Domain Wall Brane in Eddington Inspired Born-Infeld Gravity

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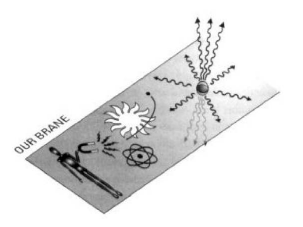
Outline

- Introduction to Brane World
- Introduction to Eddington Inspired Born-Infeld Gravity
- The Model
- Gravitational Fluctuations
- Summary

Introduction to Brane World

The prototype idea of brane world was proposed during early 1980s and made great progresses after the ADD model and RS model proposed in the late 1990s.



It suggests that the SM particles are trapped in a 4D hypersurface (called brane) embedded in a higher dimensional space-time (called bulk). It provides us new perspectives to solve the gauge hierarchy problem and the cosmological constant problem.

Based on the gravity coupled to the bulk scalar fields, the brane configuration is determined by

- the gravity theory;
- the scalar fields;
- the ways of scalar-gravity coupling.

Here, we are interested in the brane world scenario based on a new intriguing gravitational theory-the Eddington inspired Born-Infeld (EiBI) gravity.

Introduction to EiBI Gravity

1924, Eddington proposed a purely affine gravity.

$$S_{Edd}(\Gamma) = \frac{1}{\kappa b}\int d^n x \sqrt{-|bR_{MN}(\Gamma)|}, \quad (1)$$

$$\Downarrow \quad (g_{MN} = bR_{MN})$$

$$g_{MN;K} = 0, \quad \text{i.e.,} \quad \Gamma^K_{MN} = \tfrac{1}{2} g^{KL}(g_{LM,N} + g_{LN,M} - g_{MN,L})$$

$$\Downarrow \quad (\Lambda = \tfrac{n-2}{2b})$$

$$R_{MN} = \tfrac{2}{n-2}\Lambda g_{MN}$$

Eddington's theory is totally equivalent to the GR with Λ.

Eddington's theory is incomplete in which matter is not included.

Inspired by the Eddington's theory, a new theory was put forward by Bañados and Ferreira (PRL**105**(2010)011101).

Eddington Inspired Born-Infeld gravity

$$S_{\text{EiBI}}(g,\Gamma,\Phi) = \frac{1}{\kappa b}\int d^n x\left[\sqrt{-|g_{\mu\nu}+bR_{\mu\nu}(\Gamma)|}-\lambda\sqrt{-|g_{\mu\nu}|}\right] + S_M(g,\Phi), \tag{2}$$

where the dimensionless parameter λ must be nonvanishing.

By introducing an auxiliary metric

$$q_{MN} = g_{MN} + bR_{MN}, \tag{3}$$

the variation to the connection simply gives us $q_{MN;K} = 0$, i.e., Γ is just the Christoffel symbol of the auxiliary metric.

By varying the action with respect to the metric simply gives

$$\sqrt{-|q_{PQ}|}\, q^{MN} = \lambda\sqrt{-|g_{PQ}|}\, g^{MN} - b\kappa\sqrt{-|g_{PQ}|}\, T^{MN}, \tag{4}$$

The equations (3), (4) and matter field equations form a complete set of equations of the theory.

Here we note some properties of EiBI gravity.

- When $bR >> g$, $S_{\text{EiBI}} \to S_{Edd}$.
- When $bR << g$, expanding S_{EiBI} to 2nd order in b:

$$S_{\text{EiBI}} = \frac{1}{2\kappa}\int d^n x\sqrt{-|g_{MN}|}[R - 2\Lambda_{\text{eff}}] + \mathcal{O}(b^2) + S_M(g,\Phi), \quad (5)$$

where $R = g^{MN}R_{MN}(q)$ and $\Lambda_{\text{eff}} \equiv (\lambda - 1)/b$, $(b > 0)$.

- when mater is absent, the EiBI gravity is completely equivalent to the GR.
- Researches (arXiv:1006.1769,1106.3569,1201.4989) show that the cosmological singularities are prevented in this theory!

The Model

The ansatz for the most general metric which preserves 4D Poincaré invariance is

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{6}$$

The lagrangian of a scalar field is given by

$$L_M = -\sqrt{-|g_{PQ}|}\left[\frac{1}{2}\partial^K\phi\partial_K\phi + V(\phi)\right], \tag{7}$$

where the scalar $\phi(y)$ is only the function of the extra dimension y in order to be consistent with the 4D Poincaré invariance of the metric.

Then the scalar field equation is

$$\frac{1}{\sqrt{-|g_{PQ}|}}\partial^K\left[\sqrt{-|g_{PQ}|}\partial_K\phi\right]=\frac{\partial V}{\partial\phi}, \tag{8}$$

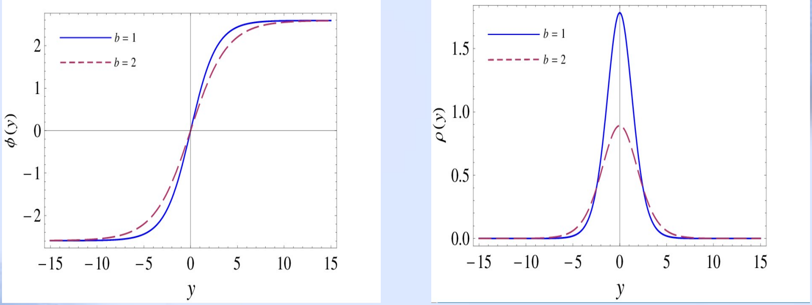
The contravariant energy-momentum tensor is given by

$$T^{MN}=\partial^M\phi\partial^N\phi-\left[\frac{1}{2}\partial^K\phi\partial_K\phi+V(\phi)\right]g^{MN}, \tag{9}$$

After introducing a condition $\phi'(y) = Ka^2(y)$, it is easy to get the solution:

$$a(y) = \text{sech}^{\frac{3}{4}}(\frac{2}{\sqrt{21b}}y),$$
$$\phi(y) = \pm\frac{7^{5/4}}{2\ 3^{1/4}\kappa^{1/2}}[i\text{E}(\frac{iy}{\sqrt{21b}}, 2) + \text{sech}^{\frac{1}{2}}(\frac{2y}{\sqrt{21b}})\sinh(\frac{2y}{\sqrt{21b}})],$$
$$V(y) = \frac{7\sqrt{21}}{24b\kappa}\text{sech}^3(\frac{2y}{\sqrt{21b}}) - \frac{\lambda}{b\kappa}.$$

As $|\phi(y\to\pm\infty)|$ approaches $v_0 \simeq \frac{2.59}{\sqrt{\kappa}}$, and $V(\phi = \pm v_0)$ are two minimums (vacua) $V_0 = -\frac{\lambda}{b\kappa}$, thus ϕ is a kink and the soluti- on depicts a domain wall configuration.



(a) $\phi(y)$ (b) $\rho(y)$

The energy density $\rho(y) = T_{MN}w^M w^N - V_0$ is calculated as

$$\rho(y) = -T^0_0 + \frac{\lambda}{b\kappa} = \frac{7\sqrt{21}}{18b\kappa}\text{sech}^3\left(\frac{2y}{\sqrt{21b}}\right). \tag{10}$$

Gravitational Perturbations

We impose the axial gauge $h_{\mu 5} = h_{55} = 0$ and the perturbed metric is simply given by

$$d\hat{s}^2 = a^2(y)[\eta_{\mu\nu} + h_{\mu\nu}(x,y)]dx^\mu dx^\nu + dy^2. \quad (11)$$

$$\Downarrow ds'^2 = \tfrac{7}{6} a^{\frac{8}{3}} (a^2 \eta_{\mu\nu} dx^\mu dx^\nu + 2dy^2)$$

$$d\hat{s}'^2 = \frac{7}{6}\, a^{\frac{8}{3}}(y)[a^2(y)(\eta_{\mu\nu} + \gamma_{\mu\nu})dx^\mu dx^\nu + \gamma_{\mu 5}(x,y)dx^\mu dy \\ +2(1+\gamma_{55}(x,y))dy^2]. \quad (12)$$

After considering the transverse-traceless (TT) components $\bar{h}^{\mu}_{\mu} = \partial_{\mu}\bar{h}^{\mu}_{\nu} = 0$, linearly perturbations of EOM gives

$$\frac{1}{2}a^2\bar{h}''_{\mu\nu} + 4aa'\bar{h}'_{\mu\nu} + \Box^{(4)}\bar{h}_{\mu\nu} = 0. \quad (13)$$

$$\Big\Downarrow dy = \frac{a(z)}{\sqrt{2}}dz$$

$$\partial_{z,z}\bar{h}_{\mu\nu} + 7\frac{\partial_z a}{a}\partial_z\bar{h}_{\mu\nu} + \Box^{(4)}\bar{h}_{\mu\nu} = 0, \quad (14)$$

$$\Bigg\Downarrow \begin{pmatrix} \bar{h}_{\mu\nu} = \varepsilon_{\mu\nu}(x)a^{-\frac{7}{2}}(z)\Psi(z), \\ \Box^{(4)}\varepsilon_{\mu\nu}(x) = m^2\varepsilon_{\mu\nu}(x). \end{pmatrix}$$

Then a Schrödinger-like equation is obtained as

$$-\partial_{z,z}\Psi(z) + U(z)\Psi(z) = m^2\Psi(z), \tag{15}$$

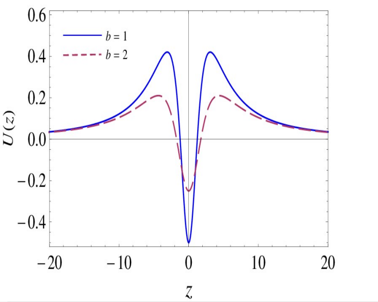
where the effective potential $U(z)$ is given by

$$U(z) = \frac{7}{2}\frac{\partial_{z,z}a}{a} + \frac{35}{4}\frac{(\partial_z a)^2}{a^2}. \tag{16}$$

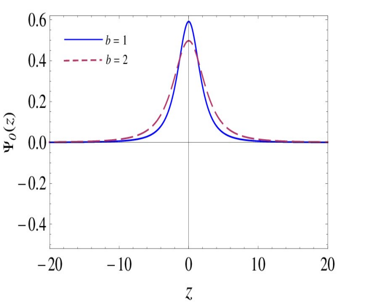
The zero mode is easily got by setting $m = 0$:

$$\Psi_0(z) = N_0\ a^{7/2}(z), \tag{17}$$

where the normalization parameter $N_0 \approx \sqrt{0.35/\sqrt{b}}$.



(c) $U(z)$



(d) $\Psi_0(z)$

Besides a bound massless mode, there exists a set of continuous massive KK modes $\Psi_m(z)$ starting at $m^2 > 0$. So this system is tachyonic free and stable under linear tensor perturbations.

$$\bar{h}_{\mu\nu} = \varepsilon_{\mu\nu}(x) a^{-\frac{7}{2}}(z)\Psi(z) \xrightarrow{\Psi_0 = N_0 a^{\frac{7}{2}}(z)} \bar{h}^{(0)}_{\mu\nu}(x) = N_0 \varepsilon_{\mu\nu}(x)$$

This mode propagates only on the brane and provides the gravitational fields in the low energy effective theory.

$$\begin{aligned} d\hat{s}^2 &= a^2(y) g^{(4)}_{\mu\nu}(x) dx^\mu dx^\nu + dy^2 \\ &= a^2(y)(\eta_{\mu\nu} + \bar{h}^{(0)}_{\mu\nu}(x)) dx^\mu dx^\nu + dy^2. \end{aligned} \quad (18)$$

Then the perturbed auxiliary metric is

$$\gamma_{\mu\nu} = \bar{h}^{(0)}_{\mu\nu}, \gamma_{\mu 5} = \gamma_{55} = 0. \quad (19)$$

The 4D effective gravitational theory on the brane is given by

$$S \supset \frac{1}{\kappa b}\int d^5x\left[\sqrt{-|\hat{g}_{MN}+bR_{MN}|}-\lambda\sqrt{-|\hat{g}_{MN}|}\right]$$
$$\supset \frac{1}{2\tilde{\kappa}}\int d^4x\sqrt{-|g^{(4)}_{\alpha\beta}|}R^{(4)}, \quad (20)$$

where $R^{(4)} = g^{(4)\mu\nu}R^{(4)}_{\mu\nu}$ with $R^{(4)}_{\mu\nu}(x)$ constituting by $g^{(4)}_{\mu\nu}(x)$, and $\kappa \approx 5.09\sqrt{b}\tilde{\kappa}$.

Einstein's gravity is recovered on the brane at low energy level.

Summary

- A domain wall brane is obtained in EiBI gravity. Further, by introducing a proper Yukawa coupling with the scalar field, Dirac fermions can be localized on the domain wall.
- The linear tensor fluctuations are stable in our model. Escamilla-Rivera, et al. (arXiv:1204.1691) found that the TT tensor modes were linearly unstable to a 4D homogeneous and isotropic universe. The brane word scenario may be helpful to stabilize the models in this gravity theory.
- Einstein's gravity is recovered on the brane at low energy level.

Thank You !

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