Localization of gauge fields on two-field thick branes

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Outline

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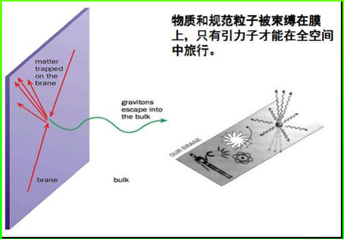
2. Solution of two-field thick M_4 branes
3. Localization of gauge fields on the brane
4. Conclusion

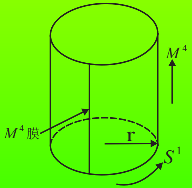
Introduction

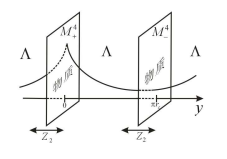
The brane world theory:

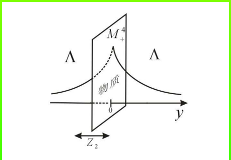
- **Motivation: strongly motivated by string theory and the M-theory.**
- **Picture: Our four-dimensional universe is a hyper-surface ("brane world") embedded in more higher dimensional space-time. All matter fields are confined to the brane, and only gravity is free to propagate in all dimensions.**



- **Models:**
  - **Thin-brane(ideal model)**
    - **ADD Model [1], RS Model [2]**
    - **Solve the hierarchy problem, the cosmological problem**



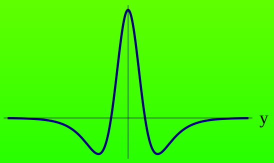




[1] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B 429(1998)263.

[2] L. Randall and R.Sundrum, Phys. Rev. Lett, 83(1999)3370.

- **Thick-branes**
  - **the energy density of the system is spread along the extra dimension**
  - **the branes can be naturally realized by one or more scalar fields**
  - **the physical length of the extra dimension is usually infinite**



- **Localization of matters on the brane:**
  - **Meaning: in order to obtain the effective action on the brane.**
  - **Relevant works: We have mainly investigated the localization of scalar, vector and fermion fields on the brane:**
    - **The scalar fields can be localized on branes of different types with exponentially decreasing warp factor;**
    - **The vector fields only can be localized on the RS brane in some higher-dimensional cases , or on the thick de Sitter brane and the Weyl thick brane;**
    - **The fermion fields have to couple with the background scalars to be localized.**

Recently we investigate the localization of the vector field in a flat thick brane model: [3]

- **Firstly, we obtained the solution of two-field thick M_4 branes. There exist a unique parameter b, which results that the extra dimension is finite.**
- **Vector fields—can be localized on the thick M_4 brane (novelty conclusion).**

[3] Chun-E Fu, Yu-Xiao Liu, Heng Guo, Bulk matter fields on two-field thick branes, arXiv:1101.0336, Physics Review **D 84** (2011) 044036.

Solution of two-field thick branes

We consider the braneworld generated by two interacting scalars:

$$S=\int d^5x\sqrt{-g}\left[\frac{1}{2\kappa_5^2}R-\frac{1}{2}(\partial\phi)^2-\frac{1}{2}(\partial\pi)^2-V(\phi,\pi)\right]. \quad (1)$$

The line-element of a 5-dimensional space-time can be assumed as:

$$ds^2=\mathbf{e}^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu+\mathbf{e}^{2B(y)}dy^2, \quad (2)$$

where $\mathbf{e}^{2A}$ and $\mathbf{e}^{2B}$ are the warp factors.

The equations of motion generated from the action are given by:

$$\frac{1}{2}\phi'^2 + \frac{1}{2}\pi'^2 - \mathbf{e}^{2B}V = 6A'^2, \quad (3)$$

$$\frac{1}{2}\phi'^2 + \frac{1}{2}\pi'^2 + \mathbf{e}^{2B}V = -6A'^2 - 3A'' + 3A'B', \quad (4)$$

$$\phi'' + (4A' - B')\phi' = \mathbf{e}^{2B}\frac{\partial V}{\partial \phi}, \quad (5)$$

$$\pi'' + (4A' - B')\pi' = \mathbf{e}^{2B}\frac{\partial V}{\partial \pi}, \quad (6)$$

where the prime stands for the derivative with respect to y.

In order to solve the equation, we use the superpotential method. Introduce the superpotential function $W(\phi)$, which is defined as $\phi' = \frac{\partial W}{\partial \phi}$, and assume the potential is:

$$V = \mathbf{e}^{-2\sqrt{b/3}\,\pi}\left[\frac{1}{2}(\frac{\partial W}{\partial \phi})^2 - \frac{4-b}{6}W^2\right], \tag{7}$$

we can find that the following first-order differential equations are the solutions of the equations of motion (3-6):

$$A' = -\frac{1}{3}W, \;\; B = b\,A, \;\; \pi = \sqrt{3b}\,A, \tag{8}$$

where b is a positive parameter.

For a specific superpotential $W(\phi)$:

$$W(\phi) = va\phi\left(1 - \frac{\phi^2}{3v^2}\right), \tag{9}$$

the solutions are found to be:

$$\begin{aligned}
\phi(y) &= v\tanh(ay), & (10)\\
A(y) &= -\frac{v^2}{9}\left(\ln\cosh^2(ay) + \frac{1}{2}\tanh^2(ay)\right), & (11)\\
\pi(y) &= \sqrt{3b}\,A(y), & (12)\\
B(y) &= b\,A(y), & (13)
\end{aligned}$$

where v, a are both positive constants.

In order to clarify this question more clearly, we would like to discuss the effect of the parameter b on the brane under the physical coordinate $\bar{y}$. Then the metric is read as:

$$ds^2 = \mathrm{e}^{2A(y(\bar{y}))}\eta_{\mu\nu}dx^\mu dx^\nu + d\bar{y}^2. \tag{14}$$

where we perform a coordinate transformation $d\bar{y} = \mathrm{e}^{b\,A}dy$. From

$$\bar{y} = \int_0^y \mathrm{e}^{bA}d\tilde{y} \to \int_0^y \mathrm{e}^{-\frac{2v^2ab}{9}\tilde{y}}d\tilde{y}, \quad \textbf{for} \quad \tilde{y} \to \infty, \tag{15}$$

it can be seen the extra dimension $\bar{y}$ is finite (with $\bar{y}_{max} = \frac{9}{2v^2ab}$).

Localization of gauge fields on the brane

We will use the conformally flat metric:

$$ds^2 = \mathrm{e}^{2A(z)}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right), \tag{16}$$

which is connected by $dz = \mathrm{e}^{2(b-1)A}dy$ with (2). For the conformally flat space-time, the extra dimension z will be infinite for $0 < b \leq 1$ and finite (with $|z| \leq z_{max} = \frac{9}{2v^2 a(b-1)}$) for $b > 1$.

The action of vector fields coupled with the dilaton is:

$$S_1 = -\frac{1}{4}\int d^5x\sqrt{-g}\,\mathbf{e}^{\tau\pi} g^{MR} g^{NS} F_{MN} F_{RS}, \tag{17}$$

where the field strength tensor is $F_{MN} = \partial_M A_N - \partial_N A_M$. The equations of motion can be obtained using the background geometry (16):

$$\begin{aligned} \frac{1}{\sqrt{-\hat{g}}}\partial_\nu(\sqrt{-\hat{g}}\hat{g}^{\nu\rho}\hat{g}^{\mu\lambda}F_{\rho\lambda}) \\ + \hat{g}^{\mu\lambda}\mathbf{e}^{-(1+\tau\sqrt{3b})A}\partial_4\left(\mathbf{e}^{(1+\tau\sqrt{3b})}F_{4\lambda}\right) &= 0, \qquad (18) \\ \partial_\mu(\sqrt{-\hat{g}}\hat{g}^{\mu\nu}F_{\nu 4}) &= 0. \qquad (19) \end{aligned}$$

With the gauge choice $A_4 = 0$ and the decomposition of the vector field:

$$A_\mu(x,z) = \sum_n a_\mu^{(n)}(x)\rho_n(z)\mathbf{e}^{-(1+\sqrt{3b}\,\tau)A/2}, \tag{20}$$

we find that the KK modes of the vector field satisfy the following Schrödinger-like equation:

$$\left[-\partial_z^2 + V_1(z)\right]\rho_n(z) = m_n^2\rho_n(z), \tag{21}$$

where the effective potential is:

$$V_1(z) = \frac{(1+\sqrt{3b}\,\tau)^2}{4}(\partial_z A)^2 + \frac{1+\sqrt{3b}\,\tau}{2}\partial_z^2 A. \tag{22}$$

Providing the orthonormality condition

$$\int_{-z_b}^{z_b} dz \rho_m \rho_n = \delta_{mn}, \tag{23}$$

we can get the 4-dimensional effective action:

$$S_1 = -\frac{1}{2}\sum_n \int d^4x \sqrt{-\hat{g}} \left(\frac{1}{2}\hat{g}^{\mu\alpha}\hat{g}^{\nu\beta} f_{\mu\nu}^{(n)} f_{\alpha\beta}^{(n)} + m_n^2 \, \hat{g}^{\mu\nu} a_\mu^{(n)} a_\nu^{(n)} \right),$$

where $f_{\mu\nu}^{(n)} = \partial_\mu a_\nu^{(n)} - \partial_\nu a_\mu^{(n)}$ **is the 4-dimensional field strength tensor.**

So with the relation $dz = e^{(b-1)A} dy$, we can get the values of $V_1(z)$ at $z = 0$ and $|z| \to z_b$:

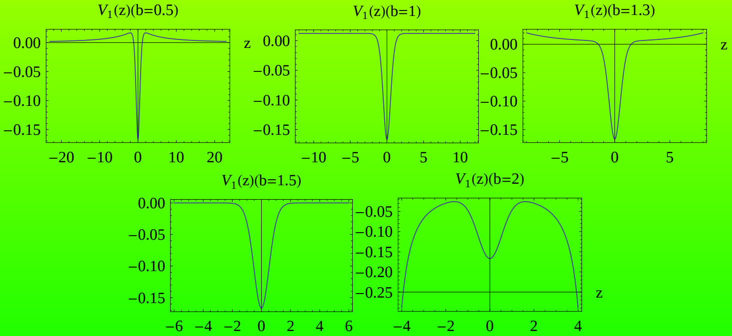
$$V_1(0) = -\frac{1}{6} a^2 v^2 (1 + \sqrt{3b}\,\tau), \tag{24}$$

$$V_1(|z| \to z_b) = \frac{v^4 a^2 (1 + \sqrt{3b}\,\tau)(3 - 2b + \sqrt{3b}\,\tau)}{81[-\frac{2v^2 a}{9}(b-1)z + 1]^2}. \tag{25}$$

In order to get a zero mode, we have to insure that the value of $V_1(z)$ at $z = 0$ is negative:

$$\tau > -1/\sqrt{3b}. \tag{26}$$

The potential $V_1(z)$ is volcano-like and PT-like ones for $0 < b < 1$ and $b = 1$, respectively. And for $b > 1$, the potential will divergent at the boundary of the extra dimension with $\tau \neq \frac{2b-3}{\sqrt{3b}}$, but vanish with $\tau = \frac{2b-3}{\sqrt{3b}}$.



On the condition (26), the zero mode for the vector field can be obtained by setting $m_0 = 0$:

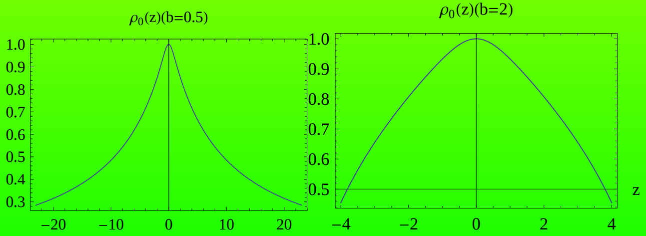
$$\rho_0 \propto \mathbf{e}^{\frac{(1+\sqrt{3b}\ \tau)}{2} A(z)}. \qquad (27)$$

In order to check whether the zero mode for the vector field can be localized on the brane, we can investigate whether it satisfies the orthonormality condition (23).

If there is no coupling between the dilaton and the vector fields, the orthonormality condition (23) becomes:

$$\int_{-z_b}^{z_b} \rho_0^2 dz = \int_{-\bar{y}_{max}}^{\bar{y}_{max}} \rho_0^2(\bar{y}) e^{-A} d\bar{y} \quad \propto \quad \int_{-\bar{y}_{max}}^{\bar{y}_{max}} d\bar{y} \quad < \quad \infty, (28)$$

form which it is clear the zero mode can be localized on the brane.



While for the case that there is a coupling:

$$\int \rho_0^2 dz = \int \rho_0^2 \mathbf{e}^{(b-1)A} dy$$
$$\rightarrow \int \mathbf{e}^{\frac{-2v^2 a}{9}(b+\sqrt{3b}\,\tau)\, y} dy \quad \textbf{for} \quad y \rightarrow \infty. \qquad (29)$$

so the condition of the localization is

$$\tau \geq -\sqrt{b/3} \qquad \textbf{for} \quad 0 < b \leq 1, \qquad (30)$$
$$\tau > -1/\sqrt{3b} \qquad \textbf{for} \quad b > 1. \qquad (31)$$

Conclusion

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- **We obtained two-field thick branes solution. There is a unique parameter b in the solution, which leads to the finity of the extra dimension.**

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- **We obtained two-field thick branes solution. There is a unique parameter b in the solution, which leads to the finity of the extra dimension.**
- **We investigated the localization of vector fields, and found the free gauge fields can be localized on the brane as the extra dimension is finite.**

Thank you very much!

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