\eta}}\right)e^{-ik\bar{\eta}}$$

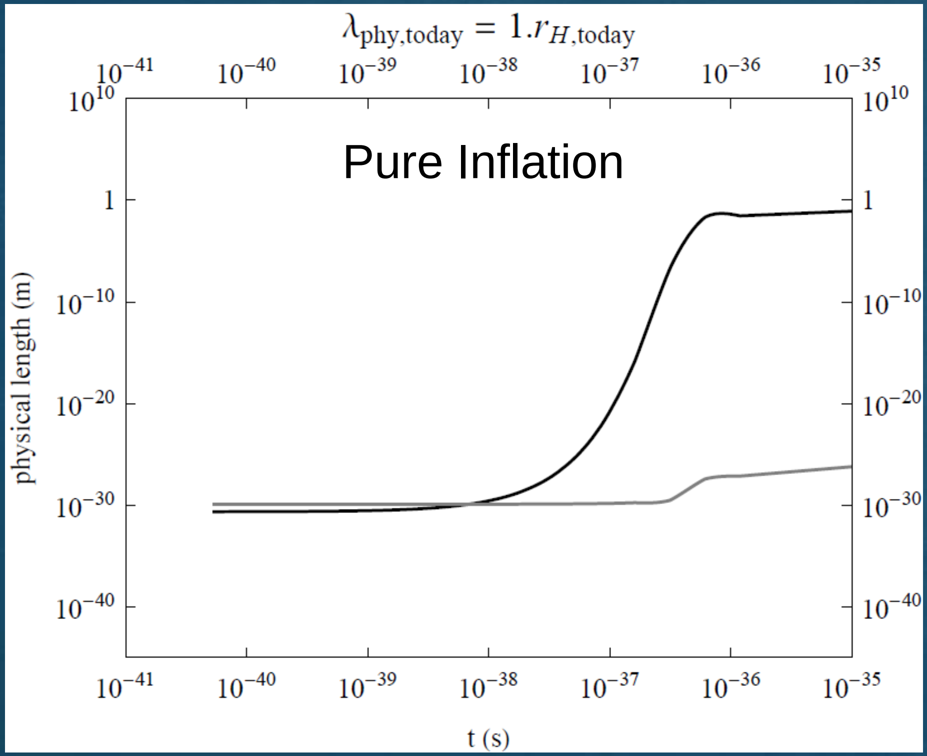
$$\Psi_k = \frac{1}{\sqrt{\pi k}}\dot{\phi}_0\left(\frac{i}{k}+\frac{3}{k^2\bar{\eta}}-\frac{3i}{k^3\bar{\eta}^2}\right)e^{-ik\bar{\eta}}$$

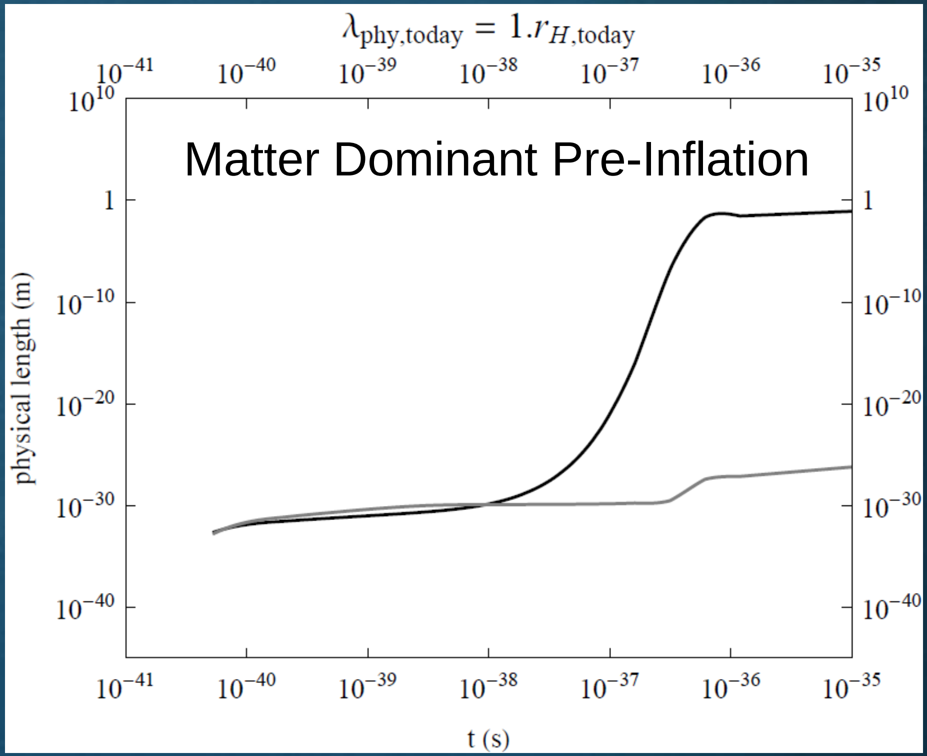
$$\bar{\eta} = \eta + \sqrt{\frac{3}{2\pi\rho_0}}$$

The Primordial Spectrum – With Pre-inflation Matter Era



Mode Evolution – Comparison Between the Pure Inflation Case and Pre-inflation Matter Era Case

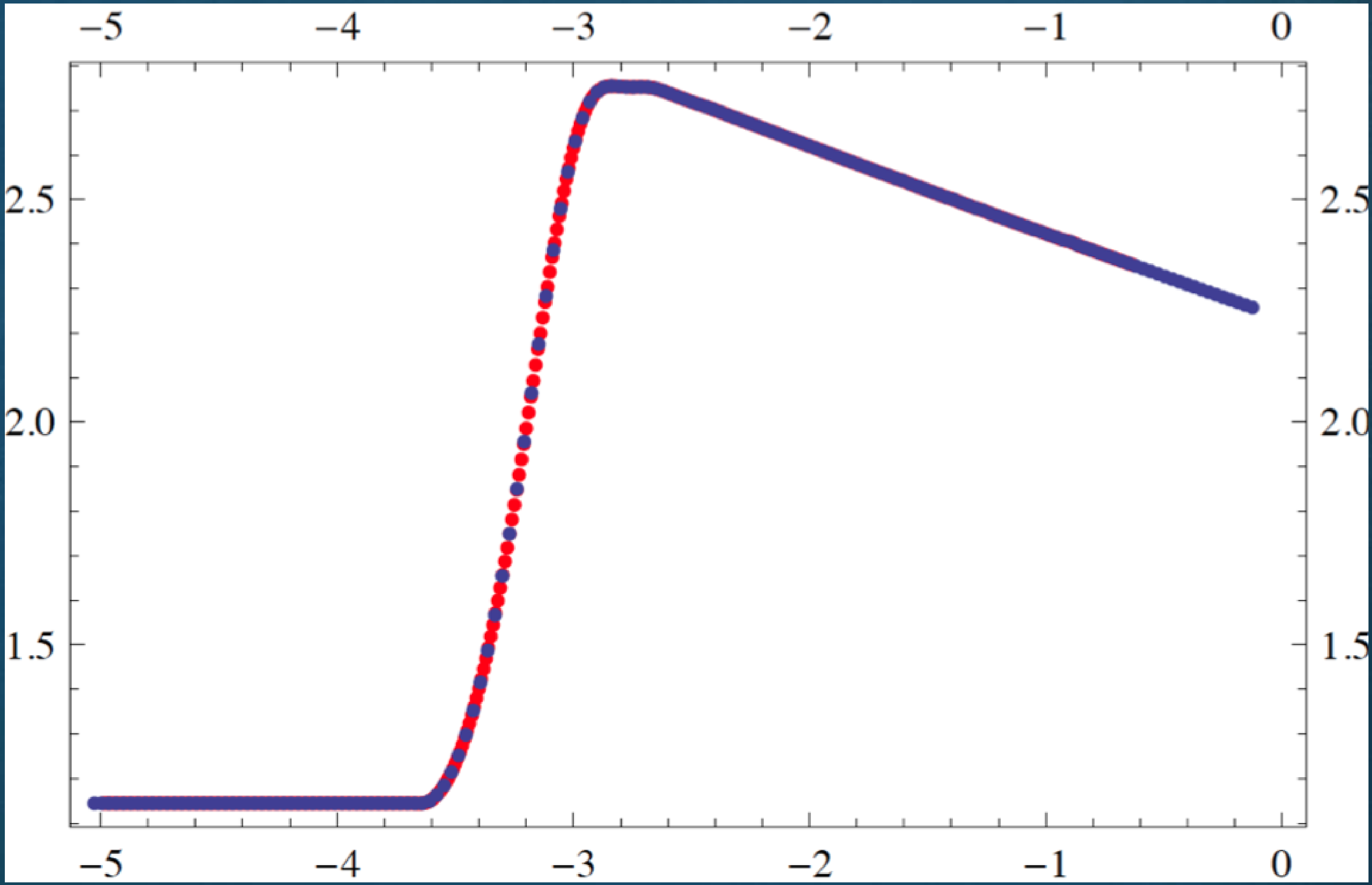




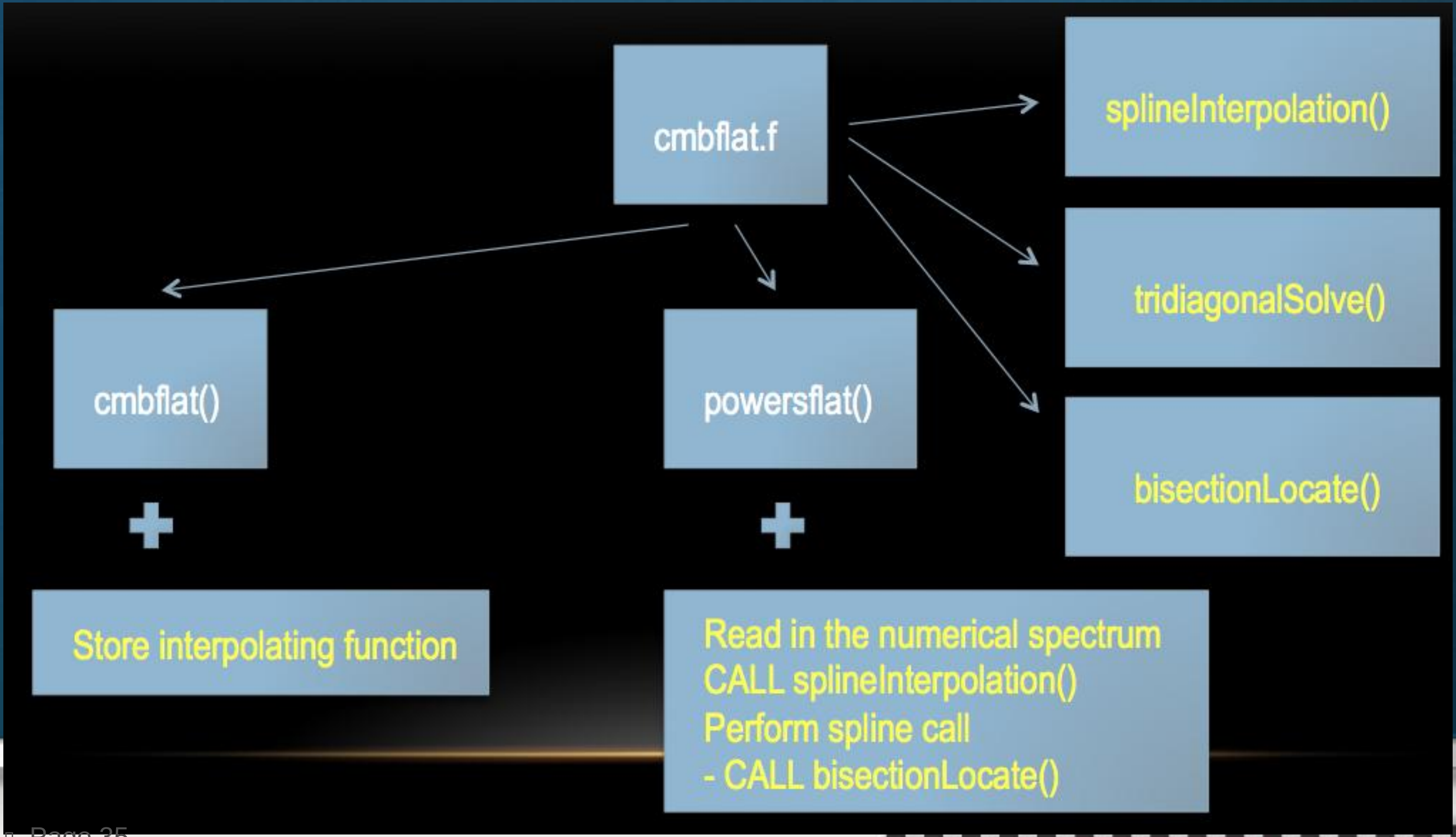
Conclusions

- An *ab initio* and unified way of calculating the primordial spectrum of the curvature perturbation is given
- The pre-inflation matter era produces a cutoff at the large scales of the primordial power spectrum, which in turn suppresses the quadrupole mode of CMB spectrum

My “Guess” to the Full-Ranged Initial Spectrum



Modify CMBFAST to Incorporate the Numerical Initial Spectrum



My “Guess” to the Full-Ranged Initial Spectrum