Teleparallel dark energy

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Based on:

CQ Geng, CC Lee, E. N. Saridakis, YP Wu PLB 704, 384 (2011)
CQ Geng, CC Lee, E. N. Saridakis JCAP 1201, 002 (2012)

JA Gu, CC Lee, CQ Geng arXiv:1204.4048

Outline

- Teleparallel Gravity
- Teleparallel Dark Energy model
- Observational Constraints
- Tracker Behavior

What is the feature of teleparallel gravity?

- An alternative theory of gravity, which is equivalent to General Relativity.
- This is a curvatureless gravity theory, and the gravitational effect comes from torsion instead of curvature.

Teleparallel Gravity
A brief introduction

- The dynamical variable of teleparallel gravity is the vierbein fields $\mathbf{e}_A(x^\mu)$, which form an orthonormal basis for the tangent space at each point x^μ of the manifold: $\mathbf{e}_A \cdot \mathbf{e}_B = \eta_{AB}$, where $\eta_{AB} = diag(1, -1, -1, -1)$.
- Notation:
  Greek indices $\mu, \nu, \ldots$: coordinate space-time.
  Latin indices $A, B, \ldots$: tangent space-time.
- The relationship between metric and vierbein fields is

$$g_{\mu\nu}(x) = \eta_{AB}\, e^A_\mu(x)\, e^B_\nu(x).$$

Teleparallel Gravity
A brief introduction

- Weitzenböck connection: a curvatureless connection

$$\overset{\mathbf{w}}{\Gamma}{}^{\lambda}_{\nu\mu} \equiv e^{\lambda}_{A}\,\partial_{\mu} e^{A}_{\nu}$$

- The torsion tensor is defined as

$$T^{\lambda}_{\mu\nu} \equiv \overset{\mathbf{w}}{\Gamma}{}^{\lambda}_{\nu\mu} - \overset{\mathbf{w}}{\Gamma}{}^{\lambda}_{\mu\nu} = e^{\lambda}_{A}\,(\partial_{\mu} e^{A}_{\nu} - \partial_{\nu} e^{A}_{\mu}).$$

- Under Weitzenböck connection, the Riemann tensor vanishes:

$$R^{\rho}_{\mu\sigma\nu} = \overset{\mathbf{w}}{\Gamma}{}^{\rho}_{\mu\nu,\sigma} - \overset{\mathbf{w}}{\Gamma}{}^{\rho}_{\mu\sigma,\nu} + \overset{\mathbf{w}}{\Gamma}{}^{\rho}_{\delta\sigma}\overset{\mathbf{w}}{\Gamma}{}^{\delta}_{\mu\nu} - \overset{\mathbf{w}}{\Gamma}{}^{\rho}_{\delta\nu}\overset{\mathbf{w}}{\Gamma}{}^{\delta}_{\mu\sigma} = 0.$$

Teleparallel Gravity
A brief introduction

- We can construct the "teleparallel Lagrangian" by using the torsion tensor,
  $\mathcal{L}_T = T = a_1 T^{\rho\mu\nu} T_{\rho\mu\nu} + a_2 T^{\rho\mu\nu} T_{\nu\mu\rho} + a_3 T_{\rho\mu}{}^{\rho} T_{\nu}{}^{\mu\nu}$.
- It is a good approach of General Relativity when we choose the suitable parameters $a_1 = \frac{1}{4}$, $a_2 = \frac{1}{2}$ and $a_3 = -1$:

$$\tilde{R} = -T - 2\nabla^{\mu} T^{\nu}{}_{\mu\nu},$$

where $\tilde{R}$ is constructed by Levi-Civita connection.

Teleparallel Gravity
A brief introduction

- The action of teleparallel gravity is

$$S = \int d^4 x e \left[\frac{T}{2\kappa^2} + \mathcal{L}_M \right],$$

where $e = det\left(e^A_{\ \mu}\right) = \sqrt{-g}$.

- Varying this action respect to the vierbein fields gives the field equation

$$e^{-1}\partial_\mu(e e^\rho_A S_\rho{}^{\mu\nu}) - e^\lambda_A T^\rho{}_{\mu\lambda} S_\rho{}^{\nu\mu} - \frac{1}{4} e^\nu_A T = \frac{\kappa^2}{2} e^\rho_A \overset{\mathbf{em}}{T}{}_\rho{}^\nu,$$

where $\overset{\mathbf{em}}{T}{}_\rho{}^\nu$ stands for the energy-momentum tensor and $S_\rho{}^{\mu\nu} = \frac{1}{4}\left(T^{\nu\mu}{}_\rho - T^{\mu\nu}{}_\rho + T_\rho{}^{\mu\nu}\right) + \frac{1}{2}\left(\delta^\mu_\rho T^{\alpha\nu}{}_\alpha - \delta^\nu_\rho T^{\alpha\mu}{}_\alpha\right)$.

- Teleparallel dark energy model is a dark energy model, which can explain the late time accelerating universe.
- This model combines quintessence model with teleparallel gravity.
- This model differs from quintessence model when we turn on the non-minimal coupling term.

- Quintessence is one of the most popular dark energy model.
- The generalized quintessence model action is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} + \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi + \xi R\phi^2\right) - V(\phi) + \mathcal{L}_M\right]$$

- Under the flat Friedmann-Robertson-Walker (FRW) background $ds^2 = dt^2 - a^2(t)d\vec{x}^2$, the effective energy and pressure density can be defined as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) + 6\xi H\phi\dot{\phi} + 3\xi H^2\phi^2,$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) + \xi\left(2\dot{H} + 3H^2\right)\phi^2 + 4\xi H\phi\dot{\phi} + 2\xi\phi\ddot{\phi} + 2\xi\dot{\phi}^2$$

Teleparallel Dark Energy

- Similar to quintessence model, we can construct teleparallel dark energy model, and the action is given by

$$S = \int d^4x e \left[\frac{T}{2\kappa^2} + \frac{1}{2} \left(\partial_\mu \phi \partial^\mu \phi + \xi T \phi^2 \right) - V(\phi) + \mathcal{L}_M \right].$$

- Variation of action with respect to the vierbein fields yields the field equation

$$\left(\frac{2}{\kappa^2} + 2\xi\phi^2 \right) \left[e^{-1} \partial_\mu (e e_A^\rho S_\rho{}^{\mu\nu}) - e_A^\lambda T^\rho{}_{\mu\lambda} S_\rho{}^{\nu\mu} - \frac{1}{4} e_A^\nu T \right]$$
$$-e_A^\nu \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] + e_A^\mu \partial^\nu \phi \partial_\mu \phi$$
$$+4\xi e_A^\rho S_\rho{}^{\mu\nu} \phi \left(\partial_\mu \phi \right) = e_A^\rho \overset{\mathbf{em}}{T}{}_\rho{}^\nu .$$

Teleparallel Dark Energy

- Again, the effective energy and pressure density under FRW metric $(e^A_\mu = \mathrm{diag}(1, a, a, a))$ are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) - 3\xi H^2\phi^2,$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) + 4\xi H\phi\dot{\phi} + \xi\left(3H^2 + 2\dot{H}\right)\phi^2.$$

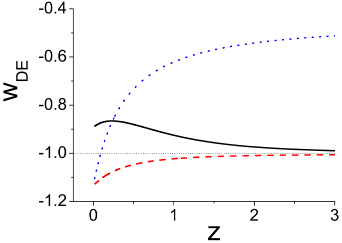
- Variation of action with respect to the scalar field gives us the equation of motion of the scalar field

$$\ddot{\phi} + 3H\dot{\phi} + 6\xi H^2\phi + V'(\phi) = 0.$$

- These equations lead to the continuity equation $\dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi = 0$, where w_ϕ is the equation of state of the scalar field, which is defined as $w_\phi \equiv \frac{p_\phi}{\rho_\phi}$.

Teleparallel Dark Energy

- In the minimal coupling case ($\xi = 0$), the teleparallel dark energy is equivalent to quintessence model
- However, these two models are different theories when we turn on the non-minimal coupling constant ($\xi \neq 0$)
- Teleparallel dark energy model can cross the phantom-divide easily.

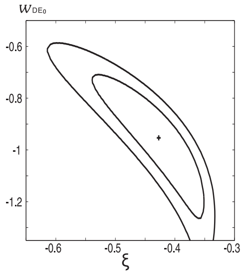


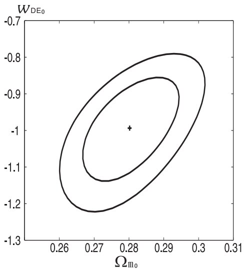
- Similar to $f(T)$ theory, this model has the local Lorentz violation problem.

Teleparallel Dark Energy: Observational Constraints

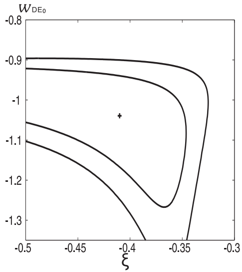
- We would like to test teleparallel dark energy model by using the SNIa, BAO and CMB data. These observational data can tell us whether this is a suitable model for dark energy or not
- We consider three kinds of potential cases:
  Power-Law potential: $V(\phi) = V_0\phi^4$
  Exponential potential: $V(\phi) = V_0 e^{-\kappa\phi}$
  Inverse hyperbolic cosine potential: $V(\phi) = \frac{V_0}{cosh(\kappa\phi)}$

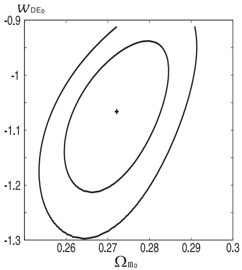
Teleparallel Dark Energy: Observational Constraints

- Potential: $V(\phi) = V_0\phi^4$
- Left: fixing $\Omega_m = 27\%$, the best fit locates at $h \simeq 0.7$, $\xi \simeq -0.42$, $w_\phi \simeq -0.96$ and $\chi^2 \simeq 543.9$
  Right: fixing $\xi = -0.41$, the best fit locates at $h \simeq 0.7$, $\Omega_m \simeq 28.0\%$, $w_\phi \simeq -0.99$ and $\chi^2 \simeq 544.5$
- 

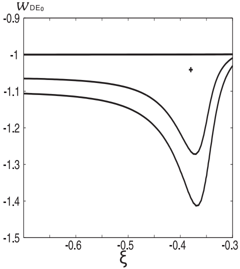


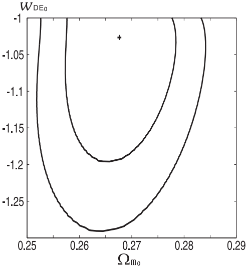
Teleparallel Dark Energy: Observational Constraints

- Potential: $V(\phi) = V_0 e^{-\kappa\phi}$
- Left: fixing $\Omega_m = 27\%$, the best fit locates at $h \simeq 0.7$, $\xi \simeq -0.41$, $w_\phi \simeq -1.04$ and $\chi^2 \simeq 544.3$
  Right: fixing $\xi = -0.41$, the best fit locates at $h \simeq 0.7$, $\Omega_m \simeq 27.1\%$, $w_\phi \simeq -1.07$ and $\chi^2 \simeq 544.6$
- 



Teleparallel Dark Energy: Observational Constraints

- Potential: $V(\phi) = \frac{V_0}{cosh(\kappa\phi)}$
- Left: fixing $\Omega_m = 27\%$, the best fit locates at $h \simeq 0.7$, $\xi \simeq -0.38$, $w_\phi \simeq -1.05$ and $\chi^2 \simeq 544.8$
  Right: fixing $\xi = -0.41$, the best fit locates at $h \simeq 0.7$, $\Omega_m \simeq 26.7\%$, $w_\phi \simeq -1.03$ and $\chi^2 \simeq 545.1$
- 



Summary

- Teleparallel gravity is an alternative gravity theory of the universe.
- Teleparallel dark energy model is equivalent to quintessence model happens at the minimal coupling case ($\xi = 0$), but it has a different behavior when we include a non-minimal coupling term ($\xi \neq 0$).
- We show that the equation of state of teleparallel dark energy model can cross the phantom-divide easily.
- The observational constraints show a good result on this model. This model is suitable for the late-time accelerating universe.

Basic Idea and Features

- Potential-free: $V(\phi) = 0$.
- Analytic solutions in the radiation (RD), matter (MD), and scalar field (SD) dominated eras.
- The tracker behavior for w_ϕ in the RD and MD eras.

Tracker Behavior
Basic Idea and Features

- The field equation of gravity and scalar field lead to

$$\ddot{\phi} + 3H\dot{\phi} + 6\xi H^2\phi = 0\,,$$

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\left(\rho_\phi + \rho_{\rm m} + \rho_{\rm r}\right),$$

$$\dot{H} = -\frac{\kappa^2}{2}\left(\rho_\phi + p_\phi + \rho_{\rm m} + 4\rho_{\rm r}/3\right).$$

- The effective energy and pressure density are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 - 3\xi H^2\phi^2\,,$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 + 3\xi H^2\phi^2 + 2\xi\frac{d}{dt}(H\phi^2)\,.$$

Tracker Behavior
Analytic Solutions in RD and MD eras

- $H = \alpha/t$, i.e. $a(t) \propto t^{\alpha}$, with α constant:

  $$\phi(t) = C_1 t^{l_1} + C_2 t^{l_2},$$

  where $C_{1,2}$ are constants and

  $$l_{1,2} = \frac{1}{2}\left[\pm\sqrt{(3\alpha-1)^2 - 24\xi\alpha^2} - (3\alpha - 1)\right].$$

- For $\xi < 0$, the power-index l_1 is positive and l_2 is negative, corresponding to increasing and decreasing modes, respectively.
- Considering only the increasing mode, i.e., $\phi(t) = C_1 t^{l_1}$.

Tracker Behavior in RD and MD eras

- We can solve the analytic solution in RD and MD eras:

$$\begin{aligned} w_\phi &= \frac{1}{3}\left(2-\sqrt{1-24\xi}\right),\ \frac{1}{2}\left(1-\sqrt{1-32\xi/3}\right), \\ \rho_\phi &\propto a^{-5+\sqrt{1-24\xi}},\ a^{(-9+\sqrt{9-96\xi})/2}, \end{aligned}$$

for the RD ($\alpha = 1/2$) and MD ($\alpha = 2/3$) eras, respectively.

Tracker Behavior
Analytic Solutions in SD era

- Setting $\rho_{\rm r} = 0$:

$$\frac{d}{dt}\left[F(\phi)a^3 H\right] = \frac{\kappa^2}{2}\rho_{\rm m}a^3 = \frac{3}{2}H_0^2\Omega_{\rm m0}\,,$$
$$F(\phi) \equiv 1+\kappa^2\xi\phi^2.$$

- Setting $\rho_m = 0$ and combining with field equation, we can solve the analytic solution:

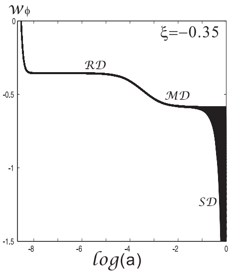
$$\phi(a) = \pm\frac{\sin\theta}{\sqrt{-\kappa^2\xi}}\,,$$
$$w_\phi(a) = -1-\sqrt{-32\xi/3}\tan\theta\,,$$
$$\theta(a) \equiv \sqrt{-6\xi}\ln a + C_3\,,$$

where C_3 is the integration constant.

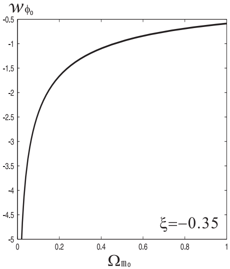
- There exists one kind of finite-time future singularities, called "sudden singularity".

Tracker Behavior
Numerical Result

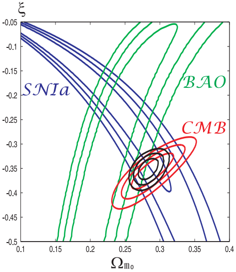
- The tracker behavior in RD (MD) eras, and the singularity happens in SD era.

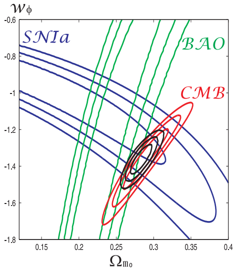


- The present $(z=0)$ values of (Ω_m, w_ϕ) with $\xi=-0.35$.



- Combining SNIa, BAO and CMB data:





Summary

- Analytic solution exist in this potential-free case.
- Equation of state (w_ϕ) has a tracker behavior and only depend on ξ in RD and MD eras.
- The final energy density depends on ξ and initial condition parameter C_1.
- The observational data can be fitted well but the concordance region for all data is only at the 3σ level.