

Standard Model Phenomenology

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Neutral Current Phenomenology

A set of new reactions are mediated by neutral Z boson– neutral current reactions. Their discoveries in 1970's lend a strong support to standard model.

Weak interaction of Z boson is of the form,

$$\mathcal{L}_Z = \frac{g}{\cos \theta_W} Z^\mu J_\mu^Z$$

where J_μ^Z is weak neutral current

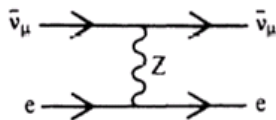
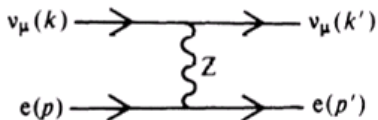
$$J_\mu^Z = \sum_i \bar{\psi}_i \gamma_\mu [T_3(\psi_i) - \sin^2 \theta_W Q(\psi_i)] \psi_i$$

Here ψ_i is either left-or right-handed field and T_3 and Q are their weak isospin and electric charge, e.g. for u_L

$$T_3(u_L) - \sin^2 \theta_W Q(u_L) = \frac{1}{2} - \sin^2 \theta_W \frac{2}{3}$$

We can compute the cross sections for those reactions mediated by Z boson..

- 1 $\nu_\mu + e \rightarrow \nu_\mu + e$ and $\bar{\nu}_\mu + e \rightarrow \bar{\nu}_\mu + e$
The contributing diagrams are .



The differential and total cross section in the lab frame,

$$\frac{d\sigma(\nu_\mu e)}{dy} = \frac{8G_F^2}{\pi} m_e E_\nu (g_L^\nu)^2 \left[(g_L^e)^2 + (g_R^e)^2 (1-y)^2 \right] \quad \text{with } y = \frac{E_e}{E_\nu}$$

and

$$\sigma(\nu_\mu e) = \frac{8G_F^2}{\pi} m_e E_\nu (g_L^\nu)^2 \left[(g_L^e)^2 + \frac{1}{3} (g_R^e)^2 \right]$$

where

$$g_L^\nu = \frac{1}{2}, \quad g_L^e = -\frac{1}{2} + \sin^2 \theta_W, \quad g_R^e = \sin^2 \theta_W$$

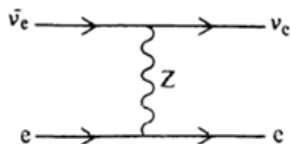
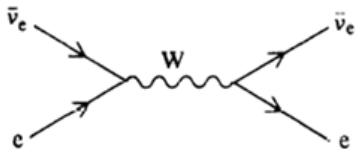
are the weak neutral couplings of ν and e .

Similarly,

$$\sigma(\bar{\nu}_\mu e) = \frac{8G_F^2}{\pi} m_e E_\nu (g_L^\nu)^2 \left[\frac{1}{3} (g_L^e)^2 + (g_R^e)^2 \right]$$

1 $\nu_e + e \rightarrow \nu_e + e$ and $\bar{\nu}_e + e \rightarrow \bar{\nu}_e + e$

The contribution diagrams are



Here, there is also contribution from W-exchange diagram. The resulting cross sections are,

$$\sigma(\nu_e e) = \frac{8G_F^2}{\pi} m_e E_\nu (g_L^\nu)^2 \left[(1 + g_L^e)^2 + \frac{1}{3} (g_R^e)^2 \right]$$

$$\sigma(\bar{\nu}_e e) = \frac{8G_F^2}{\pi} m_e E_\nu (g_L^\nu)^2 \left[\frac{1}{3} (1 + g_L^e)^2 + (g_R^e)^2 \right]$$

Note that there are terms linear in g_L and its sign can be determined. This determines the weak mixing to be

$$\sin^2 \theta_W \approx 0.23$$

There are other neutral current effects in $e^+e^- \rightarrow \mu^+\mu^-$, $\nu p \rightarrow \nu X$, $ep \rightarrow ep$ reactions. These all have been measured and results agree with the theoretical quite well

W and Z gauge bosons

The basic features of standard model is the existence of 3 massive gauge bosons, W^+ , W^- , and Z .

1 Masses

The masses of W and Z

$$M_W = \frac{1}{2} \left(\frac{e^2}{\sqrt{2} G_F} \right)^{\frac{1}{2}} \frac{1}{\sin \theta_W} = \frac{37.3 \text{ GeV}}{\sin \theta_W}$$

$$M_Z = \left(\frac{e^2}{\sqrt{2} G_F} \right)^{\frac{1}{2}} \frac{1}{\sin 2\theta_W} = \frac{74.6 \text{ GeV}}{\sin 2\theta_W}$$

From measurement of θ_W from the neutral current reactions, we can predict

$$M_W \approx 80 \text{ GeV}, \quad M_Z \approx 90 \text{ GeV} \quad \text{for} \quad \sin^2 \theta_W \approx 0.23$$

In the early 1980's, W and Z bosons are found in the experiments in CERN and their masses are found to be

$$M_W \approx 80.4 \text{ GeV}, \quad M_Z \approx 91.2 \text{ GeV}$$

The good agreement of with theoretical predictions lends another strong support for the standard model.

2 W decays

The weak interactions of charged gauge boson W is of the form,

$$\mathcal{L}_W = \frac{g}{2\sqrt{2}} W_\mu^+ [(\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\gamma^\mu(1 - \gamma_5)] \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} + (\bar{u}, \bar{c}, \bar{t})\gamma^\mu(1 - \gamma_5)U \begin{pmatrix} d \\ s \\ b \end{pmatrix} + h.c.$$

The decay rate for $W^+ \rightarrow e^+ + \nu$ is

$$\Gamma_e = \Gamma(W^+ \rightarrow e^+ + \nu) = \frac{G_F}{\sqrt{2}} \frac{M_W^3}{6\pi} \approx 0.25 \text{ GeV}$$

The same decay rate applies to other leptonic modes

$$W^+ \rightarrow \mu^+ \nu_\mu, \quad \tau^+ \nu_\tau$$

The hadronic decay modes can be described in terms of quarks

$$W^+ \rightarrow u\bar{d}, \quad u\bar{s}, \quad u\bar{b}$$

$$W^+ \rightarrow c\bar{d}, \quad c\bar{s}, \quad c\bar{b}$$

Use unitarity relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$, to get

$$\Gamma(W^+ \rightarrow ud, us, ub) = 3\Gamma_e$$

factor 3 comes from the color degree of freedom. Similarly $\Gamma(W^+ \rightarrow cd, cs, ub) = 3\Gamma_e$

$$\Gamma(\text{total}) = 9\Gamma_e$$

The branching ratio for electron mode is

$$B(W^+ \rightarrow e^+ + \nu) \approx \frac{1}{9} = 11.1\%$$

Experimental result is

$$B(W^+ \rightarrow e^+ + \nu) \approx (10.8 \pm 0.09)\%$$

The agreement seems to be quite good. Similarly,

$$B(W^+ \rightarrow \text{hadrons}) \approx \frac{6}{9} = 66.67\%$$

agrees with the measurement of 67.6%.

3 Z decays

One unique feature is that all Z decays conserve flavors. Of particular interest is the $Z \rightarrow \nu\bar{\nu}$. These can be measured as the invisible width of Z boson. For each neutrino species

$$\Gamma(Z \rightarrow \nu_i + \bar{\nu}_i) = \frac{G_F M_Z^3}{12\pi\sqrt{2}} = 0.161 \text{ GeV}$$

From the measured invisible width of Z

$$\Gamma(Z \rightarrow \text{invisible}) = 0.499 \pm 0.0015 \text{ GeV}$$

the number of light neutrinos is limited to 3.

Higgs particle

The most important ingredient of the Standard Model is the scalar particles. The left-over scalar, called the Higgs particle. The LHC machine in CERN is built to look for such a particle. We now summarize important properties of Higgs particle.

① $M_W = M_Z \cos \theta_W$

This follows from the doublet nature of the scalar fields and is well satisfied experimentally,

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1.003 \pm .004$$

For a general $SU(2) \times U(1)$ multiplets of Higgs particle $\phi_{T,Y}$ with weak isospin T and hypercharge Y , its contribution to ρ is

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = \frac{\sum_{T,Y} |v_{T,Y}|^2 [T(T+1) - Y^2]}{2 \sum_{T,Y} |v_{T,Y}|^2 Y^2/4}$$

Here $v_{T,Y} = \langle 0 | \phi_{T,Y} | 0 \rangle$. Thus the constraint on Higgs in representation other than doublet is very severe.

2 Higgs couplings to fermions

The coupling of Higgs to fermions is

$$\mathcal{L}_Y = f_{ij} \bar{\psi}'_{iL} \phi \psi'_{jR} + h.c.$$

Spontaneous symmetry breaking is generated by

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad \text{with } v \simeq 250 \text{ GeV}$$

and Yukawa coupling becomes

$$\mathcal{L}_Y = m_{ij} \bar{\psi}'_{iL} \psi'_{jR} + \frac{f_{ij}}{\sqrt{2}} h(x) \bar{\psi}'_{iL} \psi'_{jR} + h.c. \quad \text{with } m_{ij} = \frac{v}{\sqrt{2}} f_{ij}$$

Diagonalize the mass matrices by bi-unitary transformations,

$$U m V^\dagger = m_D, \quad \text{where } m_D \text{ is diagonal}$$

Explicitly, the mass term becomes

$$\mathcal{L}_m = \bar{\psi}'_{iL} m_{ij} \psi'_{jR} = \bar{\psi}'_L U^\dagger m_D V \psi'_R = \bar{\psi}_L m_D \psi_R = m_i \bar{\psi}_{Li} \psi_{Ri}$$

Here

$$\psi_L = U \psi'_L, \quad \psi_R = V \psi'_R$$

are mass eigenstates. Since $m_{ij} \propto f_{ij}$, in the basis where mass matrix is diagonal, the Yukawa coupling is also diagonal, i.e.

$$\mathcal{L}_Y = m_i \bar{\psi}_{iL} \psi_{Ri} + \frac{m_i}{v} \eta(x) \bar{\psi}_{iL} \psi_{Ri} + h.c.$$

Higgs couplings is proportional to the mass of that fermion and coupling conserve flavors and parity. This means that Higgs will decay into the heaviest particles allowed by the kinematics.

Coupling to gauge bosons

$$\mathcal{L}_{\phi VV} = g \eta(x) [M_W W_\mu^+ W^{-\mu} + \frac{1}{2 \cos \theta_W} M_Z Z^\mu Z_\mu]$$

also are proportional to masses.

3 Mass of Higgs particle

$$m_\eta = \sqrt{2\mu^2} = \sqrt{2\lambda}v \quad v \approx 250 \text{ GeV}$$

There is no information on Higgs self coupling λ . Thus m_η is not constrained and the search is going to be difficult.

Neutrino oscillations

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

Suppose ν 's are all massive, then they are linear combinations of mass eigenstates ν_1, ν_2, ν_3

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$
$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$c_{ij} = \cos \theta_{ij}, \quad s_{ij} = \sin \theta_{ij}$$

If at time $t = 0$, ν_e is produced somehow, we can write

$$|\nu_e(0)\rangle = U_{e1} |\nu_1\rangle + U_{e2} |\nu_2\rangle + U_{e3} |\nu_3\rangle$$

The time evolution is controlled by the energy eigenvalues,

$$|\nu_e(t)\rangle = U_{e1} e^{-iE_1 t} |\nu_1\rangle + U_{e2} e^{-iE_2 t} |\nu_2\rangle + U_{e3} e^{-iE_3 t} |\nu_3\rangle$$

where

$$E_i^2 = p^2 + m_i^2$$

Assume $p \gg m_i$ we get $E_i \approx p + \frac{m_i^2}{2p}$ and

$$E_i - E_j = \frac{(m_i^2 - m_j^2)}{2p}$$

Since the linear combination is time-dependent, the content of this beam will oscillate between ν_e, ν_μ , and ν_τ . Define the oscillation length l_{ij} as

$$l_{ij} = \frac{2\pi}{E_i - E_j} \approx \frac{4\pi p}{|m_i^2 - m_j^2|} = 2.5m \left[\frac{\rho(\text{MeV})}{\Delta m^2(\text{eV})^2} \right]$$

This sets the scale for the oscillation.

Consider the case of 2 neutrino species. At $t = 0$, write

$$|\nu_e(0)\rangle = \cos\theta |\nu_1\rangle + \sin\theta |\nu_2\rangle$$

$$|\nu_\mu(0)\rangle = -\sin\theta |\nu_1\rangle + \cos\theta |\nu_2\rangle$$

Consider the case of with $|\nu_e(0)\rangle$. The time evolution is,

$$|\nu_e(t)\rangle = \cos\theta e^{-iE_1 t} |\nu_1\rangle + \sin\theta e^{-iE_2 t} |\nu_2\rangle = e^{-iE_1 t} (\cos\theta |\nu_1\rangle + \sin\theta e^{-i\frac{2\pi}{l_{12}} t} |\nu_2\rangle)$$

The probability amplitude for finding the state ν_e at time t is,

$$\langle \nu_e | \nu_e(t) \rangle = e^{-iE_1 t} (\cos^2 \theta + \sin^2 \theta e^{-i \frac{2\pi x}{l_{12}}})$$

and

$$P(\nu_e \rightarrow \nu_e) = |\langle \nu_e | \nu_e(t) \rangle|^2 = 1 - 2 \sin^2 \theta \cos^2 \theta (1 - \cos(\frac{2\pi x}{l_{12}}))$$

and probability to find other species, i.e. ν_μ is

$$P(\nu_e \rightarrow \nu_\mu) = 1 - P(\nu_e \rightarrow \nu_e) = 2 \sin^2 \theta \cos^2 \theta (1 - \cos(\frac{2\pi x}{l_{12}}))$$

Observation of neutrino oscillations in many different reactions establishes the existence of neutrino masses,

$$\Delta m_{21}^2 \simeq 7.58 \times 10^{-5} \text{eV}^2, \quad \Delta m_{31}^2 \simeq 2.35 \times 10^{-3} \text{eV}^2$$

$$.25 \leq \sin^2 \theta_{12} \leq 0.37, \quad .36 \leq \sin^2 \theta_{23} \leq 0.67, \quad \sin^2 2\theta_{13} = 0.092$$

Higgs Physics

top priority at LHC is to look for Higgs particle.

- 1 Higgs coupling to fermion is proportional to fermion mass
- 2 Higgs coupling to gauge boson is also proportional to gauge boson mass,

$$L_{HVV} = gH(x) \left[M_W W_\mu^+ W^\mu + \frac{1}{2 \cos \theta_W} M_Z Z^\mu Z_\mu \right]$$

Mass of Higgs particle can be written as

$$m_H = \sqrt{2\mu^2} = \sqrt{2\lambda}v,$$

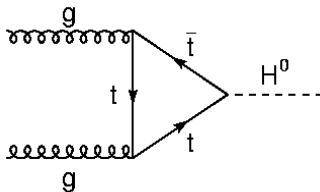
where $v = 246$ Gev is related to Fermi coupling constant G_F by

$$v = \sqrt{\frac{\sqrt{2}}{G_F}}$$

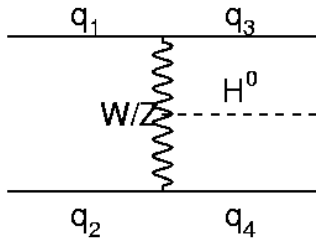
Production of Higgs Particle

Higgs coupling is strongest for t quarks. There are 4 mechanisms for Higgs productions:

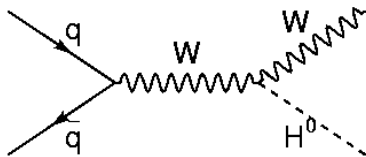
- 1 Gluon fusion through t -quark loop;



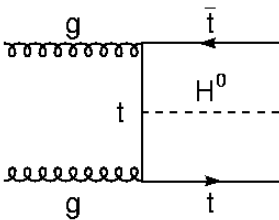
- 2 Vector meson fusion; $qq \rightarrow qqH$



- 3 Associated production of H with a gauge boson, $qq \rightarrow HW/Z$



- 4 Associated Higgs production with heavy quarks $gg \rightarrow ttH$



production cross sections for these 4 processes are shown below,

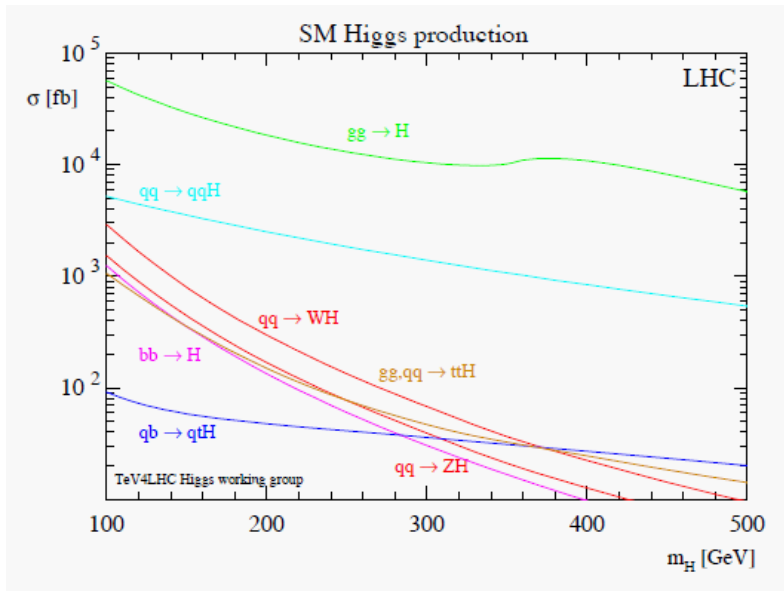


Fig: Higgs Production Cross Section for PP collision at 14 Tev

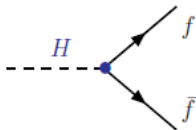
gluon fusion is dominant, vector bosons fusion comes next.

Higgs Decay

Higgs will decay into heaviest particles allowed by kinematics.

1 Decays into quarks and leptons

To lowest order, the diagram is just



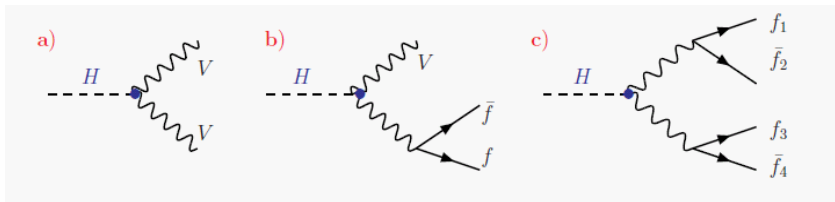
Use the Higgs coupling given in Eq(??) we can compute the decay width to give

$$\Gamma(H \rightarrow f\bar{f}) = \frac{G_F N_c}{4\sqrt{2}} M_H m_f^2 \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

where $N_c = 3$ for quarks and $N_c = 1$ for leptons.

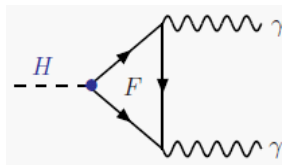
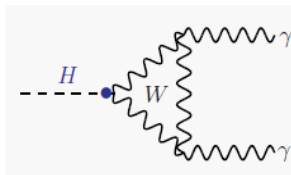
2 Decays into electroweak gauge bosons

The diagrams for decays into gauge boson pair and subsequent decay of gauge boson into leptons are given by

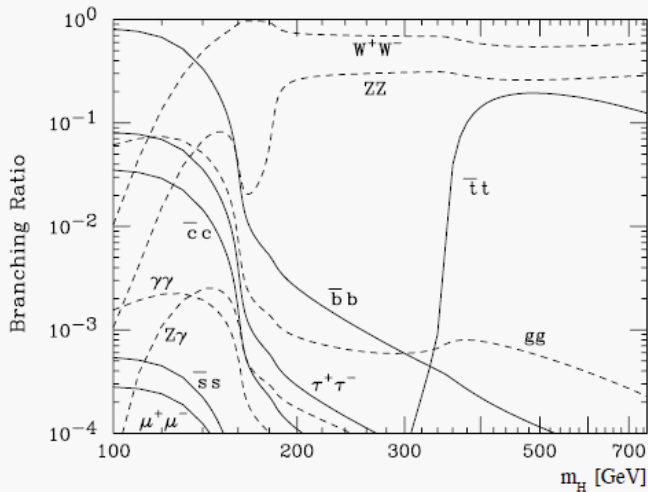


3 Decays into $\gamma\gamma$

This decay induced by the gauge boson or heavy fermion loop as shown in the graphs below can be important due to the large Higgs couplings to the heavy particles.



relative importance of each decay mode,



- decays of Higgs into WW or ZZ dominate .
- below the WW threshold $H \rightarrow b\bar{b}$ dominates.

- decay $H \rightarrow \gamma\gamma$ is of special interest due to their relatively clean experimental signature.

Outlook

- Beyond Standard Model
Dark Matter, Dark Energy
- Supersymmetry
- Grand Unification
- Quantum Gravity—superstring?